MANUAL ON
AIRSPACE PLANNING METHODOLOGY
FOR THE DETERMINATION OF
SEPARATION MINIMA

FIRST EDITION — 1998

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AMENDMENTS

The issue of amendments is announced regularly in the *ICAO Journal* and in the monthly *Supplement to the Catalogue of ICAO Publications and Audio-visual Training Aids*, which holders of this publication should consult. The space below is provided to keep a record of such amendments.

RECORD OF AMENDMENTS AND CORRIGENDA

AMENDMENTS			CORRIGENDA		
No.	Date	Entered by	No.	Date	Entered by
1	30/8/02	ICAO			

Foreword

Purpose of the manual

In September 1991, the Tenth Air Navigation Conference considered and endorsed a concept for a future air navigation system that would meet the needs of international civil aviation over the next century. The concept, which was developed by the ICAO Future Air Navigation Systems (FANS) Committee, came to be known as the communications, navigation, and surveillance/air traffic management (CNS/ATM) systems concept and involves a complex and interrelated set of technologies, dependent, to a large degree, on satellites.

In follow-up of the work of the FANS Committee and the Tenth Air Navigation Conference, several activities have taken or are taking place within and through ICAO as follows:

— the Council of ICAO has acted on the recomendations of the Tenth Air Navigation Conference to speed up the implementation of CNS/ATM;

— the global coordinated plan for transition to the CNS/ATM systems has been developed;

— the Air Navigation Commission is coordinating technical activities leading to the development of international Standards and Recommended Practices (SARPs);

— several panels of the Air Navigation Commission are developing the operational requirements and technical specifications necessary for implementation of CNS/ATM systems;

— institutional issues are being addressed by the Council of ICAO, the Legal Bureau and concerned States; and

— regional planning groups are working on strategies and analyses for their regions.

The CNS/ATM systems concept has reached a high level of understanding and acceptance, and efforts are now being directed at the implementation of a seamless, global air traffic management system. In light of this, the global plan mentioned above is being revised in a way that will present information of a practical nature to guide and assist States, the ICAO Regional Offices, regional planning groups, the avionics industry, and operators in planning for the carriage of airborne equipment required for use with CNS/ATM systems.

The primary objective of this manual is to guide airspace planners, ICAO Regional Offices and the regional planning groups and to assist them with implementation of CNS/ATM systems, particularly in relation to airspace planning, implementation of the required navigation performance (RNP) concept and area navigation techniques. This is in line with the objectives laid out by the Council of ICAO. It is envisioned that the airspace planning methodology will become a part of a larger document dealing with implementation issues.

The methodology presented in this document provides a framework by which airspace characteristics, aircraft capability and traffic demand can be assessed for the purpose of determining safe separation minima for en-route operations. The methodology has been designed to ensure that the intended safety level for a proposed airspace meets the required standard. Airspace planners will be able to assess different scenarios for airspace development. Administrations may use the methodology as a tool to assist them in determining the sequence and nature of decisions required to establish safe separation minima. However, it is recognized that, in some cases, application of the methodology may require risk analysis expertise which may not be available in all administrations. In these cases, further technical advice and support should be obtained from ICAO.

Relationship to other ICAO documents

Existing ICAO documents do not indicate methods for quantifying the effect a change of separation minima may have on air traffic safety. This document is intended for use by airspace planners as a basis for changing separation minima. It should be read in conjunction with the

International Standards and Recommended Practices, Air Traffic Services (Annex 11 to the Convention on International Civil Aviation), Attachment A — Material relating to a method of establishing ATS routes defined by VOR, paragraph 3.1, and Attachment B — Method of establishing ATS routes for use by RNAV-equipped aircraft, and the *Procedures for Air Navigation Services — Air Traffic Management* (PANS-ATM, Doc 4444), Chapter 5, Separation Methods and Minima. The *Air Traffic Services Planning Manual* (Doc 9426) provides guidance on how States should determine required levels of air traffic services for their airspace.

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Glossary

AACS	Australian Airspace Classification Scheme
AAIM	aircraft autonomous integrity monitoring
ACARS	ARINC communications addressing and reporting system
ACAS	airborne collision avoidance system
ADS	automatic dependent surveillance
AFTN	aeronautical fixed telecommunication network
AGL	above ground level
AIP	aeronautical information publication
AIRAC	aeronautical information regulation and control
ALARP	as low as reasonably practical
ANT	airspace and navigation team
APANPIRG	Asia/Pacific Air Navigation Planning and Implementation Regional Group
ARINC	Aeronautical Radio, Inc.
ARM	airspace risk model
ARSR	air route surveillance radar
ATC	air traffic control
ATCO	air traffic controller
ATFM	air traffic flow management
ATM	air traffic management
ATS	air traffic services
BASI	Australian Bureau of Air Safety Investigation
CAA	Civil Aviation Authority
CAP	close approach probability
CASA	Civil Aviation Safety Authority
CBA	cost-benefit analysis
CMA	Central Monitoring Agency
CMF	common mode failures
CNS	communications, navigation and surveillance
CPA	circular protected area
CPDLC	controller/pilot data link communications
CTA	control area
CTAF	common traffic advisory frequency
DDE	double double exponential
DME	distance measuring equipment
EATCHIP	European Air Traffic Control Harmonization and Integration Programme
ECAC	European Civil Aviation Conference

EURO-CONTROL	European Organization for the Safety of Air Navigation
FAA	Federal Aviation Administration
FANS	future air navigation systems
FCS	flight control system
FDPS	flight data processing system
FIR	flight information region
FL	flight level
FMCS	flight management computer system
FMS	flight management system
FOM	figure of merit
GNE	gross navigation error
GNSS	global navigation satellite system
GPS	global positioning system
HAZOP	formal hazard identification and analysis sessions
HF	high frequency
ICAO	International Civil Aviation Organization
IEC	International Electrotechnical Commission
IFR	instrument flight rules
IMC	instrument meteorological conditions
IOACG	Indian Ocean Air Traffic Services Co-ordinating Group
IRR	internal rate of return
ISPACG	Informal South Pacific Air Traffic Services Co-ordinating Group
LORAN	long-range air navigation (system)
LSSR	long-range secondary surveillance radar
MAPT	missed approach point
MASPS	minimum aircraft system performance standards
MBZ	mandatory broadcast zone
MNPS	minimum navigation performance specifications
MOPS	minimum operational performance specification
MWG	Mathematicians Working Group
NAT	North Atlantic
NAT SPG	North Atlantic Systems Planning Group
NM	nautical miles
NOPAC	North Pacific
NOTAM	notice to airmen

NPV	net present value		RVSM	reduced vertical separation minimum
NSW	New South Wales		SARPs	Standards and Recommended Practices
NWPIE	non-waypoint-insertion gross error		SATCOM	satellite communication
OACC	oceanic area control centre		SID	standard instrument departure
OH&S	occupational health and safety		SRD	standard radar departure
OR	operational requirement		SSR	secondary surveillance radar
OTS	organized track system		STAR	standard instrument arrival route
PANS-ATM	Procedures for Air Navigation Services — Air Traffic Management		SUPPS	Supplementary Procedures
			TCAS	traffic alert and collision avoidance system
R&D	research and development		TLS	target level of safety
RAIM	receiver autonomous integrity monitoring		TMA	terminal control area
RDPS	radar data processing system		UTC	coordinated universal time
RGCSP	Review of the General Concept of Separation Panel		VFR	visual flight rules
			VHF	very high frequency
RLRS	reduced lateral route spacing		VMC	visual meteorological conditions
RNAV	area navigation		VOR	VHF omnidirectional radio range
RNDSG	route network development sub-group		WG	working group
RNP	required navigation performance		WP	working paper
RPT	regular public transport		WPIE	waypoint-insertion gross error
RTCA	Radio Technical Commission for Aeronautics			

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Chapter 1

FACTORS AFFECTING THE DEVELOPMENT OF AN AIRSPACE PLANNING METHODOLOGY

SEPARATION CONSIDERATIONS

1.1   Separation is the generic term used to describe action on the part of air traffic services (ATS) to keep aircraft operating in the same general area at such distances from each other that the risk of collision is maintained below an acceptable safe level. Such separation can be applied horizontally and vertically. Separation in the horizontal plane can be achieved either longitudinally (by spacing aircraft behind each other at a specified distance, which may be expressed in flying time) or laterally (by spacing aircraft side by side at a specified distance from each other, or by specifying the width of the protected airspace on either side of an air route centre line). Vertical separation is achieved by requiring aircraft using prescribed altimeter setting procedures to operate at different levels expressed in terms of flight levels or altitudes.

Note.— Guidance material on reduced vertical separation minima (RVSM) is available in (Doc 9574) Manual on Implementation of a 300 m (1 000 ft) Vertical Separation Minimum Between FL 290 and FL 410 Inclusive.

1.2   The required separation between aircraft is generally expressed in terms of minimum distances in each dimension which should not be simultaneously infringed. In the case of horizontal separation, the minimum distance can be expressed in either nautical miles (NM), degrees of angular displacement or, in the longitudinal dimension, as values of either time-based or distance-based minima, by use of distance measuring equipment (DME), area navigation (RNAV), radar or automatic dependent surveillance (ADS) respectively (Doc 4444, *Procedures for Air Navigation Services — Air Traffic Management,* PANS-ATM). Vertically, the minimum is expressed in either metres, feet or flight levels.

1.3   Under some circumstances, in specified airspaces and subject to regional agreement, composite separation consisting of an element of horizontal separation combined with an element of vertical separation may be applied between aircraft (Doc 9426, *Air Traffic Services Planning Manual*, Part II, Section 2, Chapter 3 refers).

1.4   When planning airspace and air routes that are not provided with an air traffic control (ATC) service (only flight information service or air traffic advisory service), safe separation of aircraft can also be assured by the use of standard separation minima. In the event that an ATC service is subsequently introduced, the use of the same process will facilitate implementation and integration with adjacent airspace systems.

FORMS OF AIR TRAFFIC CONTROL SERVICE

1.5   To provide separation, ATC uses two forms of control: procedural and radar. Procedural control is generally understood to be the application of separation based solely on position information received from the aircraft via air-ground communications. It is envisaged that technologies utilizing ADS, where airborne navigational data are made available to ATC by data link techniques, will provide enhancements to procedural control. The introduction of ADS into the procedural ATC environment offers the potential for more frequent position updates as well as information on the future intent of the aircraft. In an environment where position reports are communicated directly from the aircraft to ATC, and where ATC is automatically kept up to date on the intentions of the aircraft, significant reductions in separation minima should be possible.

1.6   Radar control is based on radar-displayed position information. Horizontal separation is achieved by maintaining a specified horizontal distance between radar returns from different aircraft. Vertical separation may also be

applied between radar returns. This may be enhanced in areas where secondary surveillance radar (SSR) is used (it should be noted that the information on height provided in a radar environment using SSR and Mode C is a form of dependent surveillance whereby the aircraft's height is derived from the altimetry systems on individual aircraft).

EFFECT OF FORMS OF CONTROL ON SEPARATION MINIMA

1.7   There is a significant difference between the separation minima used when applying strictly procedural control methods and those used under radar control. The separation minima used under procedural control takes into account that ATC decisions are based on a "snap-shot" picture of the situation and the controller ensures that all aircraft under control are suitably separated from each other. Pilots' estimates of their flight progress must indicate that the separation established will continue until such time as ATC is in a position to again review the traffic situation. The separation minima used in this case must therefore ensure that, even in the worst case conditions (i.e. between successive snap-shots), the required minima can be maintained, or re-established should they become degraded. It should be understood, however, that the use of the procedural control method does not relieve controllers from their obligations to monitor the traffic situation continuously.

1.8   In the case of radar control, ATC is provided with frequently updated real-time information on the position of aircraft, making it possible when required to use significantly smaller separation minima. However, the minima used under these conditions must also take into account the fact that little information is provided from radar data alone on the future intent of aircraft. Further information on the determination of appropriate radar separation is provided in Annex 11 and the PANS-ATM.

1.9   *Effect of tactical radar control.* In a radar environment, when appropriate lateral spacing exists between adjacent routes, such routes may be operated by the controller as separate entities. In this case, when an aircraft is cleared and established on an ATS route:

a)   the pilot is responsible for adhering to the centre line;

b)   aircraft established on adjacent routes are separated by the appropriate spacing between the routes; and

c)   the controller's role is primarily one of monitoring the progress of cleared aircraft.

1.10   In a radar environment, when appropriate lateral spacing does not exist between routes, aircraft may be separated from those on adjacent routes, by the controller applying the minimum radar separation, specified by the ATS authority. In such cases, the use of automated warning tools, such as deviation alert and short-term conflict alert (STCA), may allow the controller to operate the routes with a degree of independence. Thereby, the controller's primary role may also become that of monitoring the progress of cleared aircraft on each route, thus allowing for more active control when required, as in the case of climbing and descending traffic. The time required, therefore, to detect and resolve a deviation and/or potential conflict will depend on a number of factors which include:

a)   controller workload;

b)   availability of automated warning tools, e.g. deviation alert, STCA;

c)   pilot/controller reaction time to initiate and execute corrective action;

d)   delays in pilot/controller communications;

e)   the resolution and accuracy of the system; and

f)   aircraft manoeuvre response time (dependent on aircraft speed and height).

1.11   The introduction of ADS into the procedural ATC environment offers the potential for more frequent position updates as well as information on the future intent of the aircraft. In an ADS environment where position reports are communicated directly from the aircraft to ATC, and where ATC is automatically kept up to date on the intentions of the aircraft, significant reductions in separation minima should be possible. The extent of separation reductions need to be determined by either collision risk modelling or the other techniques detailed in the methodology in this manual.

RESPONSIBILITY FOR NAVIGATION

1.12   The ATC system is based on the principle that the responsibility for navigation is vested with the pilot. The

ATC system does not normally assume responsibility for the navigation of aircraft except in certain prescribed instances (e.g. radar vectoring) when the air traffic controller is in a better position to obtain information on an aircraft's position relative to other aircraft, or when the air traffic controller determines the need to resolve a potential hazard.

DETERMINATION OF SEPARATION

1.13   The determination of vertical separation or time- and distance-based longitudinal separation minima should be based on the quality of information available to ATC and the pilot. Decision, coordination and transmission times may have an influence on the application of longitudinal separation minima, particularly where direct pilot-controller communications are not available. The determination of lateral separation in a procedural environment should be based primarily on the accuracy with which pilots can adhere to an assigned track. When an ATC intervention capability is available, its influence on lateral separation minima should be assessed.

MAINTENANCE OF SEPARATION MINIMA

1.14   Determination of the appropriate prescribed separation minima is a complex process and it is necessary to take into account the factors listed in 1.16, 1.17 and 1.18. Once the responsible authority establishes separation minima, it is incumbent upon ATC to ensure that these are not compromised. In addition, when evaluating airspace safety and efficiency, it is not only the minima that are important, but also how frequently separations close to the minima are applied in practice.

GLOBAL HARMONIZATION OF SEPARATION MINIMA

1.15   From the early days of ICAO, it was agreed that to facilitate global harmonization, separation minima should be established internationally and that such minima should only be changed through international agreement. Annex 11 specifies that the minima established by ICAO are published in the PANS-ATM and minima established by Regional Agreement are published in Doc 7030, *Regional Supplementary Procedures* (SUPPS). This material forms the initial source of reference material from which airspace planners may directly derive appropriate minima.

REQUIRED ELEMENTS OF AN AIRSPACE PLANNING METHODOLOGY

1.16   The primary aim of airspace system design is to provide safe aircraft operations for the intended phases of flight. This includes navigation along the intended flight path, obstacle avoidance and support of separation standards that accommodate required system capacity and safety.

1.17   Three of the main interdependent parameters that affect the achievement of such a predetermined level of airspace system safety (target level of safety — TLS) for a given traffic density are:

a)   aircraft navigation performance;

b)   ground and airborne communications performance; and

c)   surveillance performance.

1.18   These performance capabilities are used to determine airspace design (separation minima/route spacing/ sectorization), instrument procedures and air traffic control intervention capability. An increase or decrease in any single parameter may result in a corresponding increase or decrease in some or all of the other parameters. As aircraft and system capabilities improve, it is expected that corresponding improvements in system safety will be realized. The methodology for determination of en-route separation minima allows a trade-off between the system aspects of separation, navigation and intervention to ensure that an agreed TLS is satisfied.

1.19   In recent years, most of the work on separation minima between aircraft has been based on mathematical/ statistical analyses. Such work has been extremely useful in assessing the probable safety of proposed separation minima and is intended to support informed decisions based on sound operational judgement. This document includes possible methodologies to assess traffic safety in relation to separation minima. The methodology for determining separation minima is based on special mathematical models, which determine the correlation between elements such as collision risk, separation minima, airspace design, air route network characteristics, flow parameters, intervention capability and communication, navigation and surveillance

equipment performance. The airspace planning methodology is sufficiently universal to be used not only for determining separation minima, but also for safely implementing ATS upgrades in situations where separation minima are intended to remain unchanged, for example: determination of communications, navigation, and surveillance (CNS) requirements for a given TLS and separation minimum; estimation of the influence of airspace structure changes on system safety; and determination of air traffic system capacity limits.

1.20   In this document the methodology described for determining safe separation minima is an iterative method. The flow diagram in Figure 1-1 shows the relationship between the following fundamental elements of the methodology:

    a)   identification of the need for change;

    b)   determination of the proposed system;

    c)   identification of the method of safety assessment;

    d)   evaluation of the risk;

    e)   satisfaction of safety criteria;

    f)   modification of the proposed system; and

    g)   implementation and monitoring of the proposed system.

1.21   Airspace planners may use the flow diagram as a tool to assist them in determining the sequence and nature of the decisions required to derive safe separation minima or for safely implementing ATS upgrades in their airspace, described in 1.19 above. Some practical applications of how the airspace planning methodology can be used to derive air traffic system solutions are shown in Figures 1-2a to 1-2d. Detailed guidance on each of the elements of the methodology is given in the remaining chapters.
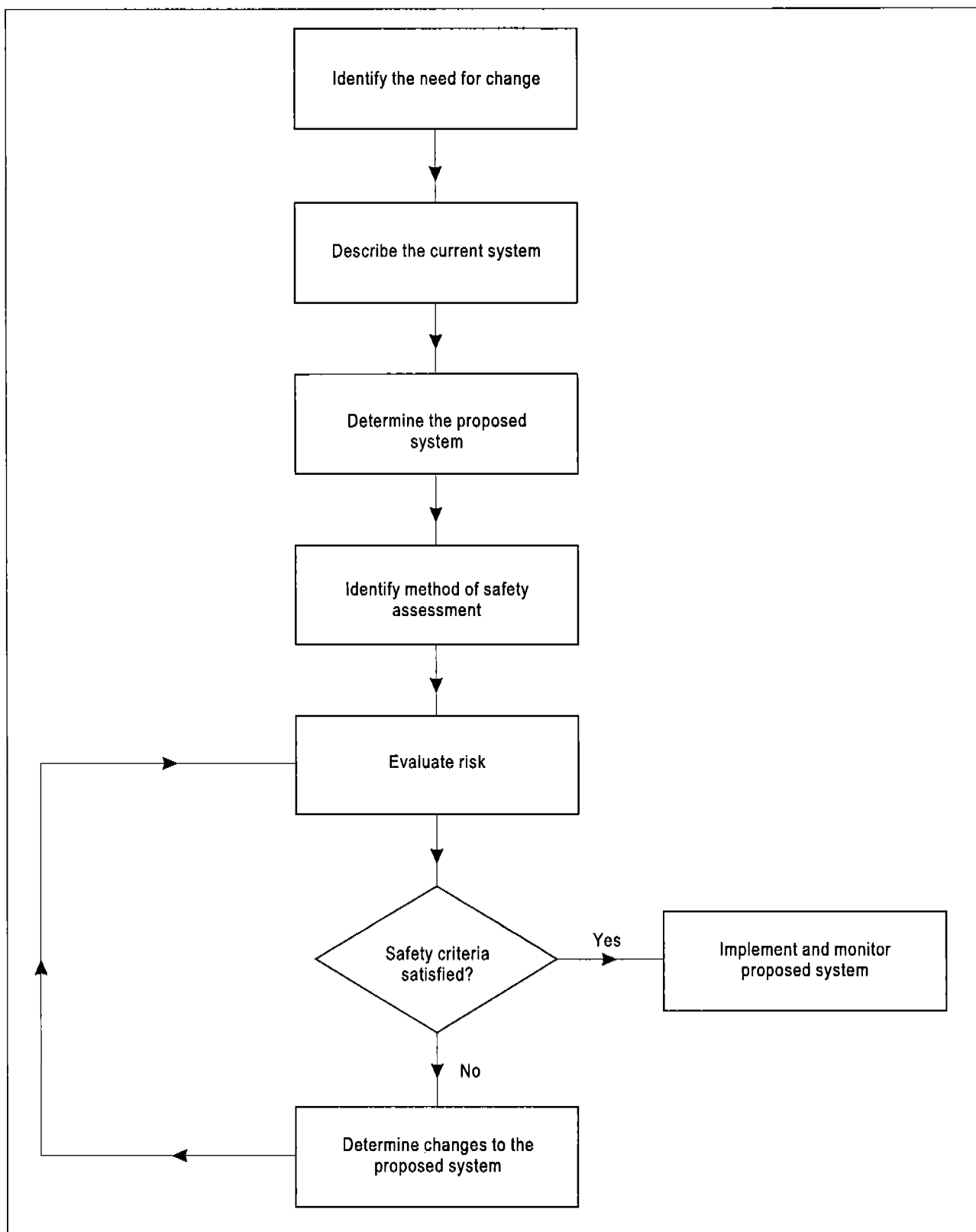
Figure 1-1. Flow diagram describing the process of determining acceptable separation minima
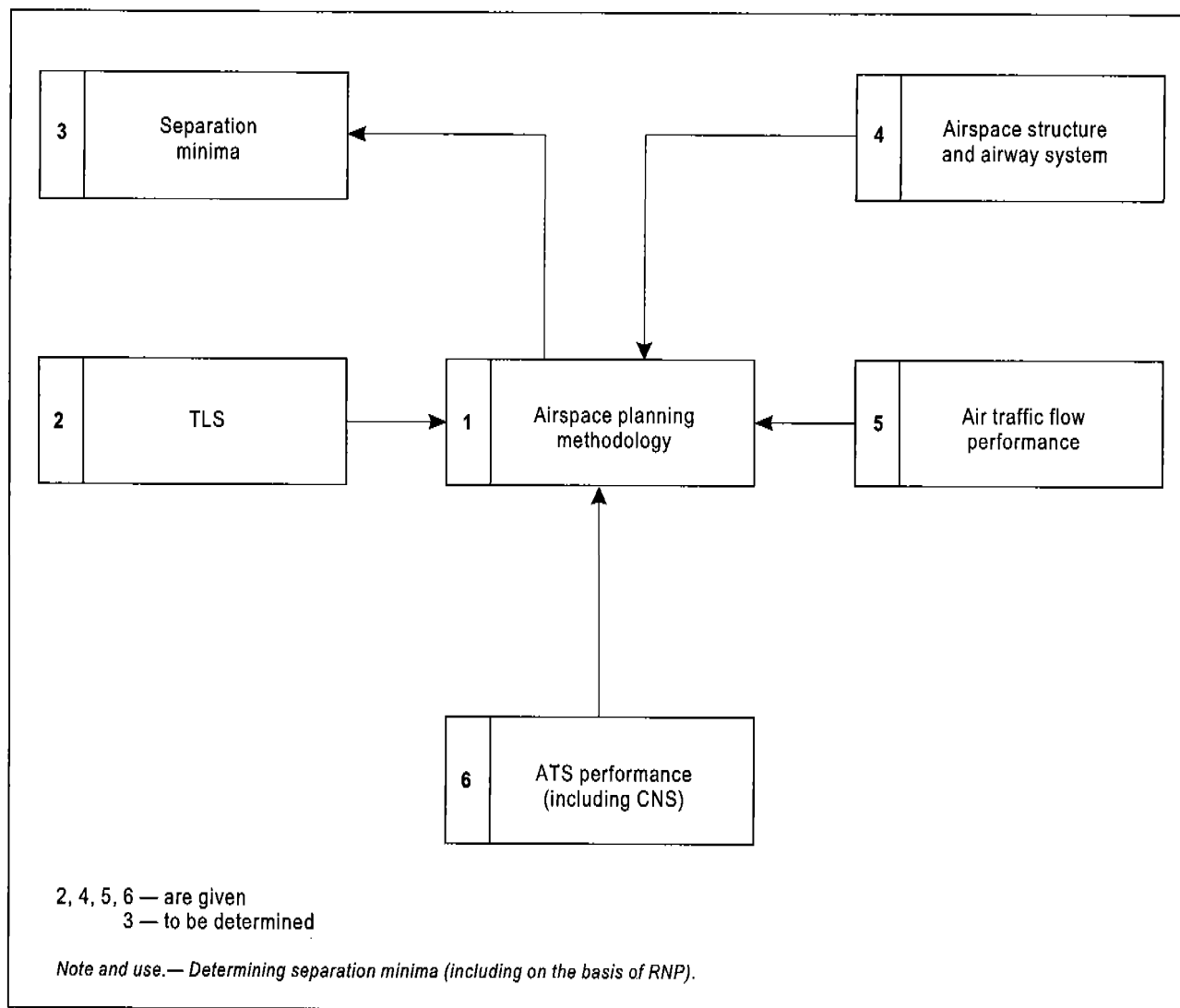
Figure 1-2a. Practical applications of the airspace planning methodology *(see Figure 1-3 for key)*
Example 1 — Determining separation minima
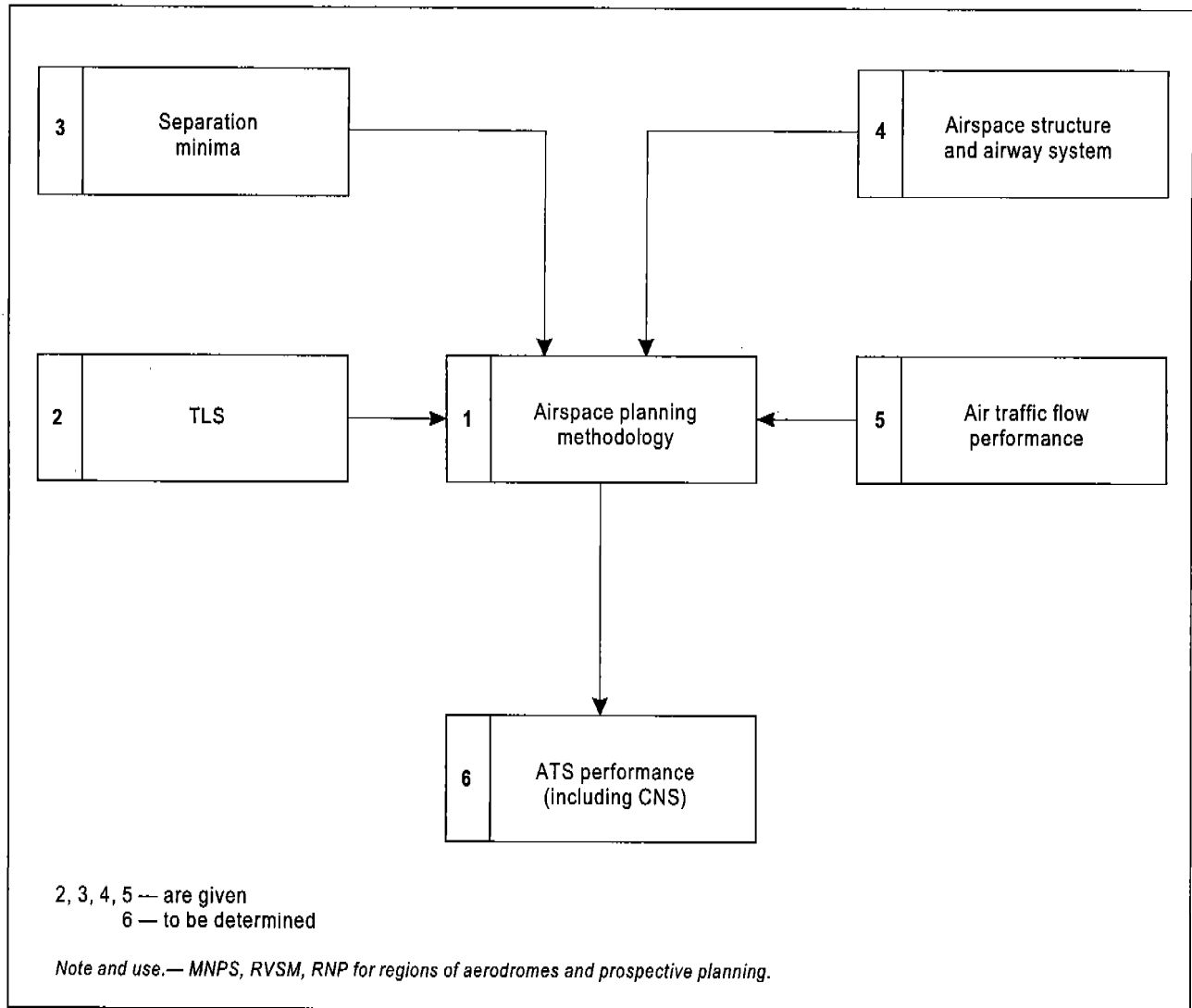
Figure 1-2b. Practical applications of the airspace planning methodology *(see Figure 1-3 for key)*
Example 2 — Determining ATS requirements on the basis of a given separation minima
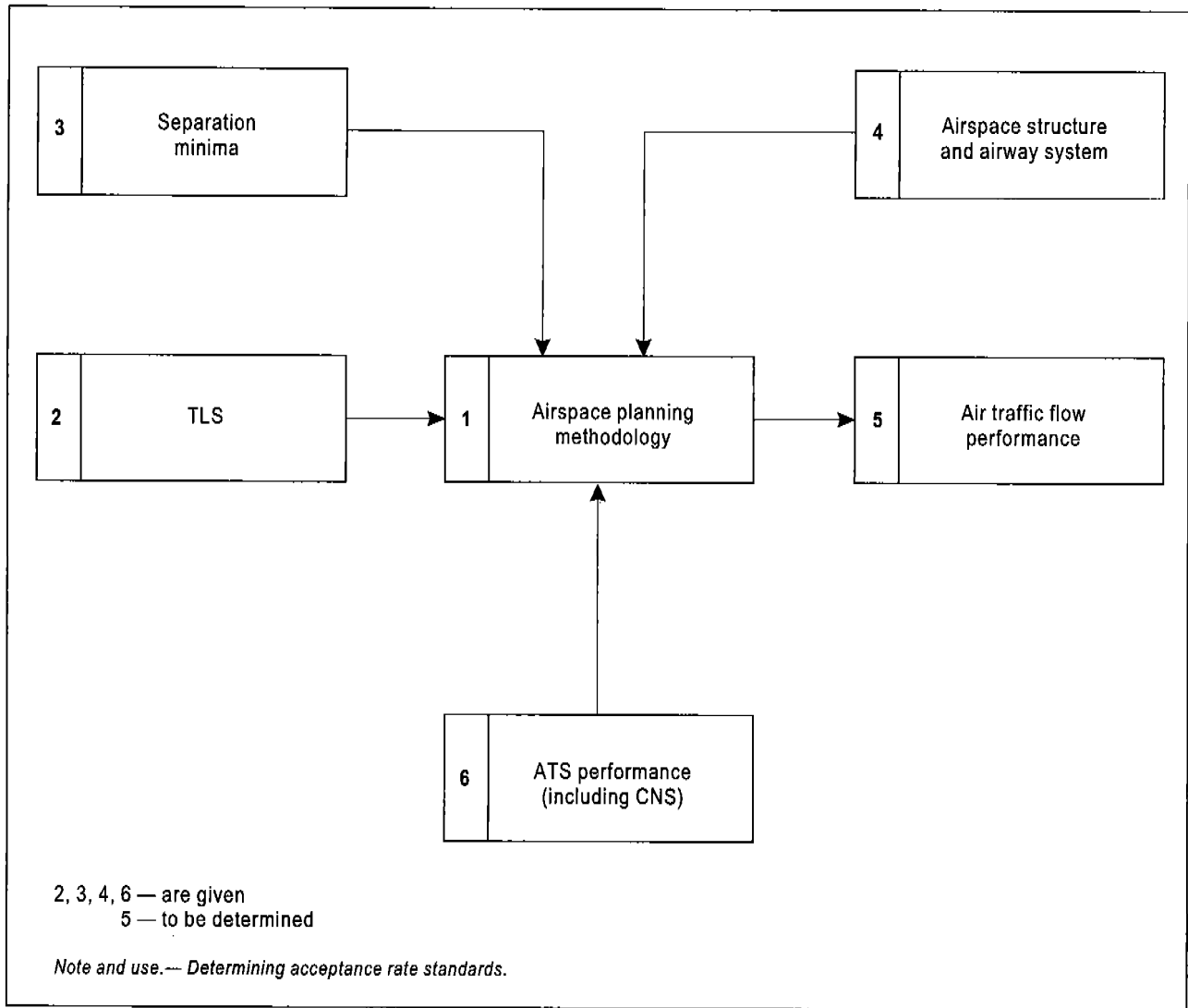
Figure 1-2c. Practical applications of the airspace planning methodology *(see Figure 1-3 for key)*
Example 3 — Determining ATS acceptance rate limits

3	Separation minima
2	TLS
1	Airspace planning methodology
4	Airspace structure and airway system
5	Air traffic flow performance
6	ATS performance (including CNS)

2, 3, 5, 6 — are given
        4 — to be determined

Note and use.— Increase of air safety and airspace acceptance rate.
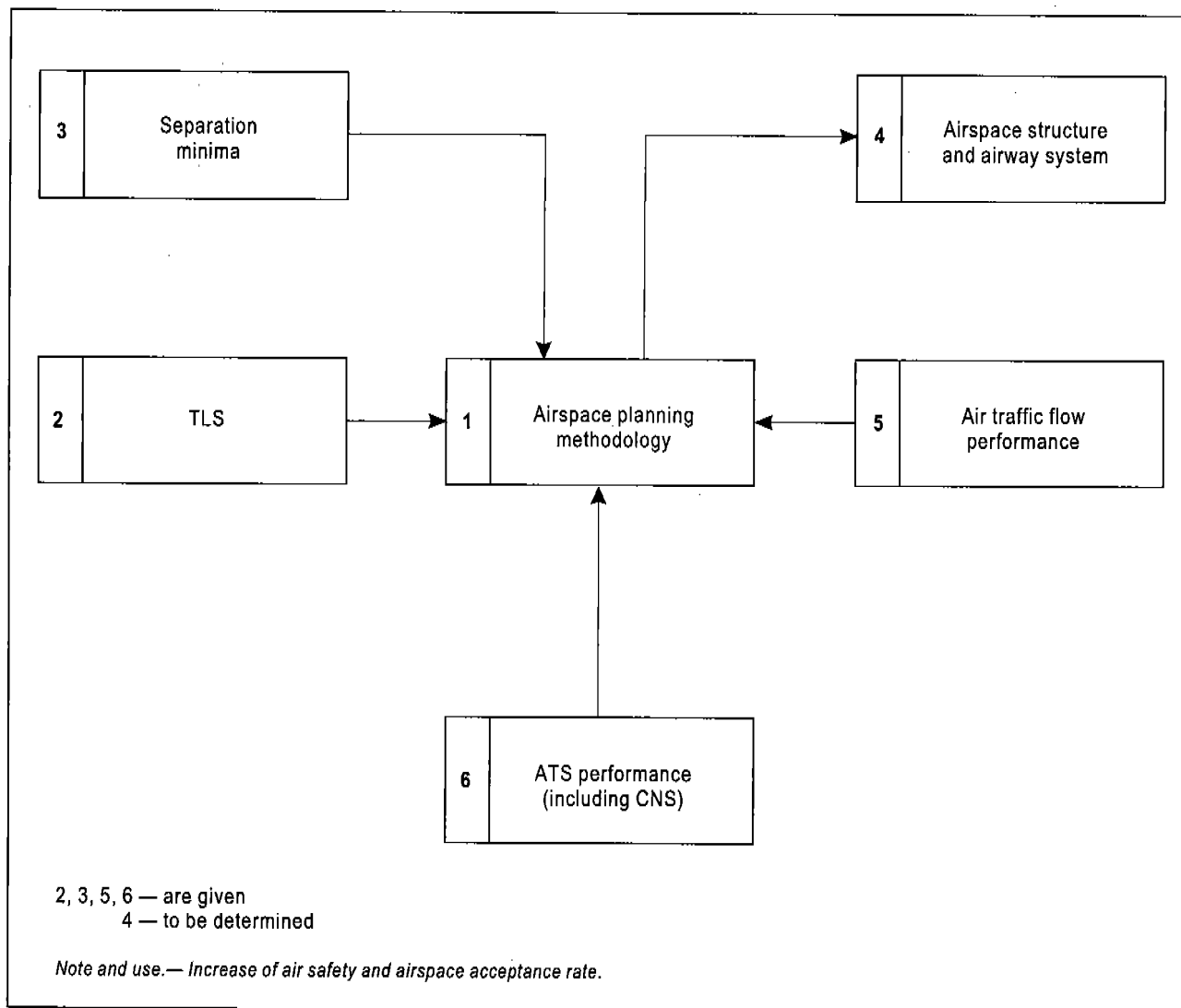
Figure 1-2d. Practical applications of the airspace planning methodology *(see Figure 1-3 for key)*
Example 4 — Organization/tactical measures for air safety
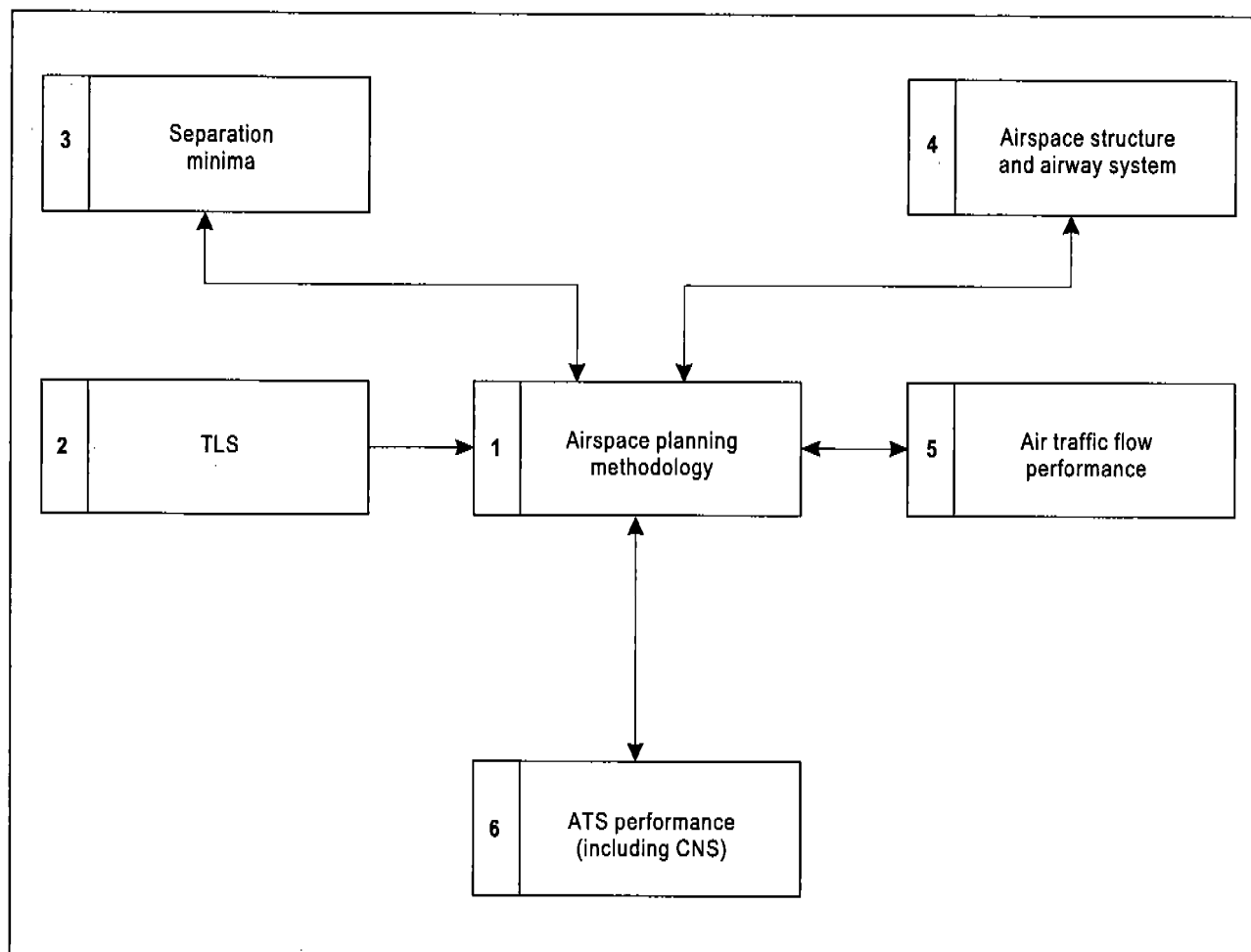
Figure 1-3. Practical applications of the airspace planning methodology — key

Chapter 2

CONSIDERATIONS IN IDENTIFYING
THE NEED FOR CHANGE

FACTORS AFFECTING THE
NEED FOR CHANGE

2.1   In most cases, the need for change in any given airspace is determined by factors such as user demand, lack of airspace capacity, and availability of improved technologies (both aircraft and ATC).

MEETING THE NEED FOR CHANGE

2.2   There are a number of possible ways of meeting an identified need for change and these include:

a)   enhancing the level of air traffic service provided;

b)   reducing separation minima;

c)   enhancing intervention capability;

d)   changing required navigation performance;

e)   revising route structure; and

f)   limiting demand on ATC services.

2.3   Chapter 7 gives more detail on these options. All of the alternatives and their interrelationships should be considered to determine which option or combination of options would best meet the identified requirements for the airspace.

2.4   After it has been determined that there is a need to change the airspace system in order to gain the desired operational benefits, the airspace planner should use up to date knowledge of current systems and new technologies to evaluate which characteristics of the airspace could be changed. Although the focus of this document is on the determination of separation minima, the airspace planner may be able to provide benefits for both ATS providers and users without reducing separation minima. An alternative choice to meet changing demands for airspace usage may be to redesign the route structure of a region. Appendix 2 briefly describes two such route reviews.

COST-BENEFIT ANALYSIS

2.5   Whatever the changes contemplated, an important factor in the evaluation process should be a cost-benefit analysis followed by a prioritization of the acceptable alternatives. It is unlikely that aircraft operators and providers of air traffic services will invest in new equipment unless cost-benefit studies have demonstrated the financial viability of such investments. In addition, many organizations will need to achieve an adequate return on investment within a relatively short time frame. Cost-benefit analysis is a complex process and this document does not provide detailed guidance on its execution. However, to illustrate the types of processes involved, Appendix 3 presents a brief outline of a cost-benefit analysis that was made for airspace changes.

Chapter 3

DESCRIPTION OF THE CURRENT
AIRSPACE AND THE CNS/ATM SYSTEMS

DETERMINING
SEPARATION MINIMA

3.1 In determining appropriate separation minima, the airspace planner must have a thorough knowledge of the existing airspace, the CNS/ATM capabilities and the airspace characteristics, which may influence the safe separation minima. There are a number of factors that may influence the separation minima and these include:

3.2 Airspace structure:

a) route structure, e.g. the use of parallel or non-parallel ATS routes and whether they are bi-directional or unidirectional;

b) existing separation minima and how often values close to the separation minima are used in practice;

c) complexity of the airspace, including inter alia:

1) traffic demand pattern,

2) numbers and locations of crossing tracks,

3) amount of traffic operating on opposite direction tracks,

4) amount of traffic which is either climbing or descending,

5) nature of the aircraft population, i.e. the diversity of traffic with respect to aircraft performance and equipage (e.g. mix of various speeds, climb performance, desired optimal flight levels),

6) peak and average traffic demands versus system capacity,

7) runway capacities and the limitations of associated ground services,

8) any adjoining special-use airspace, airspace usage and types of activities including the civil/military mix, and

9) regional meteorological conditions (e.g. the prevalence of convective storms, etc.), and

d) designated airspace classifications.

3.3 Communication capability:

a) direct controller/pilot voice communication (VHF/HF/SATCOM);

b) indirect controller/pilot voice communication (HF);

c) controller/pilot data link communication (CPDLC);

d) controller/controller voice and automated data link communication, both inter and intra ATS unit(s);

e) data link between ground ATC automation systems and aircraft flight management computers; and

f) system availability, reliability and capacity.

3.4 Surveillance capability:

a) procedural dependent surveillance:

1) content of pilot position reports, and

2) reporting intervals;

b) automatic dependent surveillance (ADS):

1) basic update rate,

2) display accuracy,

3) ADS contract (e.g. events triggering increased reporting rate),

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4) sensor accuracy,

5) system reliability, and

6) end-to-end communications time capabilities; and

c) independent surveillance (radar):

   1) type of sensor (primary or secondary),

   2) coverage area,

   3) processing and associated delays,

   4) accuracy of measured position after processing,

   5) update rate,

   6) display accuracy, and

   7) system reliability.

3.5   Aircraft navigation performance:

a) required navigation performance (RNP);

b) typical and non-typical performance (e.g. MASPS/ MOPS), (RTCA SC181 documents refer); and

c) time-keeping accuracy.

3.6   Flow management capability (ability to control traffic input to ATC):

a) strategic air traffic flow management;

b) tactical air traffic flow management;

c) ad hoc ATC "in trail" restrictions or enhancements; and

d) procedural restrictions (e.g. through local operating procedures).

3.7   Air traffic management tools to reduce controller workload or improve controller intervention capability:

a) automated controller planning tools, including conflict prediction and resolution;

b) controller displays; and

c) out-of-conformance alerts (3-D) (i.e. automatic systems which alert ATC to any deviation of an aircraft from its nominal path).

Aircraft equipped with airborne collision avoidance systems (ACAS)

3.8   It should be noted that, in accordance with the guidance given in Annex 11, the carriage of ACAS by aircraft within a region should not be used to justify a reduced separation minimum. However, the presence of such systems may be relevant when contemplating the application of reduced separations, as changes to the ACAS systems may be required in order to avoid an unacceptable rate of false alerts.

Chapter 4

DETERMINING THE PROPOSED
AIRSPACE AND CNS/ATM SYSTEMS

FACTORS TO BE CONSIDERED IN
DEFINING THE PROPOSED SYSTEM

4.1   In defining the proposed system, and thereby identifying the changes necessary to the current system, the airspace planner may find that there are several elements that can be changed, and should consider the following:

a) the benefits to be derived from a change to any one item;

b) the impact any change may have on operations and subsequent re-training, implementation time, transition periods, plus the cost of new equipment; and

c) as far as practicable, identify those changes that would provide a favourable balance of benefits in relation to the cost of implementation of the change, both to the ATS provider and user population.

4.2   The airspace planner should also keep in mind that, in some instances, changing one characteristic in the airspace will not be sufficient in itself to allow reduced separations, and additional changes to other aspects of the airspace might be needed. For example:

a) ATM requirements for aircraft to carry RNP 1 approved equipment may also require the provision of automated assistance/support tools for ATC and/or a revised navigation infrastructure;

b) when integrating ATS systems in a large region or State, the level of ATS systems improvement in a particular local area may depend on the relative

distribution of limited resources for ATS development throughout the whole region, i.e. cost-benefit criterion based decisions need to be made. An example of the distribution of limited resources related to a change in airspace planning is provided at Appendix 12; and

c) in the case of b) the potential for implementation of local reductions in separation minima should be estimated taking into account air traffic requirements in the whole region or State, i.e. the airspace should also take into consideration the capacity and procedures of adjacent airspaces and the need for a coordinated implementation.

FORECASTING TRAFFIC GROWTH,
CHARACTERISTICS AND
DISTRIBUTION

4.3   The airspace planner should carefully consider the forecast traffic growth, and the characteristics and distribution of the traffic growth throughout the airspace. Forecast traffic growth may be concentrated on selected routes and not be uniform throughout an airspace. Changes in consumer patterns, economic activity, travel preferences, new airport developments, etc. can cause simple growth to be channelled along different routes which, in turn, will affect the estimation of parameters that directly affect the level of risk in the airspace. Many aviation authorities possess dedicated traffic forecasting expertise and should be consulted for detailed information of projected traffic demand.

Chapter 5

IDENTIFYING THE METHOD OF SAFETY ASSESSMENT FOR A PROPOSED SYSTEM

SAFETY ASSESSMENT

5.1 The safety of a system depends on a number of characteristics of the airspace (see Chapter 3). When the relevant characteristics of a proposed system have been identified and quantified, there are two basic methods for determining whether the system is acceptably safe:

a) comparison with a reference system; and

b) evaluation of system risk against a threshold.

Comparison with a reference system

5.2 Comparison with a reference system is a "relative" method, i.e. all the relevant characteristics of the proposed system are compared with the corresponding characteristics of a reference system which has been judged to be safe. Provided that the proposed system can be demonstrated to be similar to or better than the reference system in all safety-related aspects, then it also may be assumed to be safe. Clearly, the most important aspect of this approach lies in the identification of a suitable reference airspace which, for minor changes, may include the current system and the demonstration that the proposed system is sufficiently similar to justify the approach. This method is discussed further in Chapter 6.

Evaluation method

5.3 The evaluation of system risk against a threshold is an absolute method where, after identification and quantification of all the safety-related characteristics of the system, an explicit relation between these characteristics and collision risk is determined and used to estimate system safety. This estimate is then compared against a maximum tolerable risk — for example, a target level of safety. The estimation of risk for any airspace is a very complex procedure and may require extensive data on all aspects of the performance of the system. The choice of a suitable value for the maximum level of risk may also be a difficult exercise.

5.4 Although the evaluation method is likely to be complex and time-consuming, it is the only choice when a radical change is planned which has not previously been tried in other regions. This approach does have the advantage that, once the safety-critical parameters are identified and their effect on collision risk modelled, it is possible to adjust the values of the various parameters to determine the most appropriate method of achieving the required improvements in the airspace. Chapter 6 describes the details of this approach.

5.5 The flow diagram in Figure 5-1 illustrates the decision-making process that should be used when deciding upon the appropriate safety assessment methodology.

5.6 A proposed system can be judged to be safe if either:

a) it has been shown to be at least as safe as a reference system which has already been judged to be safe; or

b) the quantitative estimate of risk of the proposed system is no greater than a predetermined acceptable level of risk.

5.7 If either of these conditions are met, then the airspace planner can proceed to the implementation and monitoring stage. If neither of these is the case, then the proposed system must be modified in such a way that the safety criteria will be met.
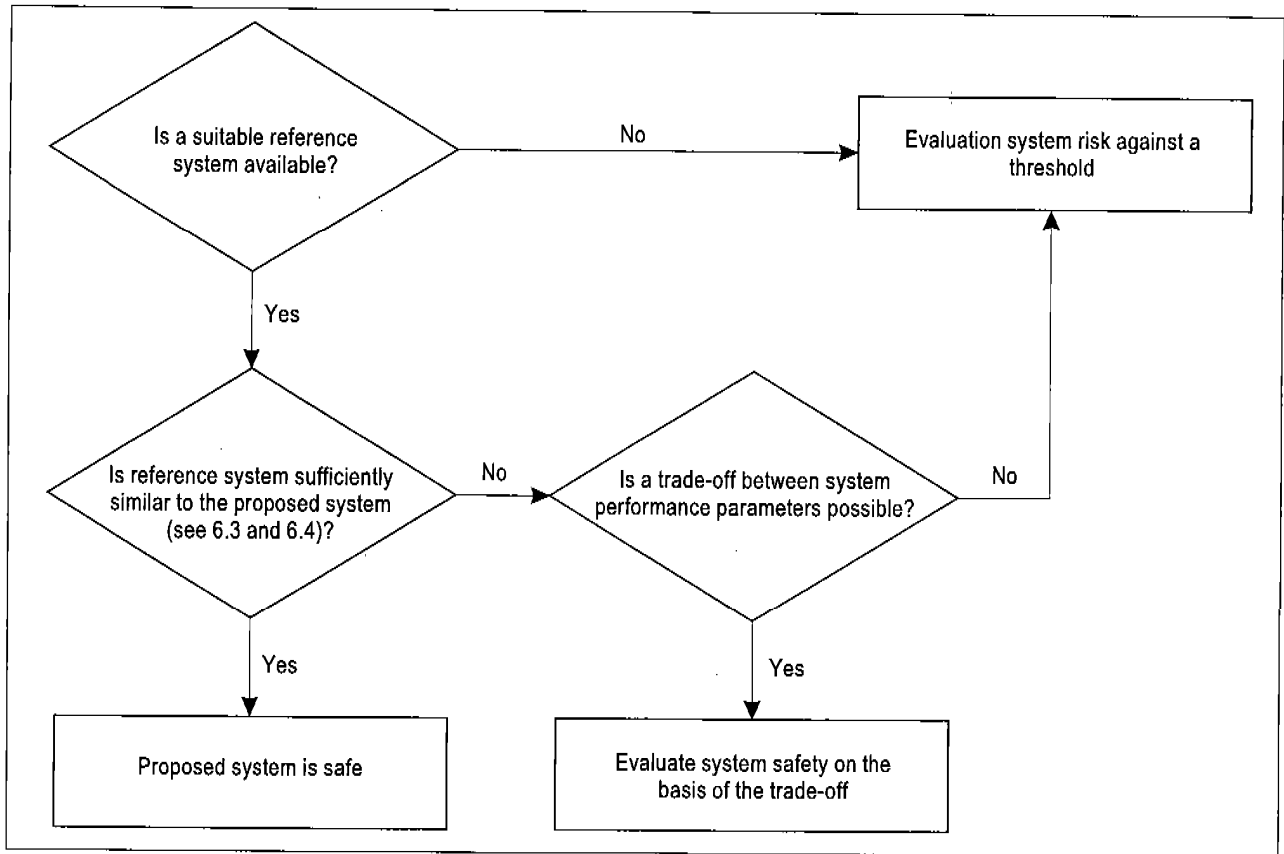
Figure 5-1. Choice of appropriate method for evaluating safety

Chapter 6

METHODS OF EVALUATING SAFETY

COMPARISON WITH A REFERENCE SYSTEM

6.1   The reference system method compares the estimated performance of the proposed system with the performance of a system which has already been judged to be acceptably safe. If the performance of the proposed system is better, or at least no worse, than the reference system in all safety-related aspects, then the proposed system can also be assumed to be acceptably safe. If the performance of the proposed system is better with regard to some factors, but worse with regard to others, then it may be possible to perform a trade-off between these factors to assess whether the proposed system will be safe. This approach does, however, need to be followed with some caution.

6.2   The first step is to choose the reference system to be used for comparison. The chosen reference system should be considered to be safe. Two options are available when selecting the reference system:

a)   refer to the system described in Appendix 4; or

b)   identify another existing system which has been demonstrated to be safe.

6.3   Whatever reference system is chosen, it must bear a sufficiently close resemblance to the proposed system for any comparison with regard to safety to be valid. The levels of air traffic service provided in the reference and proposed airspaces, as defined by the ICAO airspace classifications, should be examined. The air traffic service in the proposed system should provide at least the same level of service as the reference system.

6.4   The minimum requirements for a reference system to be considered sufficiently similar to a proposed system are:

a)   separation minima must not be less in the proposed system than in the reference system;

b)   proposed means of communication and surveillance must be no worse in terms of accuracy, reliability, integrity and availability than those of the reference system;

c)   frequency and duration of the application of minimum separation between aircraft must not be greater in the proposed system than in the reference system; and

d)   navigation performance (typical and non-typical) of the population of aircraft in the proposed system should be no worse in its effect on collision risk, in any dimension, than that of the aircraft in the reference system.

6.5   The safety of the system in Appendix 4 has been verified using collision risk modelling or through long-standing operational practice. If any other real-life operational system is taken as the reference system then more caution is needed. The difficulty lies in ensuring that the reference system has been properly judged to be safe and in assuring that all the salient characteristics of the reference system have been accounted for. For example, it would be necessary for the reference system to have demonstrated an extensive history of system safety in terms of system flight hours. However, in some cases, for example, those systems with low traffic densities, the length of time needed to accumulate a sufficiently large number of flying hours to demonstrate that the system was safe may be impractical. Therefore, other means of ensuring that the reference system is safe would have to be found.

6.6   When a proposed system requires a relatively minor change to a current operational system, and hence the reference system is actually the current operational system, the issue is whether the change would adversely alter the safety of the system. For example, in Japan it was proposed that a long-range secondary surveillance radar (SSR) system with a range of 250 NM should be used for monitoring an airspace adjacent to an oceanic transition area. To assess the feasibility of using the long-range SSR for the surveillance of that airspace, the long-range SSR system was installed

experimentally in conjunction with the conventional ARSR/SSR system with a range of 200 NM. Because all the other airspace characteristics other than the quality of surveillance available would remain constant, it was sufficient to only compare the measurement accuracy of the two radar systems. It was found that the proposed sole use of the long-range system would provide greater accuracy throughout the operating range compared to the use of the conventional system. Finally, the implementation was made on the basis that no deterioration in safety would occur. Appendix 6 describes this analysis in more detail. It should be noted that a change in separation minima would not constitute a small change.

6.7   If a reference system is available, but does not simultaneously meet all of the four criteria in 6.4, it may still be possible to evaluate the safety by means of a trade-off between the different system performance parameters. Such an analysis should be easier to perform than a full risk assessment for the system. The trade-off can be applied to all the various subsets of system performance parameters occurring in requirements a) to d) of 6.4. Two examples of particular interest are described below:

1) Consider the case where the reference system is the current system and the requirement is to examine the safety associated with a proposed reduction of lateral separation minima as a consequence of improving the navigation performance, all other characteristics remain the same, i.e. communication, surveillance and frequency of application of the separation minima. However, due to requirement a) in 6.4 (that the separation minima must not be less in the proposed system than in the reference system) the reference system is not sufficiently similar to the proposed system. In this case, it could be sufficient to only examine the interrelationship between the navigation performance and separation minima as expressed by the probability of lateral overlap.

2) Consider the case where the reference system and the proposed system have the same lateral separation minima and the same means of communication and surveillance but are different with regard to the frequency of application of the separation minima and navigation performance. On the basis of the safety of the reference system, it would be sufficient to examine the trade-off between two characteristics, occupancy and the probability of lateral overlap. An illustration of this is given in Figure 7-1 of Appendix 7.

6.8   Once a reference system has been chosen, the airspace planner must perform the following steps to evaluate the relative safety of the two systems:

1) Describe in detail the differences and similarities between the two systems. This must be done for all of the criteria listed in Chapter 3.

2) For each criterion, assess how any differences between the systems would affect the risk. This can be done using mathematical techniques and/or using operational judgement. However, particular care must be taken to identify situations where the effect on the collision risk is counter-intuitive. For example, an increase in lateral navigation accuracy, whilst decreasing collision risk in the lateral dimension, could actually increase the risk of a collision in the longitudinal or vertical dimensions. Furthermore, in cases where the risk is dominated by the occurrence of large operational errors, an increase in lateral navigation accuracy may even increase the lateral collision risk.

3) For each criterion, ensure that the proposed system is at least as safe as the reference system. Alternatively, a trade-off between factors which are worse and those which are better than the reference system may be possible.

EVALUATION OF SYSTEM RISK AGAINST A THRESHOLD

6.9   The second method of determining whether the proposed system is safe is to estimate the collision risk in the system and then compare it with a predetermined maximum tolerable collision risk. If the estimated risk is lower than the maximum tolerable risk, and is expected to remain so throughout the envisaged lifetime of the new system, then the proposed system can be judged to be acceptably safe.

6.10   The flow diagram in Figure 6-1 outlines the general principles of this approach. The process involves the following steps:

1) System definition

    The scope of the study, defining the airspace to be considered, the problems to be solved and the nature of the changes being proposed are established. Chapters 3 and 4 outline some of the aspects of the system which may need to be specified.

2) Setting evaluation criteria

    This stage involves the choice of the safety criteria against which the proposed changes will be evaluated. In the case of separation minima, this requires the determination of the maximum acceptable collision risk.

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                    │    system     │
                    │  definition   │
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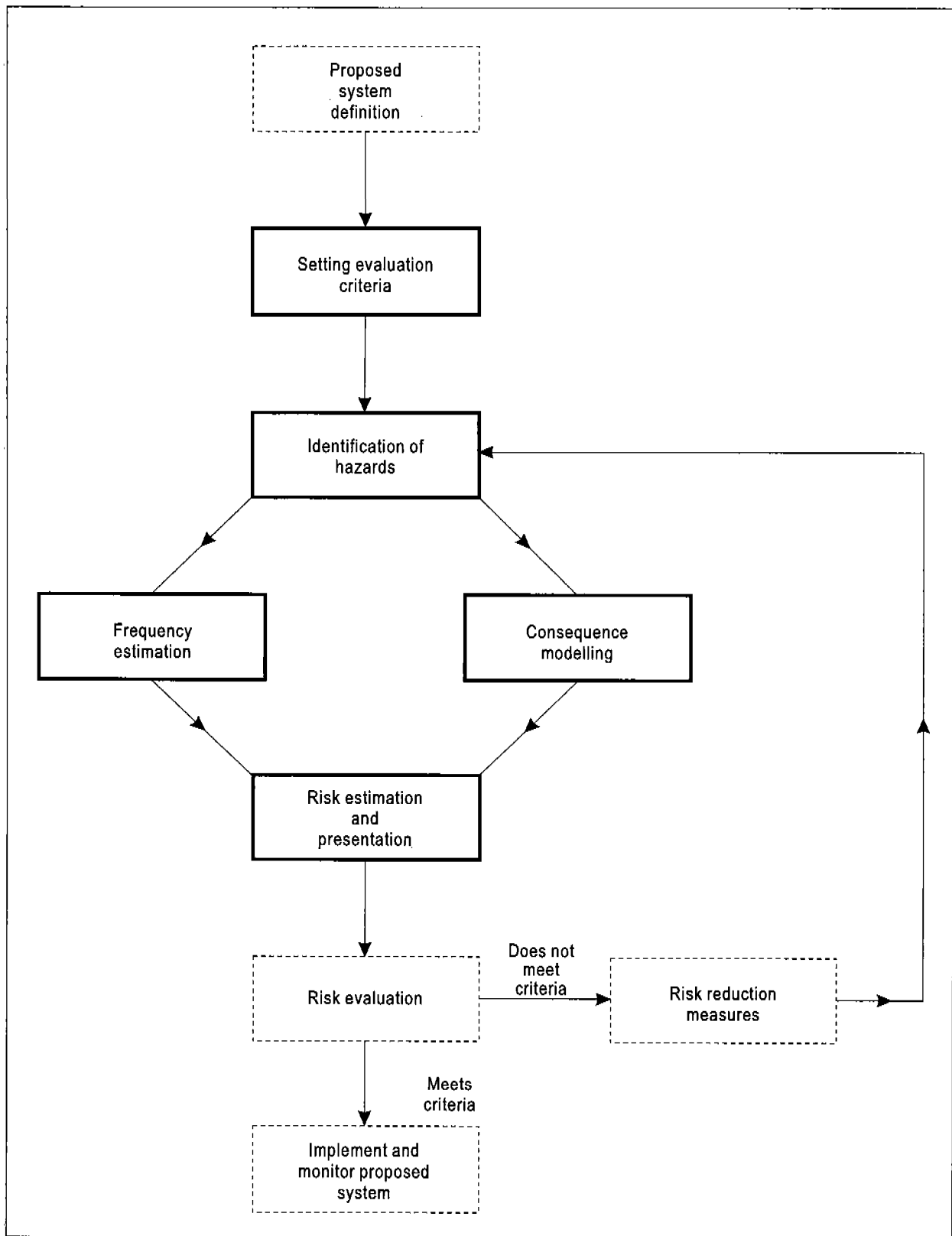
Figure 6-1. The risk evaluation process

3) Identification of hazards

The identification of all possible hazards, i.e. any events or combination of events that could possibly lead to a collision, will involve a detailed investigation of the operation of the target system as defined in step 1.

4) Frequency estimation and consequence modelling

The likelihood that each hazard could occur must be estimated. Where possible, aviation specific data should be used; however, if this is not available, it is possible to use generic data drawn from safety studies in other industries. The information gained during step 1 will be important in this process. In parallel with estimating the frequency with which hazards occur, the consequences of each of the identified hazards need to be investigated.

5) Risk estimation and evaluation

The results from the frequency estimation and consequence modelling are combined to provide overall risk estimates. The estimated risk is compared to the evaluation criteria determined at step 2.

6) Risk reduction measures

If the calculated risk does not satisfy the predetermined criteria, then it is necessary to examine how the risk can be reduced.

Proposed system definition

6.11   In order to undertake a risk evaluation, all aspects of the proposed system need to be defined. These aspects will include the system description detailed in Chapters 3 and 4. It will also be necessary to derive details of the performance of the system (either actual or projected). The quantitative data can include error rates (e.g. navigation errors), aircraft physical parameters (size, speed, etc.), expected traffic demand and measures of nominal performance (e.g. typical navigation performance). The data may be obtained from direct examination of an operational system, by forecasting the performance of a hypothetical system, or, in the case of hazard analysis, based on the expert judgement of experienced air traffic controllers.

6.12   The system parameters that have the greatest influence on collision risk may be categorized into three groups: exposure of one aircraft to other aircraft, navigation performance and effects of surveillance and communications.

1) Exposure of one aircraft to other aircraft

The exposure of the typical aircraft to other aircraft within the system is a principal determinant of the risk of collision. In order to establish lateral separations between routes within the system, this exposure is related to the frequency with which aircraft pass each other (in both the same and opposite direction) on adjacent routes. For collision risk modelling purposes, this can be translated into a value of occupancy (see Doc 9426, Part II, Section 2, Chapter 4, Appendix C).

To establish longitudinal separation minima, aircraft exposure is represented via a distribution of inter-aircraft separations. This is estimated by recording the relative frequency a given longitudinal separation is used between pairs of aircraft in a system and creating a frequency histogram.

Lateral and longitudinal exposure are both strongly influenced by the traffic flows and the route complexity for the given system. While there is no direct analytical relationship between these elements and the aircraft exposure, a simulation of the route system with the traffic distribution can yield an estimate of the exposure. If it proves impractical to mount a simulation, it may be possible to obtain an estimate for aircraft exposure by examining other similar systems. In either case, it may be necessary to impose controls and to closely monitor the performance of the proposed system after implementation in order to ensure that aircraft exposure does not exceed the original estimates.

2) Navigation performance

The navigation performance of the aircraft population has been identified as a principal influence on the risk of collision. The effect of navigation performance in each of the three dimensions, i.e. lateral, longitudinal and vertical, affect the risk of a collision and the three components need to be considered when a separation minimum is set. It is important to be aware that the risk of collision in one dimension is directly proportional to the accuracy of navigation in the other two dimensions. Navigation performance is a factor that contributes to risk and the standard of navigation performance is most important when separation standards are maintained using position information provided to air traffic control via dependent surveillance (pilot reports, ADS and, for vertical position, SSR radar).

Lateral navigation performance

The lateral navigation performance of the aircraft population determines the lateral overlap probability. This is a measure of the likelihood that two aircraft, which are nominally separated, are in fact in lateral overlap. This parameter is a key element in determining the lateral collision risk. The lateral collision risk is directly proportional to the lateral overlap probability for two aircraft nominally separated by the lateral separation minimum.

In a procedural airspace with a parallel track system and dependent surveillance, the lateral overlap probability is affected by both the typical and non-typical navigation performance. Typical performance is used here to describe the usual small errors in position which occur when navigation systems are operating correctly; the non-typical performance arises either due to navigation system failures or human error and can result in very large deviations from the correct position. The non-typical performance can be measured in terms of the proportion of flight time spent at a distance greater than half the lateral separation minimum from the correct track and by the proportion of aircraft flight time spent near to the centre line of another route.

The relative effect of these two sources of error on the lateral overlap probability may vary from airspace to airspace. For example, in the North Atlantic minimum navigation performance specifications (NAT MNPS) airspace, the lateral separation minimum is so large that the non-typical performance contributes, by far, the largest part to the lateral overlap probability and hence to the collision risk. When planning a parallel track system, great care should be exercised in establishing separations large enough to eliminate virtually all risk due to typical errors, and characterizing and then controlling the level of non-typical navigational performance.

Lateral navigation is also important when assessing the collision risk in the longitudinal dimension, although in this case it is the nominal performance that is most important. This is because if longitudinal separation is eroded between two aircraft nominally flying on the same track, a collision can occur only if the two aircraft are in lateral overlap. The longitudinal collision risk is directly proportional to the lateral overlap probability between two aircraft nominally on the same track. The effect of changing the standard deviation of the population (approximately half of the RNP value if

it is assumed that the core distribution is a Gaussian distribution) is shown in Table 6-1. It should be noted that improving lateral navigation actually increases the longitudinal collision risk.

Table 6-1. Lateral overlap probabilities RNP value

RNP value	Standard deviation of the population (NM)	Lateral overlap probability
1	0.51	0.0301
4	2.04	0.0075
5*	2.55	0.0060
10*	5.10	01.0030
12.6	6.43	0.0024
20	10.20	0.0015
* Example of a regional application		

Longitudinal navigation performance

The longitudinal collision risk is also dependent on the typical along-track navigational performance, which determines the likelihood that longitudinal separation will be lost. Therefore, it is important to constrain the along-track performance of the aircraft population. In a typical oceanic airspace, where pilot reports at waypoints are used, the maintenance of longitudinal separation is dependent not only on the ability of the pilots to determine the aircraft's longitudinal position, but also on the accuracy with which all flights in the system measure time. The accuracy of position measurement can be controlled by selecting an RNP value. The accuracy of time measurement can be controlled by specifying individual aircraft time-keeping accuracy. When both these factors are controlled, they combine to limit the variation in inter-aircraft spacing, thus also reducing the risk.

In procedural airspace the minimum longitudinal separation is often specified in terms of the minimum time between consecutive aircraft on the same track. The longitudinal separation measured in nautical miles then depends upon the speed of the aircraft concerned. Maintaining the correct longitudinal separation on long en-route tracks can be simplified by the application of speed controls,

e.g. Mach number technique, which requires all aircraft in the system to maintain constant speeds, (Doc 9426, Part II, Section 2, Chapter 2 refers). The initial longitudinal separation on entry to a track system is then based on the relative speed between each consecutive pair of aircraft and is set in order to ensure that the minimum separation on the track will not be infringed throughout the flight. The application of Mach number technique reduces the variability of spacing between aircraft and reduces the requirement for ATC intervention to correct this spacing.

Vertical navigation performance

Vertical navigation performance is determined by the altitude-maintenance capability of the aircraft population. Vertical navigation performance is not only important for setting vertical separation requirements (Doc 9574 refers), but the nominal performance also affects the risk in the lateral and longitudinal dimensions. If separation is lost in both of these dimensions between aircraft nominally at the same level, a collision will only result if both aircraft are also in vertical overlap. The collision risk in the longitudinal or lateral dimensions is therefore directly proportional to the vertical overlap probability between two aircraft nominally at the same altitude.

3) Effects of surveillance and communications

The collision risk in a given airspace is directly affected by the capability of ATC to detect aircraft on conflicting tracks and to correct the situation before a collision can occur. This intervention capability is determined by the efficiency of the surveillance and communication systems available to the air traffic controller. Safe separation minima in an airspace are closely linked to the means of surveillance and communication available to ATC. As airspaces change from strictly procedural systems, improvements in surveillance, communications and ground-based automation combine to form an enhanced decision-support system for the controller and allow progressively smaller separations to be used safely.

A principal feature of the communication links between pilot and controller, which affects the minimum separation that can be safely maintained, is the delay in transferring the desired information. The reliability, availability and integrity of the communication subsystem must also be assessed to

understand its function in the overall decision-support system. In the case where the communications link carries traffic for dependent surveillance activities, the communications performance parameters are directly related to the surveillance function, e.g. where ADS is used as a primary surveillance tool, the performance of the applied data link has a direct influence on the surveillance and intervention capability and thus on the achievable safe separation minima. Appendix 8 summarizes a method for determining lateral separation minima in an ADS-based ATC system.

Information on the status and position of aircraft is essential for ATC. The provision of this information can range from pilot reports at intervals of 30 minutes or more, to radar data updated every 4-6 seconds. Whatever the system being used, its reliability, integrity and availability must be assessed, as well as the accuracy of the information and any delays in the presentation of the information to ATC.

The delay in presenting information to the controller is related to the update rate of the surveillance system and in some cases may be produced by automation (for instance in resolving the non-synchronization of the timing of reports from different surveillance systems). In addition to presentation to the controller, some systems employ conformance checking for individual aircraft or conflict prediction for pairs of aircraft. The selection of the threshold for these decision aids and their associated alarm levels will have an effect on the system safety.

The additional margin of safety provided by controller intervention can be assessed in part by estimating the delay from the time that the controller perceives that a collision hazard exists until instructions are communicated and the aircraft responds.

Setting evaluation criteria

6.13   In order to evaluate the estimate of collision risk, this should be compared to a maximum tolerable collision risk for the system. Determining this level of risk is an independent process involving decision makers who represent State authorities, regional authorities or ICAO technical panels. The maximum tolerable risk is normally expressed in terms of a TLS. In the past, when applied to en-route collision risk, the TLS had been expressed in terms of the number of fatal accidents per flight hour, which could result from collisions between aircraft (where a collision

between two aircraft represents two fatal accidents). Although ICAO has agreed that the concept of a global TLS is valid, the metric of fatal accidents per flying hour (as applied, for example, in the NAT Region) may not be appropriate for other regions. ICAO has agreed to the development and use of different metrics, provided that it can be demonstrated that any change in separation minima or other system parameter is subject to the overriding consideration that the risk of collision as a consequence of a loss of separation, from any cause, should be lower than that of the agreed level of system safety.

6.14 The current level of risk in NAT MNPS airspace is assessed against a new TLS in the vertical plane of 5×10^{-9} fatal accidents due to collisions per system flight hour (for RVSM levels) and against a TLS of 2×10^{-8} fatal accidents due to collisions per system flight hour for non-RVSM levels. This former value takes account of the risk of collision associated with vertical navigation perform-ance as well as that associated with ATC or pilot errors (operational errors). The latter value is also used in the horizontal (lateral and longitudinal) planes. As separation reductions are introduced in the horizontal plane, the system risk in the lateral and longitudinal dimensions will also be measured against a new TLS of 5×10^{-9} fatal accidents per flight hour.

6.15 The guidance material (*Manual on Implemen-tation of a 300 m (1 000 ft) Vertical Separation Minimum Between FL 290 and FL 410 Inclusive* (Doc 9574)) sets a TLS of 2.5×10^{-9} fatal accidents per flight hour for the risk of collision associated with vertical navigational perform-ance. It should be noted that the risk arising from either ATC or pilot errors is not addressed by this TLS.

6.16 This aspect has, however, been addressed in NAT RVSM operations by applying a TLS of 5×10^{-9} fatal accidents due to collisions per system flight hour in the vertical plane as described in 6.14 above. As the implementation of RVSM on a global basis progresses, this vertical TLS may be used as a guide for other regional planning groups.

6.17 RGCSP recommends that the value of 5×10^{-9} fatal accidents per flight hour per dimension arising from collisions should be chosen as an assessment TLS for systems planned for implementation after the year 2000 in regions where the use of this metric is appropriate. Where this is not an appropriate metric, justifiable alternative values and methods of assessment should be established.

Identification of hazards

6.18 The hazard identification stage requires an examination of the events that could lead to a collision.

Both human and system failures should be considered. Hazard identification normally involves experts in all aspects of the system who identify all the possible mech-anisms that could lead to a collision. The accuracy of navigation systems, the performance of surveillance and communications, and the procedures used are all important aspects of this process. In the example of the NAT region described in Appendix 4, two principal causes that could lead to loss of separation were identified: navigation errors, which could lead to aircraft deviating from the assigned track; and misunderstandings or errors made by flight crews or ATC, which could result in aircraft following an incorrect path.

Frequency estimation and consequence modelling

6.19 The assessment of the frequency of occurrence of a hazard that could lead to the loss of separation may be based on historical observations, expert judgement, or be an aspect of the design of the system. The main difficulty with frequency estimation and consequence modelling for separation analysis is that the events of interest, i.e. events which lead to collisions, are often rare and it is difficult to obtain data based on direct observations or reports of the events. Therefore, frequency estimation is normally limited to estimating the frequency with which separation will be eroded to some extent, e.g. in the assessment of risk in the NAT region (Appendix 4) the proportion of time spent within 10 NM of an adjacent track centre line is estimated from observations. Consequence models are then used to estimate the likelihood that a collision would occur given that an aircraft flies within 10 NM of an adjacent track. The system parameters described in 6.12 form an important part of the frequency estimation and consequence modelling process.

6.20 The hazard analysis methods developed in other industries recognize the problem of assessing the frequency of very rare events. This has led to the development of techniques that can be used to quantify operational judge-ment. These techniques involve the use of panels of operational experts guided by a trained facilitator, who also use the large amount of accumulated knowledge on likely error rates in other industries. Where data on error rates due to a specific source of error are not available, this generic data may provide a guide to likely error rates. However, it should be noted that there is evidence to suggest that the error rates of trained staff in the aviation industry are less than the expected error rates in some other industries. Appendices 9 and 10 describe examples of this process in more detail.

**Risk estimation
and evaluation**

6.21   The risk estimation process involves combining the risks of collision from each of the identified hazards to obtain an estimate of the overall risk. It is important at this stage that all the risks are converted to a common metric, which is the same as that used for the evaluation criteria. As described in the preceding chapters, the system collision risk is dependent on a wide variety of factors. In order to assess the sensitivity of the risk estimate to the various parameters, it is often useful to present the risk as a function of each of the major parameters; e.g. risk versus traffic demand and risk versus lateral navigation accuracy. Where forecast values for the various parameters exist, risk versus calendar year may prove to be a useful presentation.

6.22   The process of evaluating risk primarily involves the construction of mathematical models, which use detailed information about the system to estimate collision risk (see Doc 9426, Part II, Chapter 4, Appendix B). Three examples of this process are described in Appendices 4, 5 and 11. Appendix 4 describes the assessment of 60 NM lateral separation in the NAT Region and Appendix 5 describes the assessment of 50 NM longitudinal separation in the Asia/Pacific Region. Appendix 11 considers questions related to analysing an ATS system for the purpose of compliance with target indices in respect of safety and capacity, as well as the justified selection of system component requirements. Appendices 9 and 10 describe an alternative method based on the process of hazard analysis whereby estimates of the risk are

determined from expert judgement and comparison with other similar operations. In some cases, it may only be necessary to develop and evaluate changes from previous assessments of other airspaces. In assessing those changes, it is recommended that emphasis is placed on the major system parameters mentioned within this section.

6.23   Provided the metrics of the estimate of risk and the evaluation criteria are the same, the evaluation of the risk is a straight-forward exercise involving the direct comparison of the two values. Because risk estimation has a high level of uncertainty, it may be necessary to quantify the level of statistical confidence that the true level of risk will be below the threshold value. To overcome this problem, it may be acceptable to base the estimates on conservative assumptions and thereby estimate an upper limit for the risk.

**Risk reduction
measures**

6.24   Risk reduction measures should be used when the overall risk estimate is above the predetermined threshold and when a particular element of the system is found to have a disproportionate influence on the risk, providing this can be achieved at an acceptable cost. The detailed process of risk evaluation simplifies the process of identifying effective risk reduction procedures by allowing the effect of changes to the various system parameters to be directly assessed. Chapter 7 describes some of the elements which may be considered for change.

Chapter 7

MODIFYING THE PROPOSED SYSTEM

RISK REDUCTION

7.1   If the proposed system has been found to be unacceptable in terms of risk assessment, the airspace planner may wish to return to the elements in the description of the proposed system (see Chapter 3), and look for means of risk reduction. The airspace planner may choose to:

a) change the level of air traffic services

This option may involve ATS re-sectorization aimed at exploiting the capabilities of modern CNS/ATM and aircraft technologies.

b) change the route structure

For example, if a system of bidirectional routes was chosen initially, perhaps the use of unidirectional or parallel routes may reduce the risk to an acceptable level and provide operational efficiencies (i.e. reduce ATC workload).

c) revise the proposed separation minima

If the original intention was to reduce the separation minima, the airspace planner may wish to consider smaller reductions to the separation or other ways to achieve an improvement in airspace capacity.

d) reduce the complexity of the airspace

By relocating routes it might be possible to reduce the number of crossing tracks, opposite direction tracks or the frequency of aircraft climbing and descending in the same airspace. Another option, although probably restrictive, might be to segregate traffic in terms of aircraft performance and/or equipage.

e) require a higher level of required navigation performance

Use a lower RNP type to improve the navigation performance, and/or in the case of longitudinal separation, improve the time-keeping accuracy of the aircraft by mandating carriage of time-keeping equipment meeting specified performance levels. The airspace planner should note that changing the RNP type will involve a different set of MASPS and MOPS and, as a result, may exclude some aircraft from the airspace. Additionally, a change in RNP type may require improvements to the ground infrastructure.

f) improve communication capability

Introduce more efficient communication in terms of speed or reliability, e.g. by changing from HF to VHF, or from HF to CPDLC. Improvements in communication between controllers (inter- and intra-ATS units) may reduce the collision risk by its effect on controller workload.

g) improve surveillance capability

In cases where procedural surveillance is the only available option, pilot reports could be required at more frequent intervals. Where independent surveillance (radar) is being used, an improvement to the radar system (e.g. coverage, update rate, monopulse) may also contribute to risk reduction. In areas where independent surveillance is not possible, the use of ADS may be introduced. If ADS was considered in the initial proposal, the planner may consider adjustments to the ADS contracts (e.g. increasing the basic ADS position update rate), improvements to the supporting data link communication system (e.g. a reduction in transfer delay), or an increase in the separation minima.

h)   introduce or improve flow management capability

If the initial proposal did not consider the capability of controlling the traffic demand, the introduction of air traffic flow management (ATFM) may contribute to risk reduction by reducing peak traffic demands and thereby reducing aircraft exposure to collision risk. It should be recalled, however, that the introduction of ATFM will affect capacity. Where a form of strategic ATFM is in use, improvements may be achieved by introducing tactical ATFM or by increasing effectiveness in the region of interest, e.g. by centralizing ATFM.

i)   introduce or improve air traffic management (ATM) tools

Reduction of controller workload by the introduction of out-of-conformance alerts can have a significant effect on risk, e.g. in the case of closely spaced independent tracks. Automated controller planning tools could reduce the number of potential conflicts and thereby reduce controller workload which, in turn, increases the intervention capability of the controller. Improved controller displays would help to reduce controller workload as well as improve a controller's ability to determine the potential conflict.

7.2   The airspace planner should review the options listed above, as well as any other options available. It is necessary to identify which options can be amended to still allow the planner to attain the initial objective — accommodating the needed change. In considering the options which could be changed, it should be noted that the benefits realized should still outweigh the cost to providers and users.

Chapter 8
IMPLEMENTATION AND MONITORING

GENERAL

8.1   Reduced separation minima may be implemented in three ways:

1. An airspace where a particular separation minimum is to be applied may be defined in appropriate documents with specific vertical and horizontal limits (e.g. NAT MNPS airspace).

2. ATS providers may designate a specific route or routes where a particular separation minimum is applied.

3. ATS providers may elect to apply a separation minimum on a tactical basis between aircraft which are approved to operate under that minimum (e.g. RNP-4, RVSM approved).

8.2   The introduction and continued operation of any reduction in separation minima or route spacing should be subject to the overriding consideration that the risk of collision as a consequence of a loss of separation, from any cause, must be lower than that of the agreed level of system safety (see 6.13).

THE NEED FOR SYSTEM
PERFORMANCE CRITERIA

8.3   All aircraft to which separation minima are applied must be equipped, maintained and operated in accordance with the appropriate State airworthiness and operational procedures. This may entail the development of minimum aircraft system performance specifications.

8.4   Although common ground equipment procurement processes may not be required, it will be necessary to ensure that ground equipment performance is consistent with the planned separation. With this in mind, common ground infrastructure equipment performance standards will need to be developed and the ground equipment designed and installed to these standards.

THE NEED FOR CHANGES
IN AIR TRAFFIC MANAGEMENT
PROCEDURES

8.5   Air traffic management and control procedures will need to be developed to enable full exploitation of the separation minima to be applied. The ATS provider States should be responsible for developing, either individually or on a regional basis, such procedures as are necessary to support operations in the revised airspace. This may necessitate coordination with the appropriate ICAO regional planning group.

THE NEED FOR APPROVAL

8.6   All aircraft intending to operate in a given RNP-type airspace must have operational approval from the appropriate aviation authority. It is noted that aircraft approval processes such as those detailed in the MASPS for RNP will, when ICAO-compliant, provide the service providers and airspace users with the assurance that the equipment will operate to approved tolerances.

THE NEED FOR MONITORING

8.7   Effective aircraft and operator approval processes and programmes are the principal elements, which ensure that aircraft navigation performance standards are met and aircraft safety standards are maintained. Monitoring is a quality control function that has been used to give ATS providers and users confidence that approval programmes are applied effectively by aircraft operators. Monitoring should be conducted during verification and operational trials leading to the implementation of a reduced separation standard. However, after confidence is gained that aircraft and operator approval programmes are effective, the complexity and extent of monitoring programmes may be reduced or eliminated, e.g. aircraft certification and maintenance programmes may prove to be sufficiently adequate to ensure aircraft population performance without the necessity for a specific monitoring programme.

8.8   Monitoring may be accomplished through a number of channels: specific data collections, mandatory occurrence reports, special incident reports, tactical monitoring by air traffic authorities or routine flight crew and maintenance procedures, and regional monitoring programmes that can be designed to target specific risk parameters, such as, in the MNPS airspace, the annual proportion of large errors, and the standard deviation of core performance.

8.9   Monitoring can be implemented to assess many different parameters: navigation performance, intervention performance (surveillance and communications), traffic density, effectiveness of procedures, controller workload implications or other system characteristics.

8.10   If monitoring demonstrates that performance is outside the established limits, remedial action will need to be instituted to restore the system to conformance. A number of options may be considered, namely:

a)   improving training programmes for individual operators or ATS providers;

b)   changing ATC operating procedures;

c)   limiting demand;

d)   modifying the route structure or airspace classification (level of ATS provided); and

e)   increasing separation minima.

AIR TRAFFIC CONTROL IMPLEMENTATION ISSUES

8.11   A programme of ATC implementation measures will need to be developed to facilitate reductions in separation minima in an airspace. The scope of these tasks will depend on the nature of the particular airspace under review. The following minimum considerations should be taken into account:

a)   evaluate existing surveillance and communications coverage in the airspace under review;

b)   plan route changes in the airspace under review;

c)   evaluate and provide the necessary software changes to flight data processing systems (FDPS) and radar data processing systems (RDPS);

d)   plan for the resolution of ATFM issues;

e)   plan and institute changes in sectorization, as required;

f)   provide for integration of military aircraft in the airspace under review (special civil/military procedures may be required);

g)   investigate the need for fast-time and real-time simulations;

h)   plan and carry out simulations as required;

i)   assess and provide for the interface between the modified airspace and adjacent airspace — this may require the development of special procedures in transition areas;

j)   assess the ATS resource implications, e.g. availability of trained staff; and

k)   plan, devise and complete controller training programmes as required.

TIME-SCALES FOR IMPLEMENTATION

8.12   It is essential to establish realistic implementation time-scales at an early stage in the planning process. It may be possible to implement separation minima on a localized basis with little difficulty, and therefore with short lead times. However, the impact of major reductions in separation minima and route spacing should not be underestimated. Some of the considerations which need to be taken into account when planning implementation time-scales are:

a)   aircraft re-equipage requirements and provision of necessary changes to the ground infrastructure may be complex, time-consuming and expensive to provide;

b)   human factors studies may be necessary to determine the degree of automated assistance that air traffic controllers will need in order to safely implement the proposed separation minima;

c)   the need for and provision of often scarce simulation facilities may adversely affect implementation time-scales; and

d)   in many cases it will be necessary to carry out cost-benefit studies at an early stage of the imple-

mentation planning to ensure the timely availability of funding, both by operators and administrations.

THE DECISION TO IMPLEMENT

8.13   Once the safety assessment criteria are met, the decision to implement the planned separation minimum or route spacing can be made and appropriate monitoring commenced. One means of facilitating this decision is to initiate an operational demonstration and implement as many

activities as possible that are associated with, and necessary for, the introduction of the proposed separation, except for the planned separation itself.

8.14   System performance can be monitored during this operational demonstration until sufficient confidence has been gained to verify that the enabling programmes are effective and the chosen target level of safety can be achieved. The final stage is to decide to introduce the planned reduction, having determined that there is a reasonable assurance that the process can be successfully implemented and that safety will not be compromised.

Appendix 1

A GENERAL COLLISION RISK MODEL
FOR DISTANCE-BASED SEPARATION
ON INTERSECTING AND COINCIDENT TRACKS

1. INTRODUCTION

This appendix presents a new model for the analysis of collision risk applicable to distance-based separation of aircraft on both intersecting tracks as well as identical tracks. The model is based on the well-established Reich Model (see references 19, 20, 21), but the derivation presented here is new and indicates the general applicability of the method.

2. METHODOLOGY

2.1   Suppose that a randomly chosen pair of aircraft, not necessarily at the same level, is crossing an ocean on either the same identical track or on tracks that intersect. We denote by T_C the average flying time to complete the crossing. Let the notation Prob{X} mean the probability of X occurring, and define

$$C_p = \text{Prob\{the pair collides during the oceanic crossing\}}. \tag{1}$$

2.2   As in the Reich model, we represent the aircraft by simple geometric shapes. In this appendix we will assume the aircraft are circular cylinders of diameter λ_{xy} and height λ_z. Again, as in the Reich model, we use an equivalent geometry where one aircraft, aircraft 1 in this explanation, is a cylinder of λ_{xy} radius and height $2\lambda_z$, which we denote C, and the other aircraft, aircraft 2, is a point particle, which we denote P. It is clear that for a collision to occur P must enter C through its vertical side or through the top or bottom. It is also clear that a horizontal overlap of the two aircraft occurs when P enters the infinite cylinder of radius λ_{xy} obtained by extending upwards and downwards the cylinder representing aircraft 1. Thus,

$$C_p = \text{Prob\{P enters C | P enters infinite cylinder\}} \times \text{HOP}(T_C) \tag{2}$$

where $\text{HOP}(T_C)$ denotes the probability the pair of aircraft will have a horizontal overlap during the oceanic crossing.

2.3   Now to calculate Prob{P enters C | P enters infinite cylinder}, note that ℓ_{xy}, the average horizontal path length through a cylinder of radius λ_{xy}, is given by

$$\ell_{xy} = \pi \lambda_{xy}/2 . \tag{3}$$

If P has relative speed V_{rel}^C when it enters the infinite cylinder, it takes time

$$\tau_{xy} = \frac{\ell_{xy}}{V_{rel}^C}$$

(4)

to pass through the cylinder. During this time P moves vertically a distance $z = \overline{|\dot{z}|} \times \tau_{xy}$. Thus the effective thickness of the cylinder representing a collision is

$$2\lambda_z \times \left(1 + \frac{\overline{|\dot{z}|}}{2\lambda_z} \cdot \frac{\pi \lambda_{xy}}{2V_{rel}^C}\right).$$

2.4   Thus, if the instantaneous vertical overlap probability of two aircraft of height λ_z, nominally separated vertically by distance h_z when the horizontal overlap occurs, is given by $P_z(h_z)$, then

$$\text{Prob\{P enters C | P enters infinite cylinder\}} = P_z(h_z) \times \left(1 + \frac{\overline{|\dot{z}|}}{2\lambda_z} \cdot \frac{\pi \lambda_{xy}}{2V_{rel}^C}\right)$$

(5)

since $P_z(h_z)$ may be assumed to vary linearly with λ_z over the small distances involved.

2.5   To convert from collisions per pair to fatal accidents per flight hour, we multiply by $2 \times \text{NP}$, where NP is defined as the number of pairs per flight hour. Note that for longitudinal separation calculations, the number of pairs is essentially the same as the number of aircraft, so in this case $\text{NP} = 1/T_C$. For lateral separation calculations for intersecting tracks this will not necessarily be the case. The most pairs that could be achieved would occur when aircraft from one track interweave with aircraft on the other track in such a way that for every aircraft that crosses the intersection, an aircraft on the other track crosses just before it and another aircraft on the other track is the next to cross. This situation is not very likely, and in practice NP is often significantly smaller than unity. Thus, to keep the model as general as possible, we introduce the factor NP and write the final collision risk in units of fatal accidents per flight hour as

$$\text{CR} = 2 \times \text{NP} \times \text{HOP}(T_C) \times P_z(h_z) \times \left(1 + \frac{\overline{|\dot{z}|}}{2\lambda_z} \cdot \frac{\pi \lambda_{xy}}{2V_{rel}^C}\right).$$

(6)

2.6   Note that in general V_{rel}^C will depend on the separation minimum in use, the navigational accuracy of the aircraft, the angle between the headings of the two aircraft, as well as the time between position reports, and the communication and controller intervention buffer used. V_{rel}^C is the relative speed of the two aircraft conditional on a horizontal overlap taking place. It is not correct to take this as a fixed value, and previous models (see, for example, references 1, 4, 5, 6, 10, 15, 16, 17) that used fixed values of $\overline{|\dot{x}|}$ and $\overline{|\dot{y}|}$ have been in error.

2.7   Note also that the model presented here does *not* require the two aircraft to be in level flight. All that is required is an estimate of h_z, the nominal vertical separation when the horizontal overlap occurs. If the two aircraft *are* in level flight, then h_z is just the nominal vertical separation. If h_z is not known, an overestimate of the collision risk may be obtained by using $P_z(0)$ in equation 6 instead of $P_z(h_z)$ since $P_z(0) \geq P_z(h_z)$ for any h_z.

3.   HORIZONTAL OVERLAP PROBABILITY

3.1   General case

3.1.1   Consider a general situation where two aircraft are approaching an intersection on (in general) different tracks as shown in Figure A-1-1. In the case of identical tracks, where $\theta = 0$, the "intersection" is actually a waypoint on their common track. In a procedural environment, we assume that some time prior to the leading aircraft getting to the intersection, the controller would request distances to the intersection from both pilots, the leading aircraft responding first, so the difference in the reported distances will be an underestimate of the nominal separation. We let $t = 0$ be the time the pilot of the second aircraft provides this report. In an ADS environment, we assume that the ground system or the controller measures from the possibly extrapolated positions of each of the aircraft to the intersection. Only in the case of identical tracks is it possible to measure the distance directly between the two aircraft. We let $t = 0$ be the time-stamp in the ADS position report that was last received from either aircraft. When analysing ADS separation minima, we will assume that both aircraft send their position reports at the same time. This is a conservative assumption because when the reports are not simultaneous, the ADS system needs to extrapolate only to the report time of the next aircraft of the pair to report. Since risk reduces substantially with decreasing extrapolation time, the effect of non-simultaneous reports is to reduce the risk estimate.
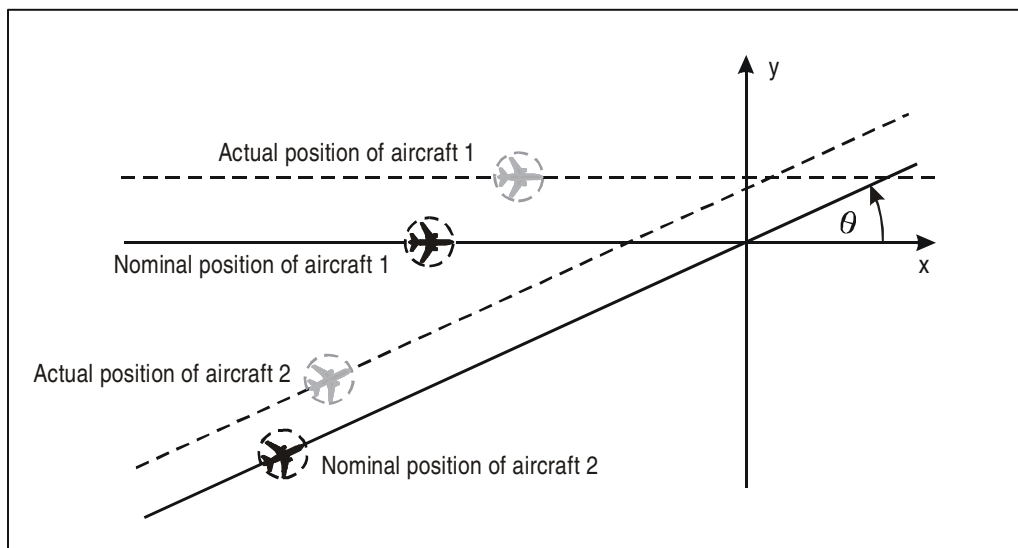
Figure A-1-1.   Nominal and actual positions of the aircraft at time $t = 0$

3.1.2   We will denote the nominal distances to the intersection at time $t = 0$ of aircraft 1 and 2, respectively, by $\hat{d}_1^0$ and $\hat{d}_2^0$. Then if ε_1^A and ε_2^A are the along-track errors of the two aircraft, and ε_1^C and ε_2^C are the cross-track errors, the coordinates of the actual positions of the two aircraft at time t will be given by

$$x_1(t) = -\hat{d}_1^0 + \varepsilon_1^A + V_1 t \tag{7}$$

$$y_1(t) = \varepsilon_1^C \tag{8}$$

and

$$x_2(t) = -\left(\hat{d}_2^0 - \varepsilon_2^A\right)\cos\theta - \varepsilon_2^C \sin\theta + V_2 t \cos\theta \tag{9}$$

$$y_2(t) = -\left(\hat{d}_2^0 - \varepsilon_2^A\right)\sin\theta + \varepsilon_2^C \cos\theta + V_2 t \sin\theta \tag{10}$$

where V_1 and V_2 are the true ground speeds of the two aircraft.

3.1.3   Now *D(t)*, the distance between the centres of the two aircraft at time *t*, will be given by

$$D(t) = \sqrt{(x_1(t) - x_2(t))^2 + (y_1(t) - y_2(t))^2} \, , \tag{11}$$

and we wish to minimize *D(t)* for $0 \leq t \leq T + \tau$, where *T* is the time between periodic reports, and τ is the communication and controller intervention buffer used previously (see references 1, 4, 5, 6, 10, 15, 16, 17). A horizontal overlap will take place when D_{min}^C, the constrained minimum of *D(t)*, is such that

$$D_{min}^C \leq \lambda_{xy} . \tag{12}$$

Thus, the horizontal overlap probability is given by

$$\text{HOP} = \text{Prob}\left\{D_{min}^C \leq \lambda_{xy}\right\}. \tag{13}$$

Note that $D^2(t)$ is a quadratic in *t* and has the form

$$D^2(t) = D_0^2 + 2Bt + V_{rel}^2 t^2 . \tag{14}$$

D_0 is the true distance between the two aircraft at time $t = 0$, where

$$D_0^2 = \Delta x_0^2 + \Delta y_0^2 \tag{15}$$

and Δx_0 and Δy_0 are given by

$$\Delta x_0 \equiv x_1(0) - x_2(0) = \hat{d}_2^0 \cos\theta - \hat{d}_1^0 + \varepsilon_x \qquad (16)$$

and

$$\Delta y_0 \equiv y_1(0) - y_2(0) = \hat{d}_2^0 \sin\theta + \varepsilon_y . \qquad (17)$$

The error terms ε_x and ε_y are defined by

$$\varepsilon_x = \varepsilon_1^A - \varepsilon_2^A \cos\theta + \varepsilon_2^C \sin\theta \qquad (18)$$

and

$$\varepsilon_y = \varepsilon_1^C - \varepsilon_2^C \cos\theta - \varepsilon_2^A \sin\theta . \qquad (19)$$

V_{rel} is the magnitude of the true relative velocity vector, given by

$$V_{rel} = \sqrt{V_1^2 + V_2^2 - 2V_1 V_2 \cos\theta} \qquad (20)$$

and B is given by

$$B = \Delta x_0 (V_1 - V_2 \cos\theta) - \Delta y_0 V_2 \sin\theta . \qquad (21)$$

D_{min}, the *unconstrained* minimum of $D(t)$, occurs when

$$t = t_{min} = -B/V_{rel}^2 \qquad (22)$$

and after some algebraic manipulation can be written

$$D_{min} = \frac{\left| \Delta x_0 V_2 \sin\theta + \Delta y_0 (V_1 - V_2 \cos\theta) \right|}{V_{rel}} . \qquad (23)$$

When $\theta = 0$ and $V_1 = V_2$, the above needs some special attention because $V_{rel} = 0$. In this case the true distance between the aircraft is equal to D_0 for all t.

3.1.4   If t_{min} is not in the range 0 to $T + \tau$, the constrained minimum of $D(t)$ will be larger than D_{min}, and because $D(t)$ is a quadratic in t, we have the following:

If $t_{min} < 0$ then $D_{min}^C = D_0$

If $t_{min} > T + \tau$ then $D_{min}^C = D(T + \tau)$.

3.1.5 Denoting the nominal ground speeds of the two aircraft by $\hat{V}_1$ and $\hat{V}_2$, we define

$$v_1 = V_1 - \hat{V}_1 \tag{24}$$

and

$$v_2 = V_2 - \hat{V}_2 . \tag{25}$$

3.1.6 Previously (see references 1, 4, 5, 6, 10, 15, 16, 17), we used the "unplanned relative velocity", v, which is just

$$v = v_1 - v_2 . \tag{26}$$

This was satisfactory because only coincident tracks ($\theta = 0$) were being considered. For the model presented in this appendix, we require individual speed differences from nominal, so we fitted the data used previously for v by the convolution of two double exponential densities with mean zero and the same scale parameter, λ_v. In order to keep the ADS model essentially the same as the procedural model, we used the larger of the parameters so obtained, and one obtained by fitting individual aircraft ground speed differences from nominal (obtained from a sample of 10 318 ADS reports during 1994 and 2000). The value chosen was

$$\lambda_v = 5.82 \tag{27}$$

3.1.7 For computational purposes, we assume ε_1^A, ε_2^A, ε_1^C and ε_2^C are double exponential random variables with mean zero and scale parameter λ_n determined from the required navigation performance value. As indicated above, we also assume that v_1 and v_2 are double exponential random variables with mean zero and scale parameter λ_v. Unfortunately, even with these assumptions it is not possible to write down a simple algebraic form for HOP given in equation 13 except for the relatively simple cases of $\theta = 0$ and $\theta = 180°$.

3.1.8 Reference 3 proposed a Monte Carlo approach to numerically calculate HOP in the general cases. The Monte Carlo method used importance sampling and took account of the symmetry of the probability density functions to speed up the computations. Because of the small probabilities involved, it was necessary to generate a large number of samples when using the Monte Carlo approach. For example, for the longitudinal separation analyses, the equivalent of approximately 10^{11} or 100×10^9 samples were used.

3.1.9 One advantage of the Monte Carlo approach is that the correct value of V_{rel}^C was estimated along with the horizontal overlap probability. This was done by assuming that at the point of horizontal overlap the aircraft each have a random lateral speed, $\dot{y}$, whose probability density function can be approximated by a double exponential probability density function. The scale parameter of this double exponential density was chosen such that the convolution of two such densities in the identical track ($\theta = 0$) case would produce a value of $\overline{|\dot{y}|} = 20$, the value that has been used in previous analyses. Note that if the random variable $\dot{y}$ has a probability density function that is the convolution of two identical double exponential probability density functions with scale parameter λ, then $\overline{|\dot{y}|} = 3\lambda/2$. Thus reference 3 chose

$$\lambda = 40/3 . \tag{28}$$

3.1.10   An alternative to the Monte Carlo approach is the numerical technique described in reference 18. The results presented later in this appendix are based on this numerical technique. Although reference 18 does not provide a value for V_{rel}^C, this is not a serious problem in practice. As will be shown below, for $\theta = 0$ it is possible to derive a theoretical value. This value will suffice for angles smaller that 15 degrees. For larger angles, V_{rel}, given by equation 20, will be accurate enough. Note that V_{rel}^C only enters into the last factor of equation 6, and in general the factor is only slightly larger than unity, so high accuracy is not necessary.

3.2   Same track longitudinal separation

3.2.1   In this case we split the oceanic crossing into m reporting periods of duration T flying hours so that $T_C = mT$. We assume that the risk of collision in each reporting interval is the same so that the total risk is just m times the risk of collision in any one interval. Assuming the two aircraft are at the same nominal level, equation 6 can be written

$$\text{CR} = \frac{2}{T} \times \text{HOP}(T + \tau) \times P_z(0) \times \left(1 + \frac{\overline{|\dot{z}|}}{2\lambda_z} \cdot \frac{\pi\lambda_{xy}}{2V_{rel}^C} \right) \tag{29}$$

where $\text{HOP}(T + \tau)$ is the horizontal overlap during a time equal to one reporting period plus the communication and controller intervention buffer τ used previously (see references 1, 4, 5, 6, 10, 15, 16, 17), and we have used $\text{NP} = 1/T_C = 1/mT$. If two aircraft have significantly different nominal speeds, the assumption that the total risk for the oceanic crossing will be m times that for one reporting period will be somewhat pessimistic because the aircraft may only constitute a pair for less than m reporting periods.

3.2.2   When the numerical calculations indicate that the risk is largest for $\theta = 0$, then it will be more accurate to use the following good approximation to the horizontal overlap probability HOP. By taking $\theta = 0$ in equations 7, 8, 9 and 10, we can obtain

$$x_1(t) - x_2(t) = \hat{d}_2^0 - \hat{d}_1^0 + \varepsilon_1^A - \varepsilon_2^A + (V_1 - V_2)t \tag{30}$$

and

$$y_1(t) - y_2(t) = \varepsilon_1^C - \varepsilon_2^C . \tag{31}$$

3.2.3   Treating the x and y directions independently and taking $t = T + \tau$, since it maximizes the risk in this case, we can approximate the horizontal overlap probability by the product of the probability the aircraft are in longitudinal overlap or out of order at time $t = T + \tau$ and the lateral overlap probability. Thus

$$\text{HOP} = \text{LOP} \times P_y(0) \tag{32}$$

where $P_y(0)$ is the lateral overlap probability of two aircraft with wingspan $\lambda_y = \lambda_{xy}$, which are nominally on the same (identical) track, and the longitudinal overlap probability, LOP, is given by

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35D

$$\text{LOP} \approx \text{Prob}\left\{x_1(T + \tau) - x_2(T + \tau) \le \lambda_{xy}\right\}. \tag{33}$$

The approximation is quite good unless $T + \tau$ is significantly smaller than the values used in section 4.

3.2.4   The nominal longitudinal separation at time $t = T + \tau$ is given by

$$\hat{S} = \hat{d}_2^0 - \hat{d}_1^0 + \left(\hat{V}_1 - \hat{V}_2\right)(T + \tau), \tag{34}$$

so

$$x_1(T + \tau) - x_2(T + \tau) = \hat{S} + \varepsilon_1^A - \varepsilon_2^A + (v_1 - v_2)(T + \tau). \tag{35}$$

3.2.5   Using this result in equation 33 and carrying out the convolution involved, assuming the distribution of $\hat{S}$ is uniform between the limits A and B, where B is very much larger than A, we obtain:

$$\text{LOP} \approx \frac{1}{4(B - A)}\left\{S_1 \lambda_n \beta^2 \exp\left(\frac{\lambda_{xy} - A}{\lambda_n}\right) + S_2 \lambda_2 (1 - \beta)^2 \exp\left(\frac{\lambda_{xy} - A}{\lambda_2}\right)\right\} \tag{36}$$

where

$$\lambda_n = \text{RNP}/2.995732, \tag{37}$$

$$\lambda_2 = \lambda_v \times (T + \tau), \tag{38}$$

$$\beta = \lambda_n^2 / \left(\lambda_n^2 - \lambda_2^2\right), \tag{39}$$

$$S_1 = \frac{A - \lambda_{xy}}{\lambda_n} + 3 + 4(1 - \beta), \tag{40}$$

and

$$S_2 = \frac{A - \lambda_{xy}}{\lambda_2} + 3 + 4\beta. \tag{41}$$

Note that, in general, the nominal longitudinal separation at time t is given by

$$\hat{S}(t) = \hat{d}_2(t) - \hat{d}_1(t) \tag{42}$$

where $\hat{d}_1(t)$ and $\hat{d}_2(t)$ are the nominal distances to the intersection at time t, given by

$$\hat{d}_1(t) = \hat{d}_1^0 + \hat{V}_1 t \tag{43}$$

and

$$\hat{d}_2(t) = \hat{d}_2^0 + \hat{V}_2 t \ . \tag{44}$$

3.2.6   We assume that if $\hat{V}_2 > \hat{V}_1$, the controller will increase the separation between the aircraft at time $t = 0$ if necessary to ensure that the aircraft will still be correctly separated after time $T + \tau$. If, on the other hand, $\hat{V}_2 < \hat{V}_1$, the leading aircraft is nominally faster than the trailing one, and the risk of collision will be substantially reduced. For these reasons, as well as the one stated above concerning the sum of the risks in each reporting interval, for calculation purposes we will conservatively assume that the nominal speeds of the two aircraft are the same. For computational purposes, we also assume that $\hat{s}(0)$ is a random variable whose probability density function is a uniform density between the distance-based longitudinal separation minimum S_x and $S_x + 250$ (nautical miles). When $\theta = 0$ this implies that A $= S_x$ and B $= S_x + 250$.

3.2.7   As mentioned previously, when $\theta = 0$ it is also possible to derive a mathematical expression for V_{rel}^C. By definition

$$V_{rel}^C = \frac{E\left(|w| \mid s + w \le \lambda_{xy}\right)}{T + \tau} \tag{45}$$

where E denotes the expected value, $s = \hat{S} + \varepsilon_1^A - \varepsilon_2^A$, $w = v(T + \tau)$ and $v = v_1 - v_2$ as in equation 26. The conditional density of w is given by

$$g\left(w \mid s + w \le \lambda_{xy}\right) = g(w) H\left(\lambda_{xy} - w\right)/\text{LOP} \tag{46}$$

where

$$g(w) = \frac{1}{4\lambda_2} \exp\left(-|w|/\lambda_2\right) \cdot \left(|w|/\lambda_2 + 1\right), \tag{47}$$

and

$$H(s) = \begin{cases} \dfrac{\lambda_1}{4(B-A)} \exp\left(\dfrac{s-A}{\lambda_1}\right) \cdot \left(\dfrac{A-s}{\lambda_1} + 3\right), & \text{for } s \le A \\[4mm] \dfrac{s-A}{B-A} + \dfrac{\lambda_1}{4(B-A)} \exp\left(\dfrac{A-s}{\lambda_1}\right) \cdot \left(\dfrac{s-A}{\lambda_1} + 3\right), & \text{for } A < s << B. \end{cases} \tag{48}$$

Therefore

$$V_{rel}^C \approx \left| \int_{v=-M}^{\infty} vg(v)H(\lambda_{xy} - v)\mathrm{d}v \right| \Big/ (T + \tau) \cdot \mathrm{LOP}, \tag{49}$$

where M satisfies $A << M << B$.

3.2.8   Although it is possible to write down an analytical expression for V_{rel}^C, the expression is quite complicated. A simpler approach that is accurate enough for this purpose is to use numerical integration with $M = S_x + 50$. We also replace the upper integral limit by zero since the contribution from positive v values is negligible. Typical values obtained using this expression are given in section 4.

3.3   Reciprocal track longitudinal separation for ADS

3.3.1   The situation considered here is when two ADS-equipped aircraft pass each other at different levels on intersecting tracks. When it is determined, by measuring between the (possibly extrapolated) position symbols and the intersection for both aircraft, that the first aircraft is at least the distance-based longitudinal separation minimum S_x further from the intersection than the second aircraft, then either aircraft may climb or descend through the level of the other. This situation is depicted in Figures A-1-2 and A-1-3. In Figure A-1-2, S_z is the applicable vertical separation minimum. In RVSM airspace, S_z will be 1 000 feet, and in other airspace S_z will be 2 000 feet. We will take a worse-case situation and assume that the level change commences as soon as the nominal distance separation is achieved. Without loss of generality we assume aircraft 2 changes level. We will also assume that the level change commences when aircraft 2 is nominally time $T_{CL}/2$ from the intersection, where T_{CL} is the time to climb or descend two vertical separation minima. Thus as aircraft 2 nominally crosses the intersection it will nominally be at the same level as aircraft 1.

3.3.2   The basic mathematics is identical to the general case detailed previously, with sign changes to $\hat{d}_2^0$, ε_1^A, ε_2^A, ε_2^C, ε_2^C, V_1 and V_2. A change from the same track model is that we do not use the communication and controller intervention buffer τ in the reciprocal track analysis. It is assumed that if the controller is unable to contact the aircraft, or if the last ADS position report from either aircraft has been lost, the controller would demand an ADS report and/or contact the aircraft. Also, because aircraft 2 could start the level change at any time between ADS position reports, we will find the point of closest approach for t in the interval $\hat{t}_2 - T_{CL}/4$ to $\hat{t}_2 + T_{CL}/4$, and maximize the risk for $\hat{t}_2 = \hat{d}_2^0/\hat{V}_2$ between 0 and T.

3.3.3   The rationale for this is that after the level change commences, and while aircraft 2 is nominally separated vertically from aircraft 1 by more than half a vertical separation minimum, the vertical overlap probability will be small enough so that, in combination with the lateral overlap probability, the risk of collision will be negligible.

3.3.4   Note that for aircraft satisfying the RVSM MASPS, the vertical overlap probability of two aircraft that are nominally separated vertically by 500 feet is approximately 5.6×10^{-4}. For aircraft not satisfying the RVSM MASPS, the vertical overlap probability of two such aircraft nominally separated vertically by 1 000 feet is
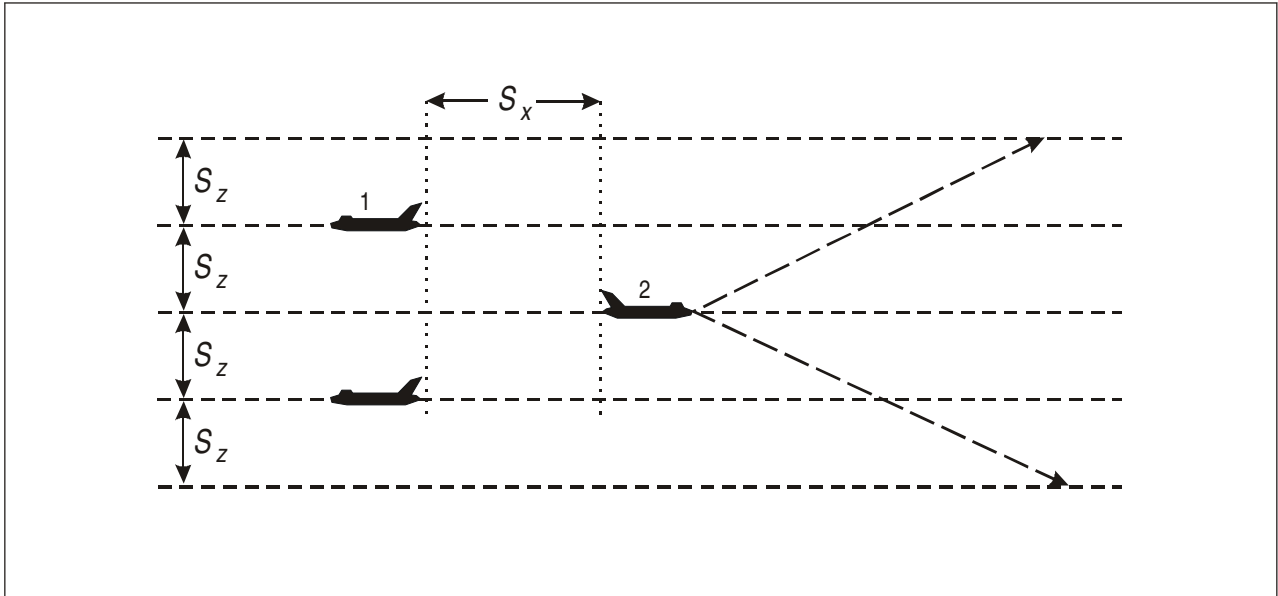
Figure A-1-2.   Side view of the reciprocal track scenario
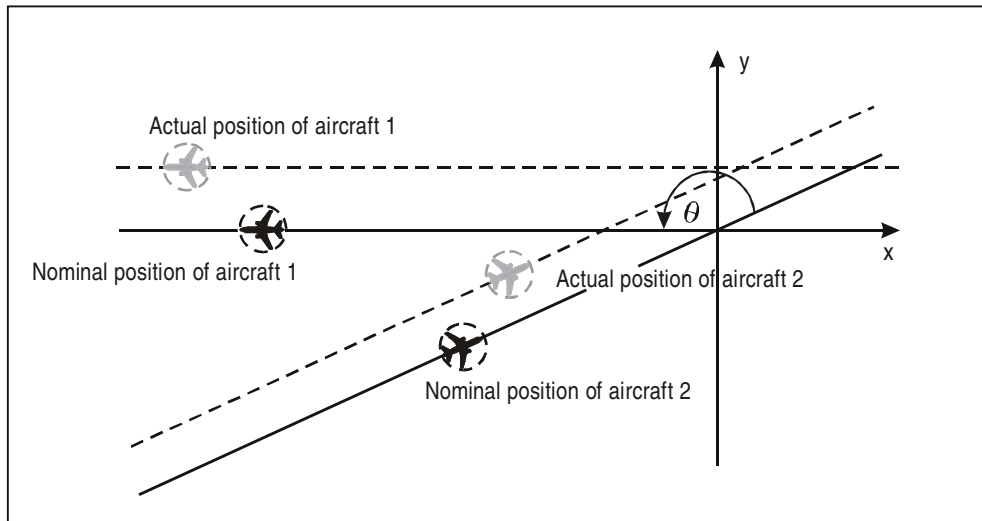
**Figure A-1-3.   Nominal and actual positions of aircraft
at time $t = 0$ (reciprocal tracks)**

approximately 9.3×10^{-6}. These values are based on modelling the vertical errors by Gaussian-double exponential mix densities as in reference 8. In the calculations we use $P_z(0)$ for the vertical overlap probability because the actual nominal vertical separation when a horizontal overlap occurs is not known, and as explained in section 2, $P_z(0)$ gives an overestimate of the collision risk.

3.3.5   A further change from the same track model is that instead of assuming the initial separation between the aircraft is a random variable with a uniform probability density, as in the previous section, we assume, as mentioned above, that the second aircraft will commence its level change as soon as the first aircraft is the longitudinal separation minimum S_x further from the intersection than the second aircraft (and getting further away).

3.3.6   The final item that requires some discussion is the value of NP, the number of pairs per flight hour. For longitudinal distance-based separation, as explained above, the appropriate value is $1/T$. We will assume for the type of procedure that we are analysing here that aircraft would not be changing levels in this manner more frequently than once every reporting period and hence use the same factor, although this is almost certainly overly pessimistic. The collision risk equation in this case is then the same as equation 29, with the changes mentioned above.

3.3.7   If, as indeed will turn out to be the case, the risk is maximized for $\theta = 180°$, then it is possible to produce a good approximation to the horizontal overlap probability HOP in a similar manner as we did previously for $\theta = 0$. By taking $\theta = 180°$ and changing the sign of $\hat{d}_1^0$ in equations 7, 8, 9 and 10, we can obtain

$$x_1(t) - x_2(t) = \hat{d}_1^0 - \hat{d}_2^0 + \varepsilon_1^A + \varepsilon_2^A + (V_1 + V_2)t \tag{50}$$

and

$$y_1(t) - y_2(t) = \varepsilon_1^C + \varepsilon_2^C . \tag{51}$$

3.3.8   Treating the x and y directions independently, and taking $\hat{t}_2 = T$, since it maximizes the risk in this case, we can approximate the horizontal overlap probability by the product of the probability the aircraft are in longitudinal overlap or out of order at time $T - T_{CL}/4$ and the lateral overlap probability $P_y(0)$. Thus

$$\text{HOP} \approx \text{Prob} \left\{ x_2(T - T_{CL}/4) - x_1(T - T_{CL}/4) \le \lambda_{xy} \right\} \times P_y(0) . \tag{52}$$

3.3.9   The nominal longitudinal separation at time $T - T_{CL}/4$ is given by

$$\hat{S} = S_x + \frac{T_{CL}}{4}(\hat{V}_1 + \hat{V}_2), \tag{53}$$

so

$$x_1(T - T_{CL}/4) - x_2(T - T_{CL}/4) = \hat{S} + \varepsilon_1^A + \varepsilon_2^A + (v_1 + v_2)(T - T_{CL}/4). \tag{54}$$

3.3.10   Using this result in equation 52 and carrying out the convolution involved in the longitudinal overlap term, we obtain

$$\text{HOP} = \frac{P_y(0)}{4}\left\{S_1\beta^2\exp\left(\frac{-\hat{S}+\lambda_{xy}}{\lambda_n}\right)+S_2(1-\beta)^2\exp\left(\frac{-\hat{S}+\lambda_{xy}}{\lambda_2}\right)\right\},\tag{55}$$

where,

$$\lambda_{n,} = \text{RNP}/2.995732,\tag{56}$$

$$\lambda_2 = \lambda_{v,}\times(T-T_{CL}/4),\tag{57}$$

$$\beta = \lambda_n^2\big/\left(\lambda_n^2-\lambda_2^2\right),\tag{58}$$

$$S_1 = \frac{\hat{S}-\lambda_{xy}}{\lambda_n}+2+4(1-\beta),\tag{59}$$

and

$$S_2 = \frac{\hat{S}-\lambda_{xy}}{\lambda_2}+2+4\beta.\tag{60}$$

3.4   Lateral separation on intersecting tracks

3.4.1   Several versions of a mathematical methodology applicable to intersecting tracks have been presented previously (see references 2, 7 and 9). Considerable debate has taken place as to the validity of those methodologies. As a result of that debate the present methodology is based on the more robust methodology presented in section 3.1.

3.4.2   Lateral separation of aircraft on intersecting tracks is based on the concept of a defined area of conflict around the intersection. The area of conflict is a quadrilateral (see Figure A-1-4), the corners of which are known as lateral separation points, defined as the points on a track where the perpendicular distance to the other track is equal to the lateral separation minimum, which we will denote S_y. Lateral separation is achieved by the controller ensuring that two aircraft will not be simultaneously within the area of conflict at the same level.

3.4.3   Suppose two aircraft are both approaching the intersection as in Figure A-1-5. We will assume that aircraft 1 will (nominally) get to the intersection first. A distance-based procedure for ensuring the aircraft are laterally separated is for the controller to ask both pilots for distances to the intersection before it is estimated that the second aircraft will get within, say, half a longitudinal separation minimum of the lateral separation point it is approaching. As with longitudinal separation, in a procedural environment, the controller should ensure an underestimate of the nominal separation by ensuring the leading aircraft responds first.
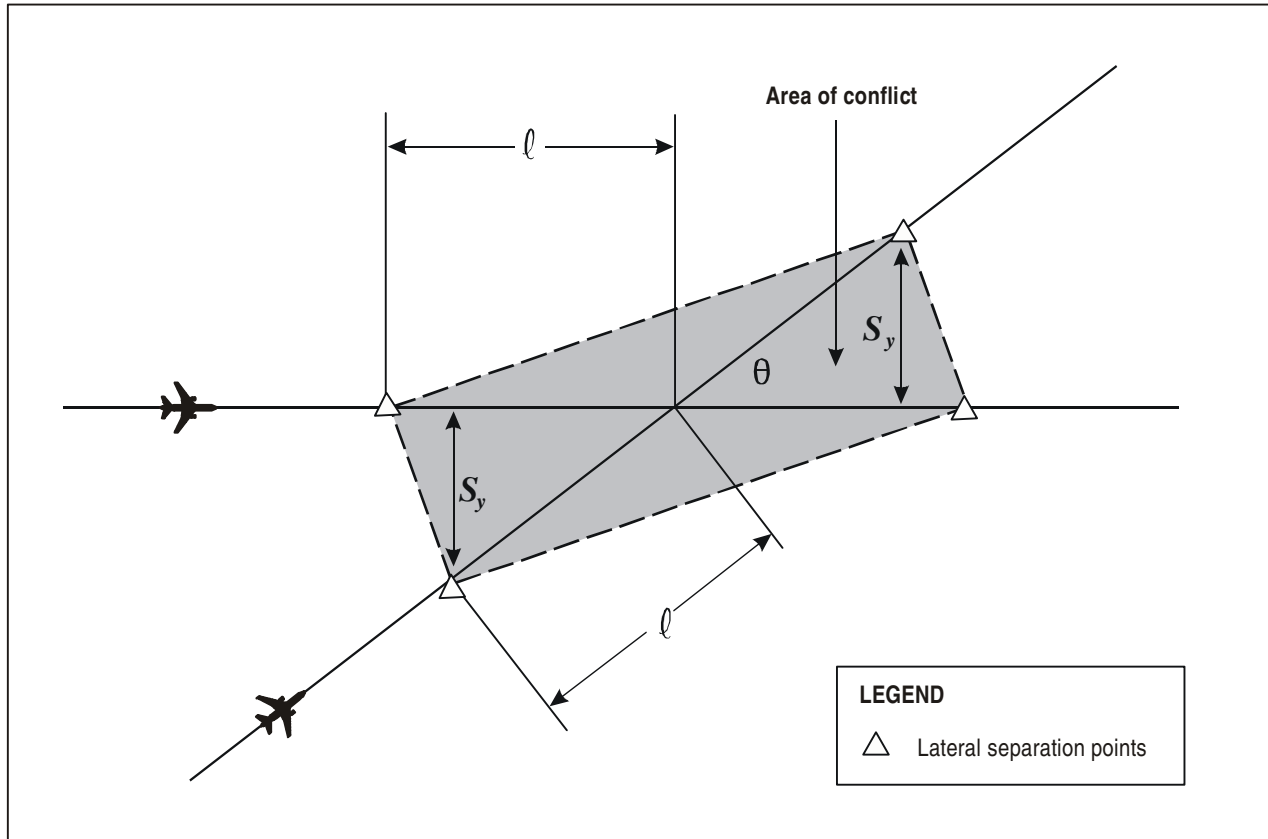
Figure A-1-4. Lateral separation points and the area of conflict

3.4.4 In an ADS environment the estimates may be based on (possibly extrapolated) position information. Based on reported or calculated distances to the intersection, the nominal ground speeds of the two aircraft, and the known reporting time, the controller calculates t_2^E, the time of entry of aircraft 2 into the area of conflict, and t_1^E and t_1^L, the times of entry and exit of aircraft 1 to and from the area of conflict. If $t_1^E \le t_2^E \le t_1^L$, then the aircraft will be simultaneously in the area of conflict at some time, and so aircraft 2 will be required to be at a vertically separated level by the lateral separation point. Note that some States, for example Australia, require the second aircraft to be at a vertically separated level by distance $S_x/2$ from the lateral separation point, or, equivalently, by distance $\ell + S_x/2$ from the intersection, where ℓ, the distance of the lateral separation point from the intersection, is determined from

$$\ell = S_y \big/ \sin \theta .$$ (61)

3.4.5 This appendix, however, does not use this extra requirement, assuming only that the aircraft will not be permitted to be simultaneously in the area of conflict at the same level. The results presented in section 4 indicate that the target level of safety will be met without it.
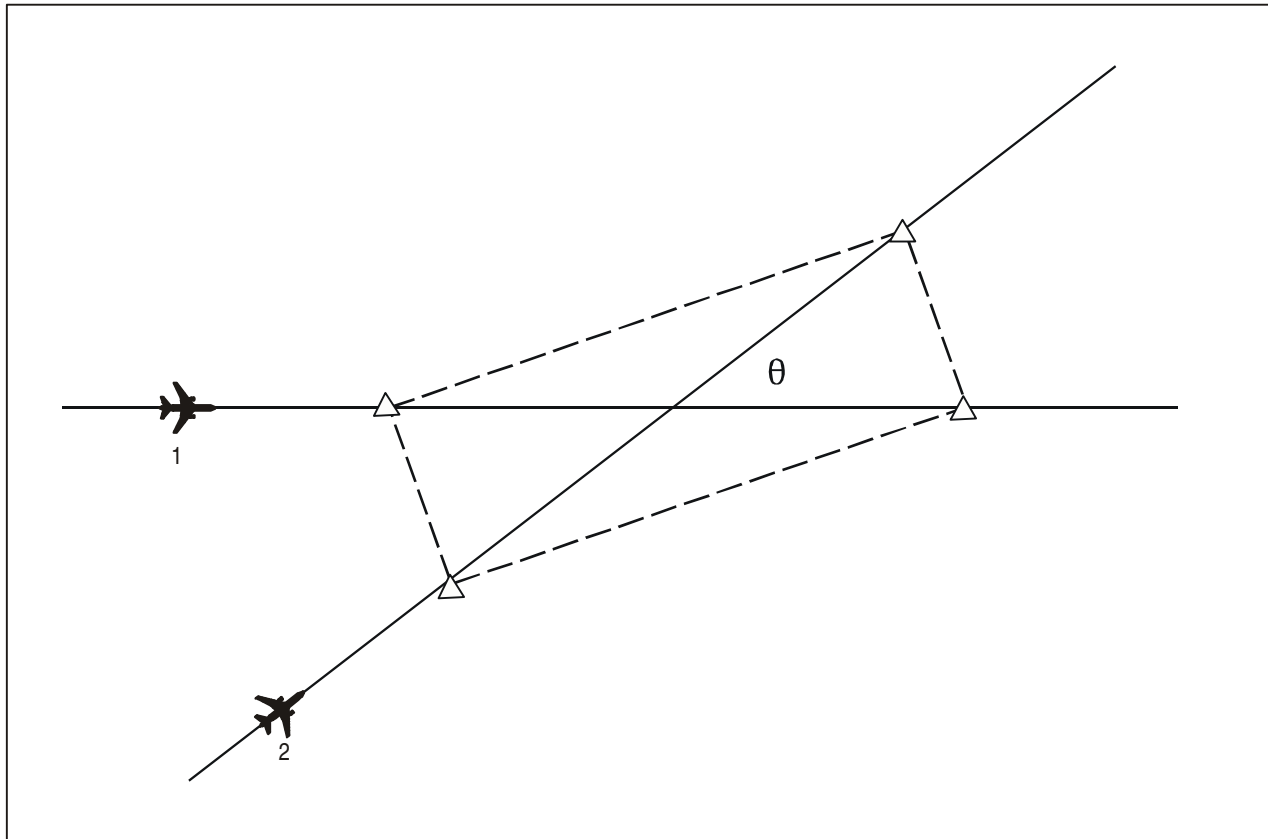
Figure A-1-5.   Both aircraft approaching the area of conflict

3.4.6   There are two different cases to deal with here:

a)   *Both aircraft approaching the area of conflict.* For the analysis of this situation, we will take as a worse case that both aircraft have equal nominal speeds and are nominally as close as possible after time $T + \tau$. Thus we assume that at time $t = 0$ that

$$\hat{d}_1^0 = \ell + \hat{V}_1(T+\tau) \tag{62}$$

and

$$\hat{d}_2^0 = \hat{d}_1^0. \tag{63}$$

Because the aircraft could report at any time prior to entering the area of conflict, we take the maximum risk value with respect to T. Note that, in reality, aircraft 2 would be required to be at a vertically separated level by distance ℓ from the intersection, but we conservatively assume that it is at the same level as aircraft 1 until it is distance ℓ from the intersection and then is instantaneously at a vertically separated level. The situation is shown in Figure A-1-5. The analysis of this situation is similar to that for longitudinal separation, except that the nominal

distances of the aircraft to the intersection at time $t = 0$ are different. The value of NP in the basic collision risk model also needs some discussion here. The worse case would be that every aircraft on one track is paired with an aircraft on the other track. Clearly they could be, but not all pairs would then be at the minimum separation considered for this analysis. In fact, on average, the difference in nominal distances of the pairs to the intersection would be at least $S_x/2$. Further, as pointed out in section 2, in practice NP is often significantly smaller than 1.

b) *One aircraft leaving the area of conflict as another is entering.* This situation is depicted in Figure A-1-6. The analysis is the same as for the basic case, but, of course, the nominal separation is different. As a worse case we will assume both aircraft are nominally at the same level and that aircraft 1 is nominally leaving the area of conflict as aircraft 2 is entering. Thus, we take

$$\hat{d}_1^0 = \hat{V}_1 T$$

(64)

and

$$\hat{d}_2^0 = \hat{V}_2 T + \ell \times \left(1 + \hat{V}_2 / \hat{V}_1\right).$$

(65)
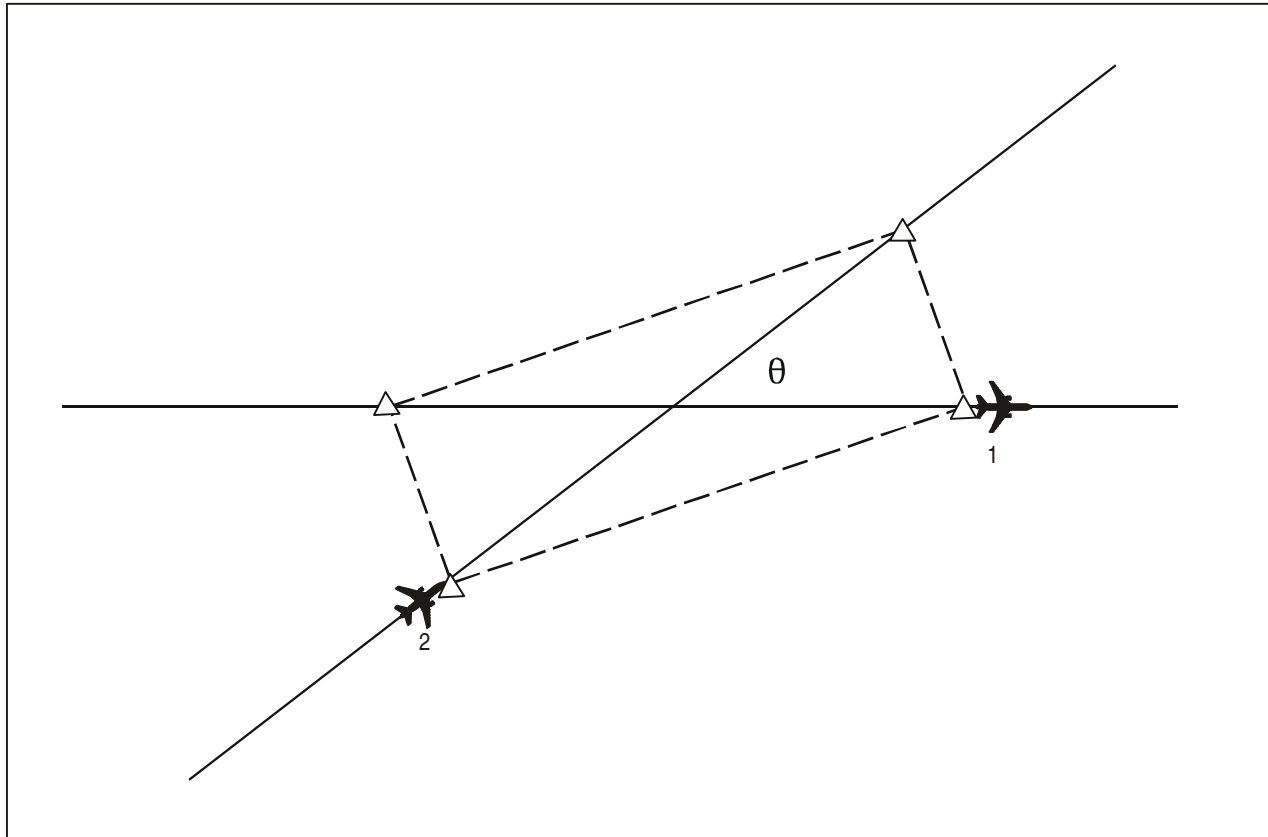
Figure A-1-6.   One aircraft leaving the area of conflict as another is entering

To maximize the risk over all possible reporting times, we assume the aircraft report when aircraft 1 is time *T* before the intersection, and we maximize the risk over two reporting periods. Note that when the angle of intersection is 45 (or 135) degrees, it typically takes less than 20 minutes for an aircraft to traverse the area of conflict when the lateral separation minimum is 50 NM, and less than 11 minutes when the lateral minimum is 30 NM.

4.   RESULTS

4.1   General

4.1.1   The results in this appendix were computed under essentially the same assumptions as for references 4, 8, 9 and 10, except for the following:

a) The Mach number technique is no longer assumed, although obviously controllers will still apply speed control to aircraft if required. Individual aircraft speed uncertainty is assumed to follow a double exponential probability density function, as explained in section 3.1.

b) Aircraft are assumed to be cylinders of diameter λ_{xy} and λ_z height. RNP 10 aircraft will be assumed to be of diameter 192.2 feet and height 54.8 feet, whereas RNP 4 aircraft will be assumed to be of diameter 231.8 feet and height 63.4 feet.

c) Because of the widespread use of RVSM, the assumed value of $P_z(0)$ has been increased to 0.48 for RNP 10 aircraft and 0.55 for RNP 4 aircraft.

d) The equivalent of $\overline{|\dot{x}|}$ and $\overline{|\dot{y}|}$, namely V_{rel}^C is used in the calculations. As noted above, in general the value depends on the separation minimum in use, the value of $T + \tau$, the RNP value of the aircraft, as well as the angle between the tracks of the two aircraft. When the risk is largest at $\theta = 0$, we use the theoretical value given by equation 49. Typical values are presented in Table A-1-1 for RNP 4. When θ is close to zero (less than 15 degrees) we use the values in Table A-1-1. When θ is not close to zero, sufficient accuracy may be obtained by using the unconditional relative velocity, i.e.

$$V_{rel}^C = \sqrt{V_1^2 + V_2^2 - 2V_1V_2 \cos \theta} \tag{66}$$

Table A-1-1. V_{rel}^C values for $\theta = 0$, RNP4

	T = 32 min, S_x = 50 NM	T = 14 min, S_x = 30 NM
τ = 4 min	91.9 kt	87.7 kt
τ = 10.5 min	80.5 kt	76.6 kt
τ = 13.5 min	76.2 kt	71.1 kt

4.1.2   Note that, in particular, we calculate the risk for two values of the communication and controller intervention buffer for procedural separation, namely $\tau = 6$ minutes and $\tau = 12.5$ minutes, and for three values for ADS, namely $\tau = 4$ minutes, $\tau = 10.5$ minutes and $\tau = 13.5$ minutes. As explained in references 4 and 10, the final risk estimate is computed as a weighted average of the risk values for the separate τ values in the following way:

     a)   Procedural separation:

        Estimate $= 0.95 \times CR(\tau = 6) + 0.05 \times CR(\tau = 12.5)$

     b)   ADS separation:

        Estimate $= 0.95 \times (0.95 \times CR(\tau = 4) + 0.05 \times CR(\tau = 10.5) + 0.05 \times CR(\tau = 13.5)$.

4.1.2.1   We also use a general value of $\overline{|\dot{z}|} = 1.5$ knots for RNP 10 analyses and $\overline{|\dot{z}|} = 1.0$ for RNP 4, except for the reciprocal track analysis where we use $\overline{|\dot{z}|} = 5.0$ since one of the aircraft is changing levels.

4.2   Longitudinal separation

The RNP 10 results presented in Table A-1-2 for procedural separation were computed assuming a reporting interval of 24 minutes, whereas the ADS results in Table A-1-3 assumed 27 minutes. The RNP 4 results in Table A-1-4 were computed using a reporting interval of 14 minutes for a 30 NM separation minimum and 32 minutes for a 50 NM minimum. These values are different to those proposed previously for several reasons. One is the change in the basic collision risk model to take proper account of non-identical tracks. The calculations that were carried out indicated that the risk generally increases with the track intersection angle θ, except for RNP 4 and the larger $T + \tau$ values, where the opposite was the case. In some cases for RNP 10 the risk near $\theta = 45$ degrees was almost three times that for $\theta = 0$. Another reason for the differences from previous results is the use of significantly larger values for the vertical overlap probability, as explained above.

Table A-1-2.   Results for RNP 10 procedural longitudinal separation
(The risk figures are in units of fatal accidents per flight hour)

Separation minimum (NM)	Required navigation performance	Maximum reporting period (min)	Risk for $\tau = 6$ min	Risk for $\tau = 12.5$ min	Weighted average risk
50	10	24	4.9×10^{-9}	5.8×10^{-9}	4.9×10^{-9}

Table A-1-3.   Results for RNP 10 ADS separation
(The risk figures are in units of fatal accidents per flight hour)

Separation minimum (NM)	Required navigation performance	Maximum reporting period (min)	Risk for $\tau = 4$ min	Risk for $\tau = 10.5$ min	Risk for $\tau = 13.5$ min	Weighted average risk
50	10	27	4.0×10^{-9}	8.2×10^{-9}	8.2×10^{-9}	4.4×10^{-9}

Table A-1-4.   Results for RNP 4 ADS separation
(The risk figures are in units of fatal accidents per flight hour)

Separation minimum (NM)	Required navigation performance	Maximum reporting period (min)	Risk for $\tau = 4$ min	Risk for $\tau = 10.5$ min	Risk for $\tau = 13.5$ min	Weighted average risk
30	4	14	3.6×10^{-10}	1.6×10^{-8}	5.7×10^{-8}	3.9×10^{-9}
50	4	32	1.4×10^{-9}	1.3×10^{-8}	2.8×10^{-8}	3.3×10^{-9}

Table A-1-5.   Results for ADS reciprocal track longitudinal separation
(The risk figures are in units of fatal accidents per flight hour)

Required navigation performance	Longitudinal separation minimum (NM)	Vertical separation minimum (ft)	Maximum ADS reporting period (min)	Time to change level (min)	Risk estimate
10	50	2 000	27	8	1.7×10^{-10}
10	50	1 000	27	4	3.2×10^{-9}
4	30	2 000	14	8	3.9×10^{-16}
4	30	1 000	14	4	1.1×10^{-12}

4.3   Reciprocal track longitudinal separation for ADS

As mentioned in section 3.3, it turns out that the risk for this case was maximized at $\theta = 180°$. Therefore we can use the results based on the analytical formula given in equation 55. The results are presented in Table A-1-4. Again, the RNP 10 results use a vertical overlap probability of 0.48 and the RNP 4 results use a

value of 0.55. Also, because one aircraft is changing levels we use a $\overline{|\dot{z}|}$ value of 5 knots. This is approximately 500 feet per minute, a typical climb performance. The figures quoted in Table A-1-5 are for nominal aircraft speeds of 300 knots because that gives the largest risk values, although it is most unlikely that both aircraft would have ground speeds this slow if they were on reciprocal tracks. The results were calculated for one reporting period for each RNP value because, the larger the reporting period, the larger the risk due to the larger extrapolation errors. It was also assumed that in RVSM airspace the level change would take 4 minutes, and in conventional airspace this would take 8 minutes.

4.4   Lateral separation

The results for both parts of this section have been calculated assuming a value for NP, the number of pairs per flight hour, of 0.5. An analysis of aircraft reporting times at intersections in the Tasman Sea area, based on both historical data from 1993 and 1994, as well as simulated data based on six weeks of flight plans from 1998 and 1999, gives an NP value of approximately 0.02 over all intersections. Thus the results presented should be conservative by a factor of approximately 25 for the Tasman and therefore should also be applicable to airspace that has significantly more traffic than the Tasman. Note that the NP factor allows for aircraft that take part in multiple pairings at various intersections as they traverse the airspace. Note also that all pairs are assumed to be at minimum separation. This again is somewhat conservative.

a)   *Both aircraft entering the area of conflict.* The results based on the methodology of reference 18 are given in Table A-1-6. Computations were carried out for angles between 15 and 135 degrees. The risk was largest at $\theta = 15$ degrees.

b)   *One aircraft entering the area of conflict as another is leaving.* The results given in Table A-1-7 were computed using a variety of combinations of aircraft speeds as shown. Calculations were carried out for angles between 15 and 135 degrees. For RNP 10 the risk was maximized at $\theta = 135$ degrees in all cases; however this was not always the case for RNP 4.

**Table A-1-6.   Results for lateral separation of aircraft on intersecting tracks,
where both aircraft are approaching the area of conflict
(The risk figures are in units of fatal accidents per flight hour)**

Required navigation performance	Lateral separation minimum (NM)	Maximum reporting period (min)	Estimated risk
10	50	24	1.1×10^{-9}
10	50	27	1.1×10^{-9}
4	30	14	4.4×10^{-13}

Table A-1-7.   Results for lateral separation of aircraft on intersecting tracks,
where one aircraft is entering area of conflict while another is leaving
(The risk figures are in units of fatal accidents per flight hour)

Required navigation performance	Lateral separation minimum (NM)	Maximum reporting period (min)	Nominal speed of first aircraft (kt)	Nominal speed of second aircraft (kt)	Estimated risk
10	50	27	300	600	2.4×10^{-9}
10	50	27	480	480	1.4×10^{-9}
10	50	27	300	300	1.7×10^{-9}
10	50	27	600	300	1.6×10^{-9}
4	30	14	300	600	6.4×10^{-11}
4	30	14	480	480	3.0×10^{-11}
4	30	14	300	300	2.2×10^{-12}
4	30	14	600	300	3.6×10^{-13}

5.   ACKNOWLEDGEMENTS

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Appendix 2
ROUTE STRUCTURE PLANNING

This appendix describes two different route reviews. The first is a description of the ATS route network improvements strategy within the context of the European ATC harmonization and integration programme. The second is a description of a review undertaken by the Civil Aviation Authority of Australia of ATS and RNAV air route structures.

PART 1: ATS ROUTE NETWORK IMPROVEMENTS STRATEGY AND METHODOLOGY WITHIN THE CONTEXT OF THE EUROPEAN AIR TRAFFIC CONTROL HARMONIZATION AND INTEGRATION PROGRAMME (EATCHIP)

1. BACKGROUND

1.1 As early as 1988, it was recognized that in the European Region, which comprises many States, some of them very small, improvements to the ATS route network needed to be made on a regional basis as opposed to each individual State working independently to address its particular problem areas. A series of meetings took place involving airspace planners from as many as fifteen States at any one time. As a result, an initial effort was agreed among all European States to create what was named the "ARN Trunk Route Network". This was an agreement to use RNAV capability to try to shorten route lengths for ATS routes above FL 300. The effort met with limited success, but served as a forerunner to more developed strategies.

1.2 Since 1992, European States within the context of the European ATC Harmonization and Integration Programme (EATCHIP), have been working to increase ATS capacity through more efficient airspace management and the optimization of the ATS route network. The work is done by a group of airspace planners from across the region who ensure coherency, compatibility and practicality in the planning and development of route network and airspace structure improvements.

1.3 This appendix explains the methodology used to develop improvements on a regional basis. It is provided as an example of how such work can be done on a regional basis in a region that consists of many States, each with its specific problems and its own civil and military airspace requirements.

2. METHODOLOGY

2.1 The method employed within EATCHIP employs five basic steps:

a) *cooperative planning* between all concerned parties — civil and military ATS providers and users;

b) agreement on a *set of regional principles and criteria,* which take into account as much as possible national requirements;

c) the definition of a *single reiterative methodology* for planning and development of improvements, i.e. a repeatable, evolutionary approach to changing the route network;

d) the *development of proposals* based on a systematic overall approach to regional planning; and

e) the use of specialist groups to *validate proposals* and to gain approval of the States.

2.1.1 Cooperative planning

2.1.1.1 Extreme care has been taken to ensure that all parties concerned are involved in the planning process, from the earliest stages until final approval. The outline planning is done by a large group, the airspace and navigation team (ANT), which consists of civil and military airspace planners, civil and military users, and other ATS experts as necessary. The ANT makes high-level decisions and assigns work to a specialist sub-group, the route network develop-

ment sub-group (RNDSG) which can, where necessary, assign specific tasks to task forces. The RNDSG consists of a select group of members of the ANT. The RNDSG works on specific improvements to the route network and proposes them to the ANT.

2.1.1.2   In both the ANT and the RNDSG, it is important that all concerned States be invited to take part. Further, both civil and military authorities of each State are requested to participate actively. General aviation, scheduled and charter operators are also key players in the process. The ICAO Secretariat is another necessary partner in the process. In Europe, the process is coordinated by the EUROCONTROL agency. In other regions, this role might be taken up by an organization agreed on by the States.

2.1.2   Regional principles and criteria

2.1.2.1   A set of agreed principles and criteria should be established at the beginning of the process. These set out clearly the objectives of the process, what measures could be used, and the commitment of all concerned to achieve the stated common objectives. It was extremely important that this was done as early in the process as possible, to ensure that all participants clearly understood their level of commitment, and that national resources were made available to the process. To the extent possible, national and subregional requirements were taken into account.

2.1.3   Definition of a single, reiterative methodology

2.1.3.1   It must be recognized that in a large region of many States, improvements to the route network are necessarily evolutionary, as opposed to a "big-bang" change. It is almost impossible to revamp an entire network, given the attendant changes to the ATC infrastructure (sectorization, frequency allocation, etc.), in a single step. Accepting this evolutionary nature of proposed changes makes it necessary that the same method of developing proposals be repeated throughout the medium and long-term planning/development process. As a result, improvements to the route network/airspace structure are constantly being reviewed and proposals are developed continuously. This process can be seen in Figure A-2-1.

2.1.4   Development of proposals to amend the route network/airspace structure

2.1.4.1   The normal process involves the EUROCONTROL agency (coordinator) and planners from the portion of the region concerned to examine identified

"trouble spots", and to develop initial recommendations. These are then reviewed and further refined by the RNDSG. In the next step the RNDSG produces its version of the proposed route network/airspace structure to the higher level planners in the ANT. The ANT then reviews the proposals, and either asks the RNDSG to reconsider some parts of the proposal, or gives its approval to the proposal. Eventually the proposals are presented, with the ANT's recommendation, to the EATCHIP Project Board, which is a high-level group consisting of representatives of States. The Project Board, a policy making group, might either ask for more information or approve the proposals. Once the proposals are approved by the Project Board, they are then put into the normal ICAO consultation process. The advantage to this approach is that all provider States and airspace users have already been "bought into" the proposals before the ICAO process begins.

2.1.5   Specialist group for validation and approval

2.1.5.1   During the process defined above, specialist groups can be called together to review proposed changes to the route network/airspace structure. Normally, a specific "trouble spot" or group of trouble spots is addressed. The specialist group should consist of national experts, users, and others who are intimately familiar with the problem. The specialist group should undertake detailed studies into the proposed changes, including, for example, the effect on adjacent airspace, ATC capabilities, and forecast traffic demand. The studies may include fast-time and/or real-time simulations. It must be kept in mind that the proposed changes will need to be integrated into the route network/airspace structure and their effect on the entirety of the airspace must be considered.

2.1.6   Implementation

2.1.6.1   As the proposed changes are agreed upon, an implementation programme should be developed. This should include agreeing on an aeronautical information regulation and control (AIRAC) publishing date and a sequence for effecting the changes if the entire proposal cannot be put in place at the same time. At all times in the programme, coherency and continuity of the route network needs to be ensured. Once the programme is agreed upon each State implements the changes on its agreed time-scale, with assistance as needed by the coordinating agency.

3.   CONCLUSION

3.1   Recognizing that national or subregional requirements are the immediate concerns of national civil and
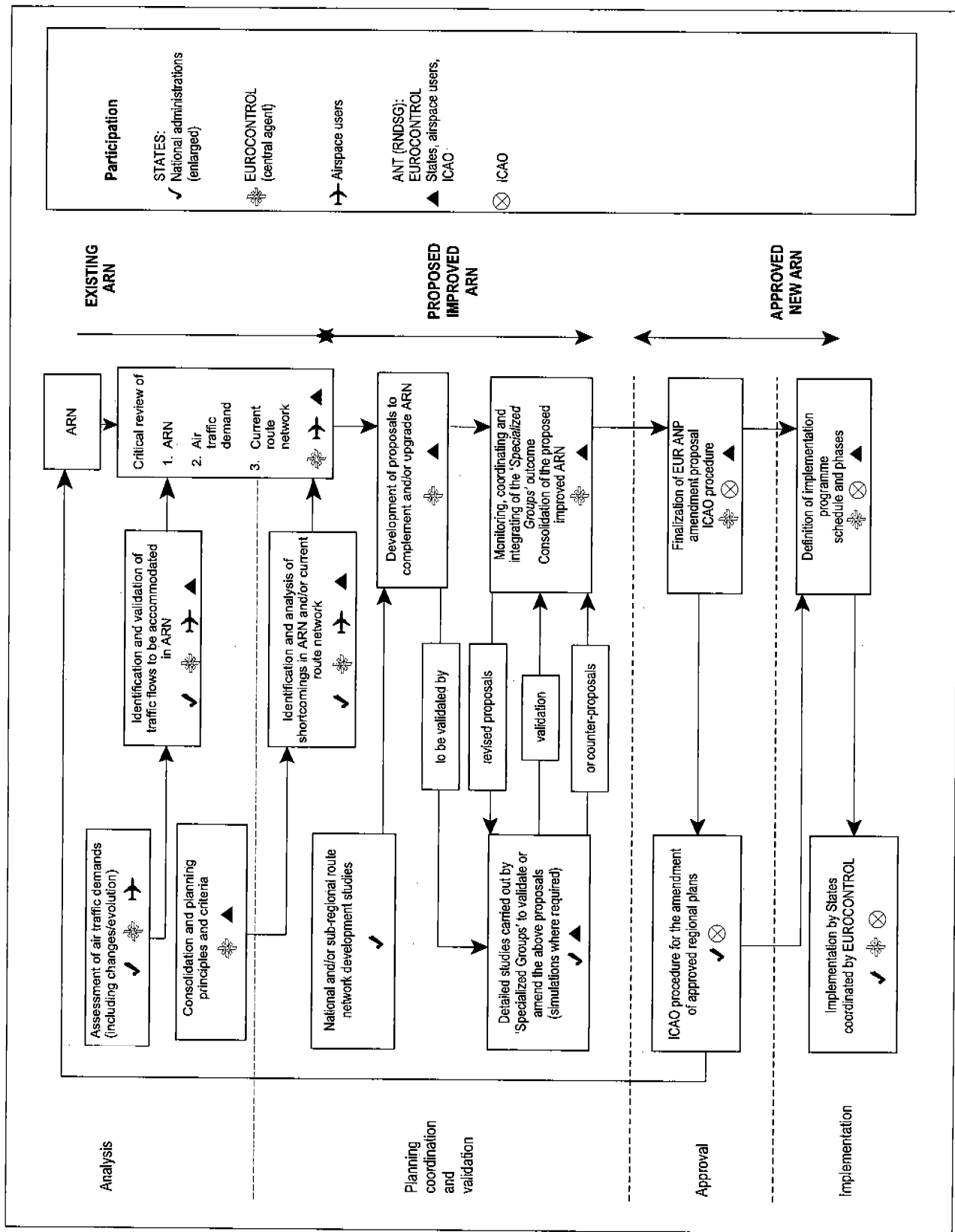
Figure A-2-1. Example of the reiteerative methodology

military authorities, the organization of a successful process is based upon recruiting the active support of all parties concerned at the beginning, and ensuring that their support and participation is assured in all steps in the process. It is important to achieve consensus in each step in the process. This consensus provides the confidence necessary to initiate the ICAO consultation process with some assurance that providers and users concerned have participated in the process leading up to an ICAO amendment proposal, are familiar with its contents, and are more likely to approve the amendment.

PART 2: ROUTE STRUCTURE PLANNING IN AUSTRALIA

1. INTRODUCTION

1.1   During late 1993, the then Civil Aviation Authority of Australia initiated a major review of its existing ATS and RNAV air route structures. Terms of reference, as outlined below, were developed and a review team was formed, comprising representatives from the Authority[1], the Department of Defence and the aviation industry, including relevant ATC and pilot industrial organizations. This review team was required to report to a Steering Committee comprising senior representatives from Air Services Australia, CASA and the Department of Defence.

1.2   At its first meeting during November 1993, the review team developed fourteen "working principles for the construction of air routes" (see 3 below), to form the basis of its redesign task. These principles were subsequently adopted by the ICAO Asia/Pacific Air Navigation Planning and Implementation Regional Group (APANPIRG) as "working guidelines for the construction of air routes", and have since been used to assist in similar reviews of air route structures over areas such as the South China Sea.

1.3   The new ATS and RNAV route structure for Australian domestic and oceanic air space, and for the Tasman Sea area between Australia and New Zealand, was designed using a "clean-sheet approach" and was successfully implemented, together with a revised ATC sectorization plan, on 14 September 1995. The new routes were

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1.   During July 1995, the responsibility for the provision of services and safety regulations were vested, by legislation, in two separate organizations, namely Air Services Australia, and the Civil Aviation Safety Authority (CASA).

implemented in accordance with a detailed "operational transition plan", which also detailed flight planning restrictions. After a two-week period, with the agreement of the ATC units and operators involved, this transition plan was cancelled and the significant safety and efficiency benefits of this new route structure began to be realized.

2. TERMS OF REFERENCE

2.1   The terms of reference for the review were:

a)   undertake a joint review of the ATS and RNAV route structures depicted in the Australian Aeronautical Information Publication (AIP);

b)   identify deficiencies in the existing ATS and RNAV route structures, and their depiction in AIP, and make recommendations, as may be required, for the early implementation of improvements in such route structures that provide:

1)   further opportunities for industry to achieve optimum performance, through exploitation of the capabilities of modern aircraft systems,

2)   further opportunities to implement:

i)   laterally separated one-way routes, e.g. Great Australian Bight route structure, and

ii)   flexible or dynamic tracking, e.g. South Pacific flex tracks,

3)   efficiencies for ATS, by taking account of present and future airspace management arrangements, including ATC sectorization, ATS system capabilities and separation minima, and

4)   an opportunity to rationalize the existing route structure and provide an improved chart presentation;

c)   consult with industry consultative forums, international and domestic operators, the Department of Defence, relevant divisions within Air Services Australia (e.g. facilities and engineering divisions), the regulatory authority, and industrial organizations; and

d)   consider recommendations from the independent consultant's report on route structure demand and capacity, together with the applicability of air route

and airspace management initiatives being pursued by forums such as the ICAO APANPIRG, the ICAO Indian Ocean Air Traffic Services Co-ordinating Group (IOACG) and the Informal South Pacific Air Traffic Services Co-ordinating Group (ISPACG).

3. WORKING PRINCIPLES FOR THE CONSTRUCTION OF AIR ROUTES

3.1 The fourteen working principles developed by the review team were as follows:

a) Air routes will satisfy appropriate ICAO Standards and Recommended Practices (SARPs).

b) Where possible, routes should be established to increase efficiency, reduce complexity and provide additional benefits to users.

c) Separation assurance principles should apply:

1) routes should be established with sufficient separation to operate independently,

2) where possible, routes in a radar environment should be procedurally (laterally) separated, and

3) segregated tracks should be established on medium/high density routes and be determined by set criteria.

d) Where required, routes should be constructed to support terminal area management procedures, e.g. SIDs/SRDs/STARs and flow management techniques, as applicable.

e) Holding patterns should be laterally separated from other tracks and tolerances captured within a single sector.

f) A maximum of two routes containing high density traffic should be blended at a single point. Inbound tracks should be blended at <90 degrees. Up to three low density traffic routes may be blended at a single point.

g) Multiple crossing points involving major traffic flows should be avoided.

h) En-route crossings should be minimized. Where crossings are inevitable they should, where possible, be established for cruise configuration. Such crossings should occur, wherever possible, within radar coverage.

i) Airspace sectorization should take account of the route structure and workload considerations. If necessary, airspace should be re-sectorized to accommodate changes to the air route configuration.

j) Routes should be constructed so as to reflect the optimum navigational capabilities of the principle users (e.g. RNAV or conventional).

k) The prime determinant should not be the minimum number of track miles. A small increase in track miles may optimize traffic flows, avoid unpredicted delays or avoid holding requirements.

l) Due allowance should be given to existing and future flight data processing/radar data processing capability (i.e. notification of messages for auto hand-off, etc.).

m) A periodic safety audit and review process of routes should be conducted to test both demand against capacity criteria and the principles. This should ideally be done in parallel with the annual sectorization review.

n) Routes that can no longer be justified should be deleted.

4. MAJOR FEATURES OF THE REVISED ROUTE STRUCTURE

4.1 The major features of the revised ATS and RNAV route structure included:

a) improved safety;

b) greater flexibility in flight planning long-haul domestic and international operations;

Note.— Industry studies indicated that benefits in excess of A$20 million annually would result, primarily from annual fuel-burn savings of approximately 25 000 tonnes.

c) significant efficiencies for ATC through a reduction in workload problems caused by lateral separation constraints on portions of the previous route structure;

d) introduction of a greater number of laterally separated one-way routes, providing "racetrack" patterns;

e) more appropriate representation of the type of operations that do operate in the upper airspace, i.e. RNAV equipped operators; and

f) AIP/MAP chart presentation that depicts both RNAV and ATS routes on the same chart.

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Appendix 3

COST-BENEFIT ANALYSIS STUDIES

1. INTRODUCTION

1.1   This appendix provides a brief description of the process undertaken for the initial cost-benefit analysis (CBA) of reduced vertical separation minima (RVSM) and reduced lateral routes spacing (RLRS) within European airspace. Tentative conclusions and recommendations are included at the end.

2. DEFINITION OF THE SCOPE AND PURPOSE OF THE CBA STUDY

2.1   Prior to beginning a CBA study it is essential to define what is to be analysed, the breadth of the study, and the final aim of the study. This involves specifying a framework and determining whether valid analyses can be undertaken from the data available.

2.2   Once the scope and purpose of the study have been decided, the next step is to construct a model, which accurately reflects the current environment and the environment that would exist if the proposed changes (in this case RVSM or RLRS) were to be introduced. An initial assessment of this model is made to determine if indications are positive enough to continue the study.

3. FORMULATION OF THE BASELINE ("DO NOTHING" REFERENCE) AND OPERATIONAL SCENARIOS

3.1   An important part in performing a CBA study is to determine what the prevailing costs and conditions will be in the future if the system is permitted to continue unchanged. It is important that the traffic forecasts are as accurate as possible for the period under consideration. In the case of RVSM versus RLRS in European airspace, the "do nothing" scenario assumed:

a)   that aircraft have at least RNP-5 navigational capabilities;

b)   that aircraft will not have RNP-1 navigational capabilities;

c)   that RVSM will only be present within the NAT;

d)   the vertical separation will be 2 000 ft for aircraft above FL 290 and the longitudinal separation between aircraft will be 5-10 NM; and

e)   that those improvements, which have been agreed upon on a regional basis for European ATS, will be completed by 1998, but neither RVSM nor RLRS will be in place.

3.2   Whilst defining alternative scenarios, the involvement of operational ATS and technical and airline operations (airspace user) personnel is of major importance. The scenarios considered within the study were:

a)   the introduction of RVSM:

1)   1 000 ft vertical separation will be applied to aircraft between FL 290 and FL 410,

2)   aircraft will have RNP-5 navigational capabilities,

3)   the agreed improvements, on a regional basis, to the European ATS will be introduced,

4)   MASPS for altimetry systems will be in place,

5)   ATS will monitor the height-keeping performance of the aircraft population,

6)   there is no requirement for RNP-1 in ECAC, and

7)   RVSM will be introduced between 1999 and 2003 and will continue until at least 2015;

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b) the introduction of full RLRS:

1) the carriage of RNAV equipment approved for RNP-1 will be mandatory on all routes in the area,

2) RLRS will be introduced to the ECAC upper airspace and the feeder routes to the TMAs,

3) EUROCONTROL equipment standards are in place,

4) the route spacing between centre lines will be 7-10 NM,

5) RVSM will not be in place, and

6) RLRS will be introduced between 2003 and 2007 and will continue until at least 2015; and

c) the introduction of minimal RLRS:

1) the mandatory carriage of RNP-1 approved equipment will be restricted to high-density routes,

2) there will be simple scaling down of full RLRS option with a percentage of the routes being provided solely for RLRS traffic, and

3) minimal RLRS would be introduced between 1998 and 2002 and would continue until at least 2015.

4. ANALYSIS OF BENEFITS

4.1   Once the scenarios have been defined, it is necessary to list the benefits to be expected from the proposed changes. In this step it is important that all interested parties (ATS providers, users, military, technical personnel, industry, etc.) are consulted, since expected benefits to one part of the aviation community may not be evident to other parties.

4.2   For European RVSM/RLRS changes, the list of potential benefit categories was established for each option and included:

a) RVSM

1) reduced delays (quantified via a simple queuing theory model at macro (system) level),

2) fuel savings in upper airspace (>FL 290) because aircraft can fly at flight levels closer to their optimal flight level, and

3) fuel savings through increased access to upper airspace; and

b) RLRS

c) reduced delays (again, quantified via the queuing model), and

d) fuel savings, because the actual route provided to the aircraft will be closer to its optimum route.

5. ANALYSIS OF COSTS

5.1   Once the benefits have been defined it is necessary to determine the costs of implementing each of the scenarios. Again, the determination of the likely costs involved must include all parties concerned. For all three options, the cost elements contain:

a) ATCO employment;

b) ATS provider capital (ATM/CNS) expenditures;

c) ATS provider maintenance and support;

d) ATS provider R&D expenditures;

e) aircraft operator capital expenditures;

f) aircraft operator maintenance and support; and

g) other (e.g. crew training).

6. CHOICE OF CBA INDICATORS

6.1   Various types of indicator or accounting methods can be used to evaluate the merits of investment alternatives. States or regions considering making changes to the environment would need to consult with experts in the field to determine the best method to use. For the European case, the classical net present value (NPV) supplemented by the benefit to cost ratio was chosen. Not all applications would necessarily best be analysed using this method. For this reason, various CBA indicators are explained below.

6.2   The most widely used and best indicator is the NPV, defined as the difference between the present value of benefits and present value of costs for the entire study period. One should select the option with the highest NPV.

6.3   Present value means that streams of benefits and costs over a project's lifetime are discounted using a *discount rate,* which represents a stated rate of return (in real terms) on capital expected in that sector of activity. This discount rate is independent of inflation and determines how rapidly the value today of a future real currency unit ($) falls away through time, just as a real rate of interest determines how fast the value of a $ invested now will increase (trade-off between current and future consumption).

6.4   An alternative indicator is the *internal rate of return* (IRR). This is the discount rate at which the net present value of a project is zero; that is, at which discounted benefits equal discounted costs. The decision whether or not to proceed with a project should in principle depend on whether the IRR is higher or lower than the target discount rate. This indicator is widely used in the private sector. Caution is needed when dealing with the multiple mathematical solutions produced by the method. In addition, it is not unusual for project ranking established by the IRR method to be inconsistent with those of the NPV criterion.

6.5   Another indicator is the *pay-back period,* also used extensively in the private sector. This is the number of years required to return the original investment. By itself, the pay-back period is an unsatisfactory indicator, because (except when applied to discounted cash flows) it takes no direct account of the timing of benefits and costs, and more seriously it takes no account of any net benefits occurring after the pay-back date. Used in conjunction with NPV (project's profitability), it will show how long the initial investment will be at risk, providing an indication of the risk of the project.

6.6   In cases where alternatives do not have the same economic or operational life, projects' net cash flows can be expressed as *annual equivalent values* (AEV). A cash flow item can be translated into a constant annual value for comparative purposes. This is a simple modification of the NPV approach.

6.7   Another investment criterion is the *benefit: cost (B:C) ratio.* This is defined as the present value of the benefits divided by the present value of costs. A proposed activity with a ratio of at least one will return as much in benefits as it costs to undertake. This corresponds to having a positive or zero NPV. While the B:C ratio provides an answer to the economic question of which alternative to select, it often fails to answer the question of how to best use limited financial resources for competing and mutually exclusive options. Only if all of the alternatives have the same present value of costs (most unlikely in practice) will selecting the ratio with the highest value produce the economically correct result.

7. SENSITIVITY ANALYSIS

7.1   At this point in the study, it is necessary to determine how detailed the results should be. The most important factor to consider is the level of confidence in the predictions made in the definition of the baseline and operational scenarios. For Europe, both low- and high-benefit scenarios were used as boundaries for the results and to cover the most pessimistic and optimistic views. Sensitivity parameters included:

a) the average cost to aircraft operators of a minute of delay;

b) the average ATC delay in 1995;

c) the maximum acceptable delay;

d) the annual traffic growth rate in unconstrained traffic demand;

e) the ratio of theoretical capacity over traffic demand in 1995 ("spare capacity");

f) the growth in theoretical capacity for 1995-1999 for the "do nothing" scenario;

g) the increase in ATC theoretical capacity for the new options;

h) the economic discount rate;

i) the price of aviation fuel;

j) the percentage decrease in fuel-burn rate under the new options;

k) the percentage distribution of users between "new/old" airspace;

l) the ATS cost recovery percentage from user charges;

m) the implementation schedule change (one year speed-up); and

n) the aircraft operator's cost sensitivity to:

   1) the retirement period of non-MASPS aircraft,

   2) the average additional cost of a new aircraft meeting RNP-1 requirements,

   3) the average upgrading cost to RNP-1 requirements,

4) the size of the fleet using ECAC airspace in 1995, and

5) the proportion of new aircraft meeting RNP-1 MASPS.

8. CONCLUSION

8.1   At this point, initial conclusions can be drawn based on the preceding steps. Depending on the knowledge available at the time of the analysis, and the amount of time until the proposed change to the system, it may be possible, or even necessary, to repeat the exercise at a later date, a so-called "go/no go" decision based on better knowledge. In Europe, it has been agreed that the study will be repeated in 1997 when more information will be available, especially on the costs to operators (MASPS) and to ATS providers in terms of height monitoring requirements.

8.2   It should be noted that some conclusions can only be considered provisional until they have been validated by, for example, simulations. As an example of the types of conclusions that can be drawn, the tentative conclusions in the European study are:

a)   delay reductions dominate in both the RVSM and RLRS cases. However, RVSM costs are lower and therefore RVSM is preferable to RLRS;

b)   fuel-burn savings are only secondary, and far below the savings made from reducing delays;

c)   delay benefits are large but widely dependent on the methodology used for the analysis (e.g. queuing model, cost of delay), the traffic growth and the assumptions on ATCO productivity gains (which are still to be validated by simulations);

d)   comparison of steady-state conditions show that, all other things being equal, RVSM is much more cost-effective than RLRS;

e)   assuming a continued increase in air traffic demand, RVSM will imply a significant ATCO personnel

increase and cost (up to 60 per cent of the additional costs) to deliver the increase in capacity;

f)   the benefits will only materialize if the terminal/ airport can accommodate the traffic growth. The real cost of airports' infrastructure was not accounted for in this study whereas all en-route benefits are theoretically captured;

g)   potential non-delay fuel savings from RVSM are much larger than from RLRS; and

h)   the costs of RVSM are lower than full RLRS (up to 50 per cent less). This is mainly due to the need for expensive RNP-1 equipage and certification costs with RLRS.

9. RECOMMENDATIONS

9.1   Each State or region that contemplates changing the ATM system would need to define the preferred next step(s) to take, i.e. what should be done with the conclusions. For example, in Europe, the next steps are:

a)   to validate the capacity increases assumed for the ATC system for all of the options (by simulations and other types of work). Only theoretical (paper) benefits have been derived so far. The potential for real and massive user benefits is nevertheless there, if the analysis is proven to be right;

b)   to produce a better (less crude) approach for modelling delays, their interaction with demand, and the impact on aircraft operator revenues and costs. Currently there is no accepted methodology to perform this task;

c)   to favour RVSM versus RLRS based on the results; and

d)   to extend the CBA to groups of users (other than commercial aircraft) or stakeholders to determine if any implications have been overlooked for these groups.

Appendix 4

THE INTRODUCTION OF A REDUCED LATERAL SEPARATION INTO THE NAT AIRSPACE

1. INTRODUCTION

1.1   The North Atlantic (NAT) minimum navigational performance specification (MNPS) airspace was introduced in 1977 as a result of a need for increased capacity. This increase in capacity was achieved through the application of reduced lateral separation. This appendix describes:

a) the steps undertaken prior to the introduction of a reduced lateral separation within the North Atlantic (NAT) airspace; and

b) the work undertaken on a yearly basis to ensure that the lateral separation applied continues to meet the target level of safety (TLS).

1.2   Section 2 provides a description of the NAT region and the formation of the North Atlantic Systems Planning Group (NAT SPG). Section 3 details the development of the MNPS for the NAT with the introduction of 60 NM lateral separation described in Section 4. Section 5 details the monitoring of the NAT airspace to ensure that the system continues to meet the target level of safety. In Section 6 a description of the models used to assess the safety of the lateral separation standard is provided.

2. BACKGROUND

2.1 The NAT region

2.1.1   The NAT track system provides the main east-west air corridor between Europe and North America. Due to passenger demands, time zone differences and airport noise restrictions, much of the NAT air traffic contributes to one of two flows — a westbound flow departing Europe in the morning and an eastbound flow departing North America in the evening. The effect of these flows is to concentrate most of the traffic unidirectionally with peak westbound traffic operating between 1130 and 1900 UTC and peak eastbound traffic operating between 0100 and 0800 UTC.

2.1.2   Due to variations of the weather patterns in the region, in particular the location of the eastbound jetstream, the most efficient route across the NAT varies on a daily basis. In order to provide the best service to the bulk of the traffic, a system of organized parallel tracks is constructed every 12 hours to accommodate as many aircraft as possible on or close to their optimum path. This system is known as the organized track system (OTS). Because of the constraints of large separation criteria and a limited economical vertical height band (FL 310-FL 390), the airspace is relatively congested during peak hours.

2.2 Organized track system

2.2.1   The OTS is constructed every 12 hours to accommodate as many aircraft as possible on or close to their minimum cost path. After determination of basic minimum time tracks, with due consideration to the airlines' preferences and any airspace restrictions in place, the OTS is constructed by the appropriate Oceanic Area Control Centre (OACC). Gander OACC is responsible for publishing the night OTS (eastbound) and Shanwick OACC the daytime system (westbound).

2.2.2   Once constructed, the OTS is published via the aeronautical fixed telecommunication network (AFTN) to all interested parties in Europe and North America.

2.2.3   Each OTS track is described in terms of way-points occurring every ten degrees of longitude (20° W, 30° W, 40° W, etc.), which are crossed at whole degrees of latitude. In addition to the 10-degree way-points, the entry and exit points at the oceanic boundaries are specified. The set of OTS tracks forms an essentially parallel structure with the most northerly daytime track

being designated track "A", the next most northerly track "B", etc. (Reference 2)[1]. In the night time the most southerly track is designated track "Z" and the next most southerly "Y", etc. All routes must be set in such a way that the separation minima are satisfied.

2.2.4 In some cases, the location of an aircraft's origin and/or destination would mean that using the OTS would require an unnecessary diversion. Such aircraft fly routes, known as "random" routes, which either remain clear of the OTS or only partly coincide with it. Random routes are described in a similar format to OTS tracks. Those aircraft flying completely in accordance with the OTS comprise the OTS traffic. Aircraft whose routes deviate from the OTS comprise the random traffic.

2.3 PROCEDURES

2.3.1 Whilst the eastern and western oceanic boundaries are covered by radar, the majority of the NAT region is outside radar coverage. Aircraft within MNPS airspace are therefore controlled procedurally.

2.3.2 Before entering the airspace, an aircraft must gain clearance for its requested route, speed and flight level from ATC. Its requested details are checked to see if it would, at any point in the crossing, result in a conflict with aircraft that have already been cleared. Any such conflicts must be resolved before the aircraft is given its clearance. The way-points of the cleared route are entered manually into the navigation system by the aircrew.

2.3.3 During the flight, the aircraft must, at each way-point, report to ATC its position, flight level and the time of crossing the way-point. The type of communication used is HF voice. ATC use this information to check that the aircraft is maintaining its cleared route. Occasionally an aircraft will request an en-route re-clearance. The aircraft is re-cleared only if it will not conflict with any other traffic at any point during the remainder of its crossing. Formal procedures, e.g. reading back of route details, for all communications are adhered to. For greater detail on the NAT MNPS environment, the reader is referred to References 1 and 2.

3. DEVELOPMENT OF THE NAT MNPS

3.1 In 1964 the lateral separation of tracks in the NAT region was 120 NM with a longitudinal separation of

15 minutes and a vertical separation of 1 000 ft below FL 290 and 2 000 ft above FL 290. It was proposed at this time that the lateral separation be reduced to 90 NM as a means of improving track system economics and availability. The proposal was not adopted due to a lack of confidence that adequate collision risk analyses had been undertaken.

3.2 ICAO set up the NAT SPG in 1965. The initial purpose of the NAT SPG was to undertake a study of lateral separation and to develop the framework of a decision-making process for subsequent reductions in separation standards. During its first meetings (in the period 1966-68) the NAT SPG, in conjunction with other ICAO groups, developed a method for assessing the safe separation between tracks in the NAT organized track system. In this method, data collected in the NAT region are used as an input to a mathematical model known as the Reich model (see Section 6), which calculates the relationship between collision risk and separation. On the basis of other, worldwide, information the NAT SPG also proposed a value for the maximum acceptable collision risk, known as the target level of safety (TLS). By comparing the collision risk calculated for a certain lateral separation with this TLS, it is possible to establish whether a proposed separation standard can be considered sufficiently safe.

3.3 The data used to assess the lateral performance of aircraft in the region are collected using radar near the airspace boundaries. It is not possible to observe the navigation performance throughout a flight; only the end-point lateral deviation error can be observed.

3.4 As a result of the data collection undertaken to estimate the systems risk and observation by ATC, it was found that there were a number of occurrences where aircraft deviated by a substantial amount from their assigned tracks. It was determined that there were a number of causes for such deviations (Reference 3), termed gross navigation errors (GNEs). These were:

a) ATC system loop errors: An ATC system loop error is any error caused by a misunderstanding between the pilot and controller regarding the assigned flight level, Mach number or route to be followed. Such errors can be caused by airline dispatchers incorrectly interpreting the NAT track signal, by errors in coordination between ATC units, and by misinterpretation by pilots of an oceanic clearance or re-clearance;

b) way-point insertion errors: These occur when pilots inadvertently input an incorrect way-point into the flight management system (FMS). This may be either because of incorrect data entry or because the pilot misunderstood the clearance and correctly

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1. All references are listed at the end of this appendix.

input the wrong information (a form of ATC systems loop error); and

c) equipment failure: These occur when there is either a complete failure of the navigation system or a partial failure, which leads to a degradation in track-keeping accuracy.

3.5   Reviewing the results of data collections in the NAT region, the NAT SPG found that although a reduction of the lateral separation might be feasible, a minority of the aircraft observed were responsible for the majority of the GNEs. Therefore, a reduction in the lateral separation for the complete population of aircraft did not appear feasible. As a result, it was proposed that a minimum navigation performance specification (MNPS) airspace be set up, which would accommodate the majority of the aircraft using the NAT, but would exclude those aircraft that did not perform to the desired standard.

3.6   The objectives of an MNPS are to facilitate either:

a) the selection of a separation minimum compatible with the needs of a particular airspace in terms of traffic densities and collision risk levels; or

b) the reduction of an existing separation minimum on the basis of navigation performance and traffic density, whilst maintaining an acceptable level of safety; or

c) the continued use of an existing separation minimum, in a situation where air traffic is increasing, whilst maintaining an acceptable level of safety.

3.7   At its eleventh meeting (Paris, May 1975) the NAT SPG discussed setting up an MNPS for aircraft operating in the OTS. As the use of the long range air navigation (LORAN) system would be drastically curtailed by the end of 1977, the specification was for aircraft using inertial navigation systems (with the view that operators using LORAN A would take this into account when replacing their equipment). It was agreed that the new specification would be designed for an OTS with a planned lateral separation of 60 NM and there would exist the capability to go to a 30 NM and 1 000 ft composite system.

3.8   It was proposed that the MNPS be established and a monitoring programme developed to verify that the specification was being met. The proposed performance specification was designed to ensure an acceptable level of lateral collision risk in the track system operating environment for at least a ten-year period from the time of adoption.

The collision risk would be periodically re-estimated during that period in order to assess the impact of system performance parameters upon safety.

3.9   The MNPS was developed during 1975 and adopted by ICAO in May of 1976. It became effective on 29 December 1977 concurrent with the decommissioning of the LORAN A chain in the NAT region.

3.10   The MNPS airspace is that portion of the NAT airspace between FL 275 and FL 400 from latitude 27° N to the North Pole, bounded in the East by the eastern boundaries of control areas Santa Maria Oceanic, Shanwick Oceanic and Reykjavik and in the West by the western boundary of control area (CTA) Reykjavik, the western boundary of CTA Gander Oceanic and the western boundary of CTA New York Oceanic excluding the area west of 60° W and south of 38° 30' N. The MNPS specifies that:

a) the standard deviation of the lateral track errors shall be less than 6.3 NM;

b) the proportion of the total flight time spent by aircraft 30 NM or more off track shall be less than 5.3×10^{-4}; and

c) the proportion of the total flight time spent by aircraft between 50 and 70 NM off track shall be less than 13×10^{-5}.

3.11   These performance requirements were derived on the basis of assumptions about traffic levels and were designed to ensure that the TLS would be met.

4. INTRODUCTION OF 60 NM SEPARATION

4.1   In August 1978, the NAT SPG Mathematicians Working Group (MWG) convened and reviewed the core lateral navigation performance and GNEs (i.e. those lateral deviations greater than 30 NM) occurring in MNPS airspace. It concluded that:

a) the standard deviation of the core lateral track errors was well within the MNPS specification;

b) the number of errors greater than or equal to 30 NM was approximately the maximum permitted by the MNPS; and

c) the number of errors between 50 and 70 NM was greater than the maximum permitted by the MNPS criteria.

4.2 Therefore the MWG could not recommend a reduction to 60 NM lateral separation due to the level of GNEs which were present at that time. The meeting further concluded that it would be advisable to continue monitoring and evaluating GNEs to permit a further analysis. On 30 October 1980, following successful efforts to reduce the number of large errors, 60 NM lateral track spacing was implemented in the NAT track system.

5. MONITORING

5.1 To ensure compliance with any MNPS, States need to establish procedures for the systematic and periodic monitoring of the achieved navigation performance. The purpose of this monitoring is to provide confirmation that the performance required by the MNPS criteria is being maintained and that the level of safety is acceptable (i.e. meets the TLS).

5.2 The monitoring function covers four areas:

a) the acquisition of the monitoring data (on a routine basis);

b) action to be taken by ATC when observing a GNE and the consequential follow-up action required by the operator and/or State concerned;

c) the issue at periodic intervals of a summary of radar-observed deviations to all interested States and international organizations to highlight the general situation existing in the NAT region; and

d) the conduct of other non-routine data collections on navigational performance.

5.3 For NAT MNPS airspace, the responsibility for the monitoring resides with the Central Monitoring Agency (CMA). Each year the NAT SPG uses the data collected by the CMA to produce an estimate of the risk associated with the NAT MNPS system. A description of the manner in which these estimates are currently calculated can be found below.

6. REICH MODEL

6.1 The NAT SPG uses the Reich model (Reference 4) to estimate the risk of collision in NAT MNPS airspace; this model considers the risk of collision due to the loss of separation between two aircraft flying on nominally parallel tracks. The form of the model described here is used to assess lateral collision risk.

6.2 To simplify the mathematics it is assumed that each aircraft is a rectangular box with average dimensions λ_x, λ_y, λ_z. These dimensions represent the average length, width and height of the aircraft population respectively. The risk of collision between two such boxes is mathematically equivalent to the risk of collision between a point and a box of dimension $2\lambda_x$, $2\lambda_y$, $2\lambda_z$. It should be noted that as the aircraft population changes these dimensions should be re-estimated. It is recommended that aircraft dimension values be re-estimated after a maximum period of five years (Reference 5).

6.3 The number of collisions per unit time is given by the expression:

$$C = N_x P_y P_z + N_y P_x P_z + N_z P_x P_y$$

where N_r is the frequency with which separation shrinks to less than λ_r in the rth dimension;

P_r is the probability of loss of separation in the rth dimension; and

r can be the x (longitudinal), y (lateral), or z (vertical) dimension.

Therefore, the number of collisions per unit time is:

C	=	$\sum$ all dimensions	frequency with which separation between two aircraft is lost in one dimension	×	probability that separation in the other two dimensions has simultaneously been lost

6.4 Let the relative velocity of the two aircraft in the rth dimension be denoted by $|\dot{r}|$. Then, the time taken for the point to pass through the box in the rth dimension would be $2\lambda_r/|\dot{r}|$.

6.5 The probability P_r of overlap in the rth dimension at any moment in time is equal to the average time spent in overlap in that dimension. That is:

$P_r =$	frequency of overlap in rth dimension	*	average time in overlap per overlap
	unit of time		

6.6 If the unit of time is taken to be 1 hour and the dimensions and relative velocities are measured in nautical miles and knots respectively, then:

$$P_r = \frac{N_r * 2\lambda_r}{|\dot{r}|}$$

and so:

$$N_r = \frac{P_r * |\dot{r}|}{2\lambda_r}$$

Hence the number of collisions, C, per unit time can be expressed as:

$$C = P_x P_y P_z \left[\frac{|\dot{x}|}{2\lambda_x} + \frac{|\dot{y}|}{2\lambda_y} + \frac{|\dot{z}|}{2\lambda_z} \right]$$

6.7   In the case of collisions due to the loss of lateral separation, the notation for P_x, P_y and P_z can be expanded to $P_x(O)$, $P_y(S_y)$ and $P_z(O)$. $P_x(O)$, the probability of longitudinal overlap, depends upon the amount of traffic in the system. If the number of aircraft within a distance S_x of each other on adjacent tracks is denoted by E then P_x is equal to $E\lambda_x/S_x$ (Reference 3). E is known as the systems occupancy and provides a measure of the traffic density. A collision could occur between aircraft travelling in either the same direction or in opposite directions. In this case the relative longitudinal velocities would be different, $|\dot{x}_s|$ and $|\dot{x}_o|$ say, as would the traffic densities E_s and E_o where "s" represents same direction traffic and "o" represents opposite direction traffic.

6.8 $P_y(S_y)$ is the lateral overlap probability, i.e. the likelihood that any two aircraft which have been assigned the correct lateral separation are in fact not separated laterally.

6.9 $P_z(O)$ is the vertical overlap probability, i.e. the probability that two aircraft nominally at the same level are in vertical overlap.

6.10   The equation for the lateral collision risk thus becomes:

$$C = P_y(S_y) P_z(O) \frac{\lambda_x}{S_x} \left\{ E_s \left[\frac{|\dot{x}_s|}{2\lambda_x} + \frac{|\dot{y}|}{2\lambda_y} + \frac{|\dot{z}|}{2\lambda_z} \right] + E_o \left[\frac{|\dot{x}_o|}{2\lambda_x} + \frac{|\dot{y}|}{2\lambda_y} + \frac{|\dot{z}|}{2\lambda_z} \right] \right\}$$

where C measures the expected number of fatal accidents per aircraft flying hour.

6.11   Apart from $P_y(S_y)$, E_s and E_o, all of the parameters of the above equation are more or less stable with time. Table A-4-1 presents the estimates of the parameter values for lateral collision risk given a 60 NM lateral separation minimum. These values refer to the parameters used in 1995.

6.12   The parameters $P_y(S_y)$, E_s and E_o tend to vary with time and, in the NAT region, are measured regularly.

6.13   The system occupancy (E_s and E_o) is estimated from flight plan data for a set of sample days throughout the year (Reference 3). The lateral overlap probability, $P_y(S_y)$, is estimated from ATC observed GNEs from continually monitored "windows" of airspace at the eastern and western boundaries of the ocean. This is because most of the ocean is outside radar coverage; although deviations from track that occur over mid-ocean may be reported, it cannot be assumed that such information is complete.

Table A-4-1. Values used for lateral collision risk parameters

Parameter	Estimated value
S_y	60 NM
S_x	120 NM (15 mins)
$P_z(O)$	0.39
λ_x	0.0306 NM
λ_y	0.0272 NM
λ_z	0.0086 NM
$|\dot{x}_s|$	13 kt
$|\dot{x}_o|$	960 kt
$|\dot{y}|$	80 kt
$|\dot{z}|$	1.5 kt

6.14   In the development of the Reich model it was initially assumed that the data collected from the sample portions were representative of the rest of the airspace. Consequently, the lateral overlap probability was estimated directly from the observed distributions. That is, the distribution of deviations as measured at the boundary were considered to be exactly the same as that measured at any other point over the ocean. This approach was known as Model 1 (Reference 3). As more information was amassed it was considered that the distribution of deviations observed at the boundary was unlikely to be representative of the actual distribution. Therefore, in order to make the lateral overlap probability more accurate certain observed deviations were weighted, with the aim of making the weighted distribution more accurate. This approach became known as Model 2. Full details of both of these models and the calculations employed whilst generating the parameters can be found in Reference 3.

6.15   Once values are available for each of the parameters, these can be input into the collision risk model to obtain an estimate of the risk of the system. For future systems, estimated values for occupancy, traffic density, etc. can be used. After the risk of the system has been calculated it can be compared against a target level of safety to determine if the system is likely to meet the required safety requirements. This exercise is undertaken on a monthly basis for the NAT, and the results are reviewed annually by the NAT SPG.

REFERENCES

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2. North Atlantic MNPS airspace operations manual, Sixth Edition. Published on behalf of the North Atlantic Systems Planning Group by the Civil Aviation Authority, London, December 1993.

3. L. Davies, A. Sharpe. Review of the North Atlantic lateral collision risk model. Air Traffic Control Quarterly, Vol. 1(3), 1993, pp. 225-253.

4. P.G. Reich. Analysis of Long Range Air Traffic Systems — Separation Standards Parts I, II and III. Journal of the Institute of Navigation, Vol. 19, 1966.

5. Summary of discussions and conclusions of the Thirty-First Meeting of the North Atlantic Systems Planning Group, NAT SPG/31, June 1995.

Appendix 5

ASSESSMENT OF LONGITUDINAL SEPARATION
IN THE ASIA/PACIFIC REGIONS

1. INTRODUCTION

1.1 This appendix describes the methodology used by the Member States of the Asia/Pacific Air Navigation Planning and Implementation Regional Group (APANPIRG) to determine distance-based longitudinal separation minima for use in all areas within the Asia/Pacific Regions.

1.2 The methodology was presented by Australia to the Review of the General Concept of Separation Panel (RGCSP) in a series of papers in 1993 and 1994 (References 1 to 5)[1]. It is usable for the evaluation of longitudinal distance-based separation standards for existing RNAV routes and RNP airspaces or routes subject to procedural ATC control, or in an ADS environment. The data collection on which the analysis is based included data from the South Pacific Region and the Oakland Flight Information Region (FIR) and took place during the twelve months from September 1993 to the end of August 1994.

1.3 The method builds on the well-established theory put forward by Reich (References 6, 7, 8), but is distance-based and, in line with the concepts of RNP, uses the 95 per cent along-track and cross-track containment figures as the starting point for aircraft navigation system performance.

1.4 This assessment is the result of a mathematical analysis and does not include operational and technical considerations, which would enable the introduction of distance-based longitudinal separation minima. This is particularly important in respect of separation minima of less than 50 NM, where it is essential to ensure that the assumptions of the analysis are achievable from an operational and technical point of view.

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1. All references are listed at the end of this appendix.

2. ASSUMPTIONS

2.1 The analysis assumed that the following conditions apply to the use of longitudinal distance-based standards:

a) when aircraft are at, or are expected to reduce to, the minimum separation applicable, speed control techniques, including assigning Mach number, should be applied;

b) direct pilot/controller communications must exist;

c) in an ADS environment, separation is established by reference to a display system, which allows the controller to assess the distance between the aircraft. Separation exists if the observed longitudinal separation on this display is equal to or greater than the appropriate minimum;

d) in a procedural RNAV environment, separation is established by asking the aircraft to report in turn their RNAV distance to a common way-point. The reporting order must be leading aircraft first. Separation exists provided that the difference in the reported distances is equal to or greater than the appropriate minimum;

e) separation must be checked sufficiently often to ensure that the distance between any two aircraft will not be reduced to less than the minimum. The maximum permitted time interval between reports varies with the separation minimum being used, and whether or not ADS is being used, and is specified later in the presentation of results;

f) if the minimum separation could be infringed upon during the next reporting period plus 15 minutes, the controller must take action to maintain the minimum or to establish some alternative form of separation;

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g) if the controller is unable to contact an aircraft when a distance check is due, action must be taken to establish some alternative form of separation within a prescribed time interval. Appropriate values for this time interval are discussed in the paper. If a periodic ADS position report is not received by the controller within 3 minutes of the time it is due, a one-off ADS report will be demanded. If ADS surveillance cannot be re-established within a further 5 minutes, the controller must commence immediate action to establish some alternative form of separation; and

h) if separation is based on the use of ADS position reports originated at different times, the aircraft involved and the ground system(s) must use a common time reference, e.g. global positioning system (GPS).

3. THE SAFETY ASSESSMENT CRITERIA

3.1 The safety of the proposed standards was assessed against a target level of safety (TLS) of 2×10^{-8} fatal accidents per flying hour. The Tasman Sea area between Australia and New Zealand is a typical region of airspace where the proposed standards could be applied. This airspace generates approximately 30 000 flying hours per year. In this airspace, the agreed TLS corresponds to an aircraft loss frequency of one aircraft every 1 667 years.

4. OVERVIEW OF THE METHODOLOGY

4.1 The following is a description of the mathematical model used to analyse collision risk.

4.2 Assume that two aircraft are initially separated by some distance S when the separation standard is established. The first step in the methodology is to calculate the probability of collision for this particular pair between the time the separation is established and the time that it must next be checked.

4.3 The planned separation for the aircraft at the time the next check is due will be some value greater than the minimum. It is, however, quite probable that the distance at this time will be different from the planned distance, due to a combination of factors such as errors in the indicated Mach number, speed variations within the normal limits of the flight control systems, and differences in the wind being experienced by the aircraft.

4.4 Data were collected on changes in the relative velocity of pairs of aircraft. From the distribution of these velocities, it is possible to calculate, for any given starting separation, the probability of any particular value of final separation.

4.5 The separations referred to above are based on reports of position from the aircraft navigation systems, therefore they are only nominal separations. If we know the distribution of aircraft navigation system errors, it is possible to calculate a collision probability for each case. This can be derived from the RNP for the airspace or route where the standard is to be applied.

4.6 Rather than base the calculations solely on the time interval to the next distance check (referred to as the update interval), the model uses the concept of a communication and controller intervention buffer, to allow for the possibility that, when the next distance check is obtained, it may become apparent to the controller that separation will be lost, requiring the controller to establish some alternative form of separation. This buffer is added to the time over which the collision risk is calculated, so the calculated collision risk is that applying to the case where intervention is necessary. Since intervention will be necessary in only a small number of cases, the collision risk calculated in this way will be greater than the true collision risk. Once data on the frequency of intervention is available, it will be possible to model this more precisely. However, as the method employed in this analysis will overestimate the true collision risk, this method is a conservative approach.

5. COMMUNICATION AND CONTROLLER INTERVENTION BUFFER

5.1 The elements to be considered in the communication and controller intervention buffer (represented by τ) are:

a) the time for the controller to recognize the potential conflict and to devise an alternative means of separation (assumed in this case, as we are considering a procedural oceanic environment, to be achieved by a change of level);

b) the time taken to communicate the instructions to the pilot;

c) the time for the pilot to react and initiate an appropriate manoeuvre; and

d) the time for the aircraft to achieve a change of trajectory sufficient to ensure that a collision will be averted.

5.2   One of the requirements for the use of procedural RNAV standards is direct controller/pilot communications. In the future, controller/pilot data link communications (CPDLC) may be used to satisfy this requirement. The value of τ will need to include the delays in the CPDLC system and also, when ADS is being used to separate aircraft, the ADS data link and display system. Since the estimated delays for CPDLC are greater than the delays with direct voice communication, a value of τ appropriate for the CPDLC environment was used for both the procedural RNAV and ADS cases.

5.3   Although the various relevant panels of ICAO had not finalized design standards and performance specifications for all of the various components of CPDLC at the time this analysis was done, it was possible to give an estimate of τ using the factors that were likely to have a major effect. This estimate takes into account the fact that the controller is making decisions using information on the computer screen that is not current, and, in addition, that any necessary commands to the pilot will take some time to reach the pilot and be acted upon.

5.4   The assumptions used in the derivation of the values for τ were based on the following understanding of the operation of the ADS system. Although, it is based on the initial ADS implementation known as the FANS 1 upgrade for the Boeing 747-400 aircraft, the Airbus FANS A implementation is expected to be similar. It was assumed that when an ADS position report was due, the aircraft's flight management computer system (FMCS) would obtain the necessary data, construct the report, and pass the report to the ARINC communications addressing and reporting system (ACARS), which would then transmit the report. The document (Reference 9) identifies the following contributing factors in the expected transport delay:

a)   time to accept message from FMCS;

b)   time spent in ACARS queue;

c)   time to route message to selected data pathway (radio or satellite);

d)   time for the selected transmitter to transmit the message;

e)   time to transmit message from ground station to service provider's processor;

f)   time to route message from service provider to ground end system router; and

g)   time to route message from ground end system router to application.

5.5   If we denote by R the maximum age of the data before leaving the aircraft, and by D the maximum downlink time, then we can equate $R + D$ with the time for all of the above steps. It is assumed that the air traffic controller will be using a screen, displaying positions derived from ADS position reports, and that the display will be updated every X seconds, that it will take the controller C seconds to recognize a potential conflict and a further M seconds to compose a message. Let the maximum uplink time be denoted by U and let P be the time it takes the pilot to respond to the message and to initiate a change of trajectory. Finally, let A be the time it takes the aircraft to achieve alternate separation. For oceanic operations it was assumed that this alternate separation would involve a change of level of 500 ft. This leads to the following expression for τ:

$$\tau = R + D + X + C + M + U + P + A.$$

5.6   Although it was not possible to give precise values for the various components making up τ, it was nevertheless possible to give approximate upper bounds. Early FAA documents (References 10 and 11) had concluded that $R + D$ was likely to be less than 61 seconds. The Boeing document (Reference 9) concluded: "It is currently felt that these steps" (5.4 a) through g) above) "can occur in about 1 minute 95 per cent of the time". The analysis assumed a value of 61 seconds for $R + D$. The same FAA analysis gave an approximate 95 per cent upper bound for U (the maximum uplink time) as 107 seconds. It also presented evidence to suggest that the value of P (the time it takes the pilot to respond to an ATC message) is likely to be greater than 22 seconds. Discussions with staff from the then Australian CAA's Directorate of Air Safety Regulation led to the adoption of a value of 45 seconds for P. Similar discussions within the then Australian CAA, including air traffic services and technical staff, led to the adoption of values of 60 seconds for X (the update rate of the ATCOs aircraft position screen), 30 seconds for C (the time required by the controller to recognize a potential conflict) and 30 seconds for M (the time taken by the controller to compose a message). The value of A (the time it takes the aircraft to achieve alternative separation) may be estimated by assuming the inertia of the aircraft will result in a 15-second lag and that the aircraft will climb at 500 ft per minute for 500 ft, resulting in a value of 75 seconds for A. These values gave:

$$\tau = 408 \text{ sec}$$

5.7   Again, a somewhat conservative value of 7 minutes for τ was used in the estimation of collision risk.

6. AUTOMATIC DEPENDENT SURVEILLANCE

6.1 The ADS system allows aircraft to automatically report their positions, as derived from their on-board navigational systems, via communication links to ground-based ATC. The study assumed that positions of ADS-equipped aircraft would be displayed to the controller on a screen similar to present radar displays. In spite of this superficial similarity to radar, the initial application of ADS will be to monitor procedural separation standards. The expected update rates in oceanic airspace will not be sufficient to allow radar-like control procedures to be used.

6.2 Aircraft equipped with ADS will generally also have the capability for CPDLC, and will use this to receive instructions from ATC. Since CPDLC will be the primary means of communication with aircraft, the communication and controller intervention buffer must be based on its use.

6.3 The use of ADS adds an additional source of error which is not present in the procedural RNAV case. This is the extrapolation error, when aircraft are projected forwards for display on the basis of an ADS report received at some time in the past.

6.4 Extrapolation errors

6.4.1 In the analysis of procedural separation standards, it was assumed that both aircraft in the pair reported more or less simultaneously, or at least in such a way as to give the controller an underestimate of the separation. This is not the case when ADS is being used. The information that is displayed to the controller will always be out of date, and even when different aircraft are reporting at the same rate they will never send ADS reports simultaneously. It is even possible for two adjacent aircraft to be reporting at different rates. In the collision risk modelling it was assumed that the worst case would occur when the ADS report of one aircraft was just received and that of the other was due almost immediately. The data pertaining to this latter aircraft would then be almost one reporting interval old. It was assumed that the ground system would carry out some form of extrapolation using an estimate of the aircraft's ground speed. Thus, it was necessary to take account of the possible prediction error in the estimation of collision risk.

6.4.2 In a full ADS implementation an automatic ADS report will be generated if an aircraft's ground speed varies by more than 20 knots. Since this trigger is not implemented in Boeing's FANS 1 upgrade, it was assumed that the probability density function representing the change in ground speed of an individual aircraft over successive

reporting intervals is represented by a double exponential distribution.

6.4.3 The effect of the extrapolation error is to increase the uncertainty in the position of the aircraft being extrapolated. This was incorporated into the methodology by means of a modification to the distribution of navigation system errors as shown in the mathematical summary in Attachment A.

7. RESULTS

7.1 The results in this section are based on data from the 1993/94 RGCSP data collection exercise in the Pacific area. These data were used to estimate the distribution of separations, $w(s)$, and the distribution of relative velocities, $\varphi_x(v)$.

7.2 The distribution of separations

7.2.1 Data from the Pacific Area, where it was first intended to apply the new standards, was examined to determine the nature of the distributions of separation likely to be encountered in practice. The data used were aircraft position reports. The distance between successive pairs of aircraft at the same level were calculated by converting the time difference between the aircraft at the position report to a distance, using the average ground speed over the last route segment for the following aircraft.

7.2.2 Since a new separation standard was to be introduced, it was necessary to estimate the effects of this new standard on the distribution. To do this, an initial assumption was made that all aircraft would be at the same level. Then, if the longitudinal separation at a particular reporting point were to be less than the new minimum, the level of the following aircraft would be adjusted downwards to provide separation.

7.2.3 For airspace planning purposes only, the 50 NM RNAV longitudinal separation minimum was developed in accordance with a collision risk analysis, which dictates conditions under which this minima can be applied. The conditions of this analysis are:

1) the frequencies of observed or estimated separations between successive aircraft at the same level are such that no more than 4 per cent of separations will fall in any one 10 NM band, from the minimum separation of 50 NM to a separation of 100 NM; and

2) the communications facilities are such that the total controller communication and intervention buffer will not exceed 7 minutes in all cases.

7.3 The distribution of relative velocities

7.3.1   The distribution of relative velocities, $\varphi_x(v)$, was estimated from the same data. The relative velocities were calculated by comparing successive position reports of pairs of aircraft flying at the same level and within one hour of each other. Since the calculation of unintended relative velocity requires a correction for differences in the intended Mach number, pairs were rejected if the Mach number was not recorded for both aircraft at both position reports. This

latter requirement significantly reduced the size of the sample, as compliance with the requirement to report and transmit Mach number has not been good, especially earlier on in the data collection exercise. The usable sample consisted of 2 677 pairs.

7.4 Calculation of the expected number of fatal accidents per flying hour (N_{ax}) for normal operating conditions

7.4.1   The collision risk formula requires values for several parameters. The values used in the analysis are presented in Table A-5-1, and are representative of the traffic collected in the sample.

Table A-5-1. Collision risk equation parameter values

Parameter	Value (procedural RNAV)	Value (ADS)
λ_x	192.2 ft	232 ft
λ_y	167.7 ft	211 ft
λ_z	54.8 ft	64 ft
$\dot{x}$	35 kts	35 kts
$\dot{y}$	20 kts	20 kts
$\dot{z}$	1.5 kts	1 kt
$P_y(0)$ for RNP 4	0.041	0.052
$P_y(0)$ for RNP 10	0.017	NA
$P_y(0)$ for RNP 20	0.008	NA
$P_z(0)$	0.390	0.390

The results of the analyses are presented in Tables A-5-2 and A-5-3.

Table A-5-2. Collision risk estimates — procedural RNAV ($\tau = 7$ minutes)
(fatal accidents per flying hour)

Navigation system error (95% containment) (NM)	Separation standard (NM)	Maximum permitted time between reports (minutes)	Point estimate of collision risk
4	30*	15	3.12×10^{-10}
4	40*	30	6.54×10^{-10}
10	50	30	4.81×10^{-9}
20	80	55	1.74×10^{-8}
*See paragraph 1.4.			

Table A-5-3. Collision risk estimates — ADS ($\tau = 7$ minutes)
(fatal accidents per flying hour)

Navigation system error (95% containment) (NM)	Separation standard (NM)	Maximum permitted time between reports (minutes)	Point estimate of collision risk
4	30*	15	8.17×10^{-9}
4	50	30	1.47×10^{-8}
*See paragraph 1.4.			

7.5 Effect of communication failures

7.5.1 When an aircraft fails to report at a required position, or does not respond to a call from ATC, the controller is required to take action within 3 minutes to try to establish communications. This action will include calling on other frequencies, and requesting other aircraft to call the aircraft which is out of communications.

7.5.2 To allow for the effect of such a communication failure when a longitudinal distance standard is

being used, it was assumed that, if communications had not been re-established within 5 minutes after the normal 3-minute communication check time, the controller would be required to take action to achieve some alternative form of separation.

7.5.3 This gave a value of τ of 15 minutes for the communication failure case.

7.5.4 The effect on the collision risk of such a value can be seen in Tables A-5-4 and A-5-5.

Table A-5-4. Collision risk estimates — procedural RNAV (τ = 15 minutes)
(fatal accidents per flying hour)

Navigation system error (95% containment) (NM)	Separation standard (NM)	Maximum permitted time between reports (minutes)	Point estimate of collision risk
4	30	15	1.85×10^{-8}
4	40*	30	4.01×10^{-8}
10	50	30	1.91×10^{-8}
20	80	55	3.00×10^{-8}
*See paragraph 1.4.			

Table A-5-5. Collision risk estimates — ADS (τ = 15 minutes)
(fatal accidents per flying hour)

Navigation system error (95% containment) (NM)	Separation standard (NM)	Maximum permitted time between reports (minutes)	Point estimate of collision risk
4	30*	15	1.20×10^{-7}
4	50	30	5.05×10^{-8}
*See paragraph 1.4.			

7.5.5   The communication and delay buffer τ incorporates several performance parameters regarding the communication system. It is common practice to express this performance in terms of availability, continuity, end-to-end delay, integrity and reliability. These parameters are defined by the ICAO Automatic Dependent Surveillance Panel (ADSP) for data link applications, but are taken here to be general communication performance parameters.

a)   Availability. (*The ability of a system to perform its required function at the initiation of the intended operation. It is quantified as the proportion of time the system is available to the time the system is planned to be available.*) It is assumed that if at the start of the operation the communication link is not available, appropriate alternative separation will be applied.

b)   Continuity. (*The probability of a system to perform its required function without unscheduled*

interruptions during the intended period of operations.) In case of communication failure, the following times are assumed:

Detection of communication failure: 3 minutes

Attempt alternative means of communication: 5 minutes

Achieve alternative separation: 7 minutes

Therefore, a total of 15 minutes is assumed for achieving alternate separation. The effect of this on the collision risk results in a maximum permitted probability of loss of communications of 0.21 for 80 NM (i.e. a continuity of at least 79 per cent and 1.00 for 50 NM (i.e. a continuity of at least 0 per cent), provided any total communications loss does not exceed 5 minutes in duration.

c)   End-to-end transfer delay. (*The period elapsed from the time at which the originating user initiates the triggering event until the transmitted information has been received by the intended recipient.*)

Assumed end-to-end downlink transfer delay: approximately 60 seconds (95 per cent)

Assumed end-to-end uplink transfer delay: 107 seconds (95 per cent)

d)   Integrity. (*The probability that errors will be mis-detected. This may be when a correct message is indicated as containing one or more errors, or when a message containing one or more errors is indicated as being correct. Integrity relates to the trust which can be placed on the correctness of the information.*) It is assumed that the integrity is such that it would not have a significant effect on the collision risk estimate.

e)   Reliability. (*The probability that the system will deliver a particular message without one or more errors.*) It is assumed that if the rate of errors is deemed unacceptable by the users, appropriate alternative separation will be provided. Any delays due to retransmissions after the occurrence of

errors are included in the delay figures under continuity above.

7.6 Overall collision risk

7.6.1   To calculate the overall collision risk for the system in the presence of communication failures, the collision risks for the two cases (normal and communication failure) should be factored by the respective probabilities of these events. The collision risk, allowing for failures, then becomes the sum of these two terms.

7.6.2   At the stage this analysis was performed, the data on frequency of loss of communication were not available. However under the assumptions made in this analysis, a TLS of 2×10^{-8} fatal accidents per flight hour will be met provided that the frequency of loss of communication does not exceed the values presented in the last column of Table A-5-6. It was expected that the frequency of loss of communications would be substantially better than any of these values.

8. CONCLUSIONS

8.1   The separation minima in Table A-5-6 were recommended as a result of this analysis, taking into account the comments noted in 1.4.

Table A-5-6. Acceptable procedural RNAV longitudinal separation minima

Navigation system error (95% containment) (NM)	Separation standard (NM)	Maximum permitted time between reports (minutes)	Maximum permitted probability of loss of communications
10 or better	50	30	1.00
20 or better	80	55	0.21

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Attachment A to Appendix 5

MATHEMATICAL DESCRIPTION OF THE
DISTANCE-BASED LONGITUDINAL
COLLISION RISK MODEL

Let:

N_{ax} = Expected number of fatal accidents per flying hour.

Π_x = Proportion of time that a typical aircraft is in longitudinal overlap with another aircraft assigned to the same track and flight level.

$P_y(O)$ = Probability that two aircraft assigned to the same track are in lateral overlap.

$P_z(O)$ = Probability that two aircraft assigned to the same flight level are in vertical overlap.

$|\dot{x}|$ = Average along-track component of the relative velocity of two aircraft which collide due to loss of longitudinal separation.

$|\dot{y}|$ = Average relative cross-track speed for two aircraft assigned to the same track.

$|\dot{z}|$ = Average relative vertical speed of two aircraft assigned to the same level.

λ_x = Average length of aircraft using the airspace.

λ_y = Average wing-span of aircraft using the airspace.

λ_z = Average vertical dimension of aircraft using the airspace.

$t = 0$ Represents the start of a reporting period.

$t = T$ Represents the end of the same reporting period.

τ = Buffer for communication and other delays and controller action.

S = Separation standard in use.

S_t = True longitudinal separation between a pair of aircraft at time t.

$\hat{S}_t$ = Nominal longitudinal separation between a pair of aircraft at time t (that is, the separation calculated from the distances given by the aircraft navigation systems).

$\hat{S}_0$ = Nominal longitudinal separation between aircraft at time $t = 0$.

S_F = True separation at time $t = T + \tau$.

Then

$$N_{ax} = \Pi_x P_y(O) P_z(O) \left[\frac{|\dot{x}|}{2\lambda_x} + \frac{|\dot{y}|}{2\lambda_y} + \frac{|\dot{z}|}{2\lambda_z} \right] \qquad (1)$$

In Equation (1), Π_x is given by

$$\Pi_x = \frac{4\lambda_x}{S} \times \int_s w(s)U(s)ds \qquad (2)$$

where

$w(s)$ = the density function giving the probability, over the whole system being evaluated, that any selected pair of aircraft at the same level are nominally separated longitudinally by a distance s, and

$U(s)$ = the probability that longitudinal overlap occurs between times $t = 0$ and $t = T + \tau$ given an initial nominal separation $\hat{S}_0 = s$.

Let

$\psi(x)$ = Probability density function of along track navigation system errors for first aircraft.

$\varsigma(x)$ = Probability density function of along track navigation system errors for second aircraft.

$F(a)$ = Probability at any given instant in time that for a pair of aircraft the true separation is $\leq \lambda_x$, given that the nominal separation is equal to a

= $\text{Prob}\left(S_t \leq \lambda_x | \hat{S}_t = a \right)$.

$\varphi_x(v)$ = Probability density function of unplanned relative velocities in the longitudinal direction.

Note that $\varphi_x(v)$ takes account of any gross navigational or flight technical errors.

An expression for $F(a)$ in terms of the navigation system errors of each aircraft is given by

$$F(a) = \int\limits_{x=a-\lambda_x}^{\infty} \int\limits_{n=-\infty}^{\infty} \varsigma(n)\Psi(n-x)dndx \qquad (3)$$

and $U(s)$ is given by

$$U(s) = P(S_F \le \lambda_x \big| \hat{S}_0 = s) \qquad (4)$$

where we have assumed that multiple passings by the same pair of aircraft will not occur during any given reporting interval. Now,

$$U(s) = \int\limits_{v}^{\infty} \varphi_x(v)P(S_F \le \lambda_x \big| \hat{S}_F = s+v(T + \tau))dv$$

$$= \int\limits_{v=-\infty}^{\infty} \varphi_x(v)F(s + v(T + \tau))dv \qquad (5)$$

In a procedural RNAV environment, we assume that $\psi(x)$ and $\varsigma(x)$ may be taken to be double-exponential probability density functions. That is

$$\psi(x) = \varsigma(x) = \frac{1}{2\lambda}e^{-\frac{|x|}{\lambda}} \qquad (6)$$

In an ADS environment, account must be taken of the non-simultaneous nature of the position reports of pairs of aircraft. To make the ADS theory consistent with the basic procedural RNAV theory, the position of one of the aircraft in a pair must be extrapolated forward in time to coincide with the reporting time of the other aircraft. Consequently, we modify $\varsigma(x)$ by taking it to be the probability density function of the sum of two random variables, each distributed according to different double exponential densities. That is

$$\varsigma(x) = \frac{\lambda e^{-\frac{|x|}{\gamma}} - \gamma e^{-\frac{|x|}{\gamma}}}{2[\lambda^2 - \gamma^2]}, \qquad (7)$$

where γ is the double-exponential parameter corresponding to the extrapolation error.

Appendix 6

EXAMPLE OF COMPARATIVE SAFETY ASSESSMENT

1. INTRODUCTION

1.1 In Japanese airspace, a radar separation minimum of 5 NM is applied to pairs of aircraft between 40 NM and 200 NM from a radar site. The radar systems used to support this separation minimum are called air route surveillance radar (ARSR) and have been used since 1976 without any safety problems. In order to extend the range at which 5 NM separation could be used offshore in oceanic airspace, a long-range secondary surveillance radar (LSSR) system was developed. This system was designed so that the maximum range of the system would be 250 NM, and the azimuth estimation accuracy was improved through the use of a monopulse technique.

1.2 Prior to the implementation of the LSSR, a risk assessment on the feasibility of using the new system to support 5 NM radar separation was carried out.

1.3 In this scenario the traffic and procedures to be used by air traffic control would be the same. The only difference being the quality of surveillance available using the new radar. Since all other airspace characteristics remain constant, it was sufficient to compare only the measurement accuracy of the two radar systems with the reference system for this analysis being the existing system. In order to compare the quality of surveillance provided by the LSSR to that provided by the ARSR, the close approach probability (CAP), a parameter associated with collision risk, was calculated for the two systems. The CAP is defined as the probability that an aircraft pair actually overlap when the apparent distance between them is equal to the radar separation minimum.

2. METHOD OF COMPARISON

2.1 Air traffic controllers maintain the separation between aircraft based on blips/symbols of aircraft presented on radar displays. The displayed position contains the following uncertainties:

a) errors in the displayed aircraft position; and

b) discontinuity of data (update rate of 10 seconds).

2.2 The uncertainty in the displayed aircraft's position can be related to the parameters used for determining the safe radar separation minima.

2.3 Collision risk modelling could be used to explicitly evaluate the safety level associated with the application of a particular separation minimum using a radar surveillance system. However, it is not always easy to estimate all of the model parameters. In this evaluation the CAP, i.e. the probability that the aircraft pair actually overlaps when the apparent distance is equal to the radar separation minimum, D_s, was calculated. The CAP, P_{ca}, is given by

$$P_{ca} = Prob[d < D_k | D_s] \qquad (1)$$

where d is the actual separation between an aircraft pair and D_k is the size of an aircraft.

2.4 In general, for an area far from the radar site, the range measurement errors of a radar system are negligible compared with the azimuth measurement errors. Therefore, the worst case is the situation in which two aircraft are located on an arc of radius R from the radar. This situation is shown in Figure A-6-1. In this case, the CAP is the probability that the aircraft pair overlaps in the azimuthal direction when the nominal separation is D_s.
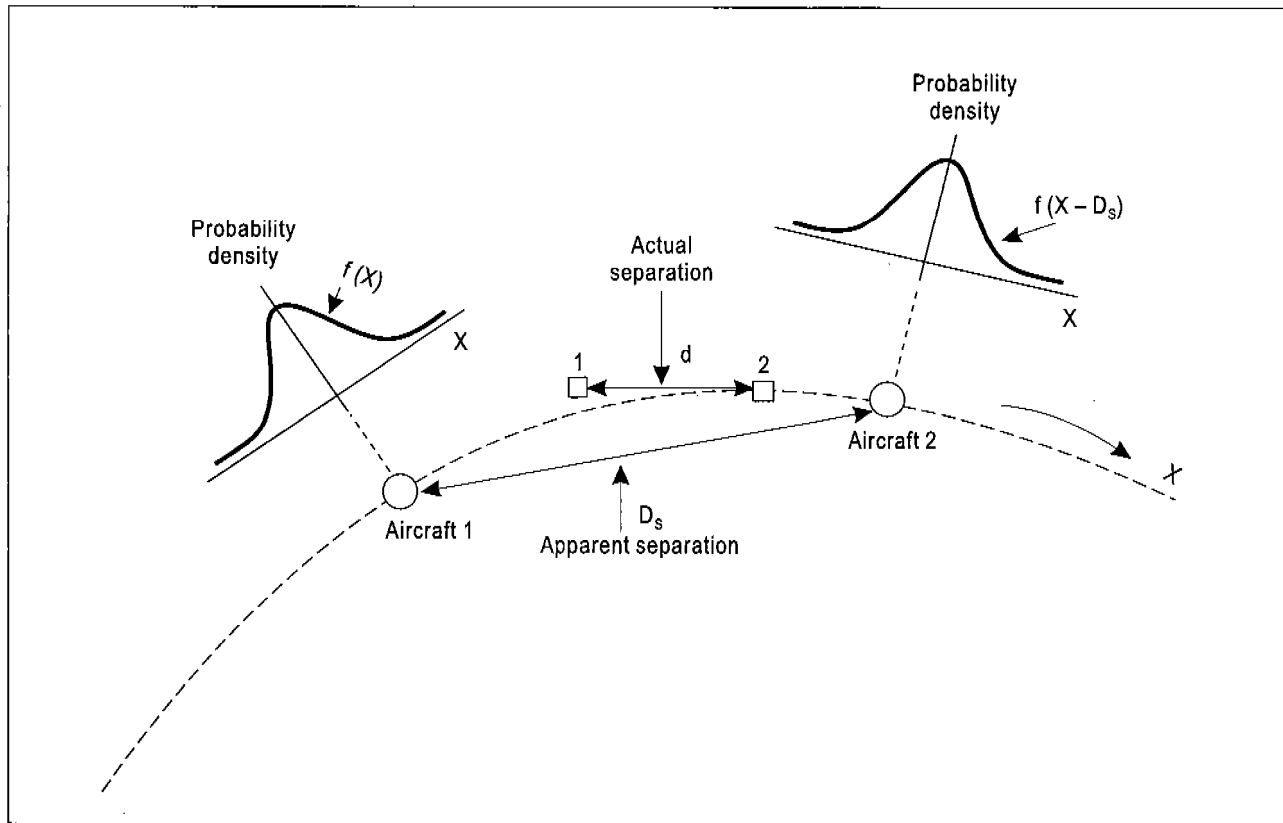
Figure A-6-1. Actual and apparent separation between two aircraft

2.5   Assuming that the azimuth errors are independently and identically distributed for each aircraft, the CAP for a pair of aircraft on the arc of radius R can be approximated by:

$$P_{ca}(R) = 2(D_k /R) \ C(D_s /R) \qquad (2)$$

where, $C(z) = \int_{-\infty}^{\infty} f(x)f(x - z) \ dx \qquad (3)$

and $f(x)$ is the probability density function (pdf) of the azimuth error.

2.6   Once $f(x)$ is given, the convolution, $C(z)$, can be calculated. Equation (2) has its maximum value at the maximum range, R_{max}.

3. ESTIMATING CAP FOR BOTH RADAR SYSTEMS

3.1 Methodology

3.1.1   The following steps were followed during the evaluation of the radar systems:

a)   establish an azimuth error model for each radar system based on empirical data. The pdf of azimuth errors is estimated from the distribution of azimuth deviations based on digitized data from the two radar systems;

b)   estimate the maximum CAP for both systems;

c)   compare the maximum CAPs of the two systems.

3.2 Data analyses

3.2.1   Two data sets, one from the LSSR to be evaluated and the other from the operational Jobon-zan ARSR/SSR were used. The data used for the analyses were for inbound aircraft flying from/on the North Pacific ATS routes. The flight patterns observed, within the two data sets, were almost identical and the area of data collection was roughly the same for both systems. Both radars were installed at almost the same site (150 m apart).

3.2.2   The conventional ARSR uses a scan-to-scan correlation technique called the sliding window technique

for obtaining an azimuth estimate of the target aircraft. The LSSR uses a monopulse technique. Table A-6-1 compares the characteristics of the two systems.

3.2.3   The azimuth errors were estimated as the difference between the observed (estimated) azimuth and the true (smoothed) azimuth. A number of pdf models were fitted to the empirical distributions with the following models being considered as the most appropriate for estimating the CAPs:

a) N-N model for the conventional radar data

$$f(x) = (1 - \alpha)N(x|\sigma_1) + N(x|\sigma_2) \qquad (4)$$

b) N-DE model for the LSSR

$$f(x) = (1 - \alpha)N(x|\sigma_1) + DE(x|\lambda) \qquad (5)$$

where,

$N(x|\sigma_i)$ is the *pdf* of the Gaussian distribution with zero mean and standard deviation of σ_i ,

$DE(x|\lambda)$ is the *pdf* of the double exponential distribution with zero mean and standard deviation of $\lambda\sqrt{2}$, and

α is a weighing coefficient ($0 < \alpha < 1$).

The model parameters were estimated by the maximum likelihood estimation method.

4. RESULTS

Table A-6-2 shows the results of the estimation. These results indicate that the $P_{ca}(R_{max})$ for the LSSR is considerably smaller than that for the conventional radar. Though these values are point estimates, a statistical approach based on the Bootstrap method also supported the results (see Reference 1 below). As the results indicated that the use of LSSR would provide greater accuracy than the use of the current system, and that no deterioration in safety would occur, the LSSR system was introduced.

REFERENCE

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Table A-6-1. Comparison of the conventional ARSR/SSR and the LSSR

	Conventional ARSR/SSR	*LSSR*
Site	Jobon-zan	Jobon-zan
Azimuth measurement	Sliding window	Monopulse
Coverage	200 NM	250 NM
Sensor	PSR/SSR	SSR

Table A-6-2. Estimated model parameters and CAPs

Data	*Conventional ARSR/SSR*	*LSSR*
Sample size	8 935	11 395
Standard deviation of azimuth errors	0.112 deg.	0.037 deg.
Best fitted *pdf* model	N-N	N-DE
Fitted model parameter	$\sigma_1 = 0.0987$ deg. $\sigma_2 = 0.1640$ deg. $\alpha = 0.164$	$\sigma = 0.0226$ deg. $\lambda = 0.0315$ deg. $\alpha = 0.5823$
Estimated $P_{ca}(R_{max})$	5.3×10^{-12} ($R_{max} = 200$ NM)	3.1×10^{-16} ($R_{max} = 250$ NM)

Appendix 7

EXAMPLE OF HOW TO TRADE OFF VARIOUS FACTORS WHEN COMPARING A PROPOSED SYSTEM WITH A REFERENCE SYSTEM

1. INTRODUCTION

1.1   This manual outlines two basic methods of determining whether a system is safe. One method involves the comparison of the proposed system with a reference system. If the proposed system bears a sufficiently close resemblance to the reference system and the reference system has been proven to be safe, it is possible to implement the proposed system without performing risk analyses. In certain circumstances, the proposed system may be similar to the reference system but some aspects may be sufficiently different to warrant an analysis of the proposed change. In these cases it may be possible to evaluate the safety of the proposed system by means of a trade-off between the different system performance parameters.

1.2   This appendix provides one example where the safety of a proposed system is assessed in this manner.

2. TRADE-OFF BETWEEN OCCUPANCY AND THE PROBABILITY OF LATERAL OVERLAP

2.1   Consider a proposed airspace, which is identical in all respects to the current North Atlantic (NAT) minimum navigation performance specification (MNPS) airspace, except that the traffic density is higher and the lateral navigation accuracy is better in terms of both the core performance and the number of large blunders which will occur. The reference system and the proposed system have the same lateral separation minima and the same means of communication and surveillance but the frequency of application of the lateral separation minima and the navigation performance will be different. Before implementing such a system it is necessary to ensure that this new system will be safe. As the proposed system is otherwise similar to the reference system, it is sufficient to consider the effects upon the risk of the increase in

occupancy and the changes to aircraft navigation performance. Within this example only the effect upon the lateral risk has been considered. It should be noted that lateral navigation accuracy also affects the longitudinal and vertical risk.

2.2   The Reich model is typically used as a means of calculating the risk of collision in the NAT MNPS airspace. The expression for the lateral risk, C, can be written as:

$$C = P_y(S_y)P_z(O)\frac{\lambda_x}{S_x}E_{tot}$$

where

$$E_{tot} = E_s\left[\frac{|\bar{\dot{x}}_s|}{2\lambda_x} + \frac{|\dot{y}|}{2\lambda_y} + \frac{|\dot{z}|}{2\lambda_z}\right] + E_o\left[\frac{|\bar{\dot{x}}_o|}{2\lambda_x} + \frac{|\dot{y}|}{2\lambda_y} + \frac{|\dot{z}|}{2\lambda_z}\right]$$

Table A-7-1 defines the various parameters employed within the equation.

2.3 E_{tot} is related to the lateral occupancies and provides a measure of the traffic density within the system. It refers to the number of aircraft with which each aircraft in the system would be expected, at any one time, to form a proximate pair. Within the NAT MNPS, two aircraft form a proximate pair if they are longitudinally within 120 NM of each other while flying nominally at the same flight level and on adjacent tracks. If the two aircraft are flying in the same direction they form a same direction proximate pair and if they are travelling in opposite directions they form an opposite direction proximate pair.

2.4   From the Reich model, it can be seen that the risk, C, is proportional to E_{tot}, $P_y(S_y)$ — the lateral overlap probability — and $P_z(O)$ — the vertical overlap probability. Therefore, to prevent the risk increasing with the increase in traffic density it is necessary for either $P_y(S_y)$ or $P_z(O)$ to be

reduced. In this case, because of the improved lateral navigation accuracy, $P_y(S_y)$ is reduced. However, in order to assess whether or not this reduction is sufficient to offset the increase in traffic density it is necessary to perform a trade-off analysis between $P_y(S_y)$ and E_{tot}.

2.5 The current maximum tolerable collision risk for NAT MNPS airspace is 2×10^{-8} fatal accidents per flight hour, per dimension. Future RVSM operations will need to satisfy the more stringent TLS of 5×10^{-9}, and it is likely that any reduction in horizontal separation minima will also be associated with a more stringent TLS. Figure A-7-1 shows values of $P_y(S_y)$ and E_{tot} which satisfy these TLS values. The area of acceptable risk lies on and below the lines shown. It should be noted that the lines represent the maximum acceptable risk and, in order to allow for future traffic growth, the estimated risk should be below these lines.

2.6 By calculating E_{tot} and $P_y(S_y)$ for a proposed system the airspace planner can determine whether the risk in the proposed system is acceptable. For example, if E_{tot} is calculated as 1200 and $P_y(S_y)$ is estimated to be 5×10^{-8} the airspace planner could conclude that the proposed system would meet a TLS of 2×10^{-8} but not a TLS of 5×10^{-9}. In order to meet a TLS of 5×10^{-9} either the occupancy would have to decrease or the navigational performance of the aircraft would have to improve.

Table A-7-1. Parameters of the Reich collision risk model

Parameter	Description
S_y	Lateral separation minimum.
$P_y(S_y)$	Probability that two aircraft nominally separated by the lateral separation minimum are in lateral overlap.
$P_z(O)$	Probability that two aircraft on the same track are in vertical overlap.
S_x	Length of longitudinal window used to calculate occupancy.
λ_x	Average aircraft length.
λ_y	Average aircraft wingspan.
λ_z	Average aircraft height.
E_s	Same direction lateral occupancy.
E_o	Opposite direction lateral occupancy.
$|\bar{x}_o|$	Average relative along-track speed for opposite direction traffic.
$|\bar{x}_s|$	Average relative along-track speed for same direction traffic.
$|\bar{y}|$	Average relative cross-track speed.
$|\bar{z}|$	Average relative vertical speed.
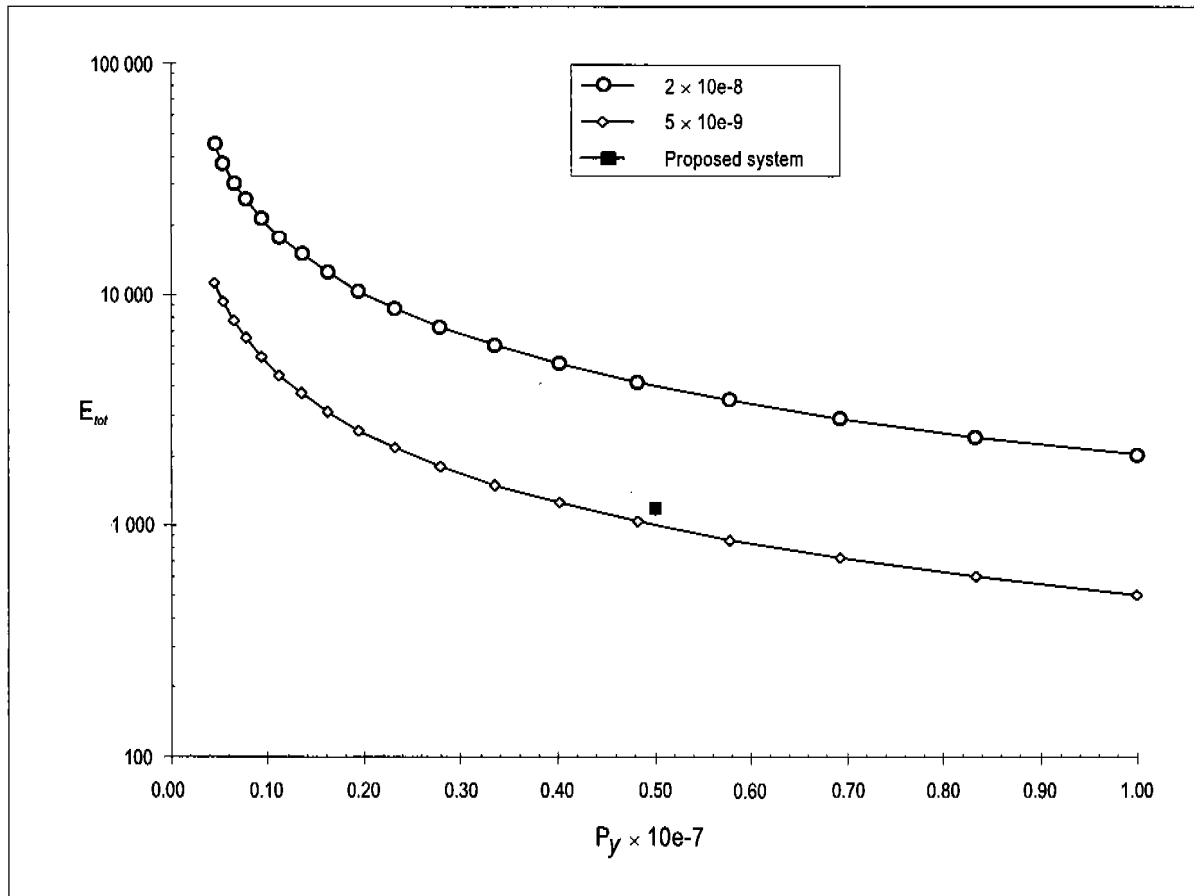
Figure A-7-1. Graph of values of P_y and E_{tot} which satisfy the TLS

Appendix 8

A COLLISION RISK MODEL FOR DETERMINING LATERAL SEPARATION MINIMA FOR ADS-BASED AIR TRAFFIC CONTROL

1. INTRODUCTION

1.1   This appendix describes a collision risk model, which can be used to determine lateral separation minima in an ADS-based air traffic control system. It summarizes two working papers (References 1 and 2)[1] which have been presented to Working Group A of the ICAO Review of the General Concept of Separation Panel (RGCSP).

1.2   The model to be described is based on the operational requirements (ORs) for a fully developed ADS system as described in Reference 3. Not all of the ORs have a direct impact on collision risk. The ORs that have been taken into account are:

OR 4	Comparison of the four dimensional (4D) profile stored in an aircraft system with the flight data stored in the flight data processing system (FDPS).
OR 8	Recognition that the aircraft has entered the ADS-ATC system.
OR 9	Confirmation that the aircraft's projected profile coincides with that stored in the FDPS.
OR 10	Verification by the FDPS that the aircraft is proceeding in accordance with ATC clearance.
OR 11	Provision to the controller of the most up-to-date traffic situation available using ADS-derived information.
OR 12	Provision of automatic position reporting in accordance with ADS agreements allocated by the ATC ground system.
OR 17	Self-monitoring and automatic reporting by the aircraft of significant flight variances.

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1. All references are listed at the end of this appendix.

OR 18   Aircraft notification of changes to position determination capability.

1.3   The model is an extension of the original Reich aircraft collision risk model in that it assumes that different types of lateral navigation errors may occur. Each of these error types produces its own probability of lateral overlap. The model examines the effect that the different ADS capabilities have on the detection and elimination or reduction of the different types of errors. This effect depends on the condition, e.g. failure state or normal operation, of the various elements of an ADS-based ATC system. Event trees provide a convenient tool for analysing all the different combinations of states which can exist. The collision risk model then calculates an overall lateral overlap probability as a weighted average of the lateral overlap probabilities associated with the individual navigation error types, with the weighting factors representing the proportion of time each error may be expected to occur in practice.

2. SYSTEM MODELLING ASSUMPTIONS

2.1 System description

2.1.1   Schematically, an ADS-ATC system can be represented as in Figure A-8-1. The FDPS is the part of the ADS-ATC unit taking care of the automation of the ADS-ATC system on the ground. Communication between the aircraft and the ADS-ATC system proceeds via data link. Each of the four surrounding elements, i.e. avionics, pilot, FDPS, and controller, can initiate messages to be transmitted via data link. Figure A-8-1 also shows a radio communication link between the pilot and the controller.

2.1.2   The following elements E_i (i=1,...,4) of an ADS-ATC system are distinguished in this paper:
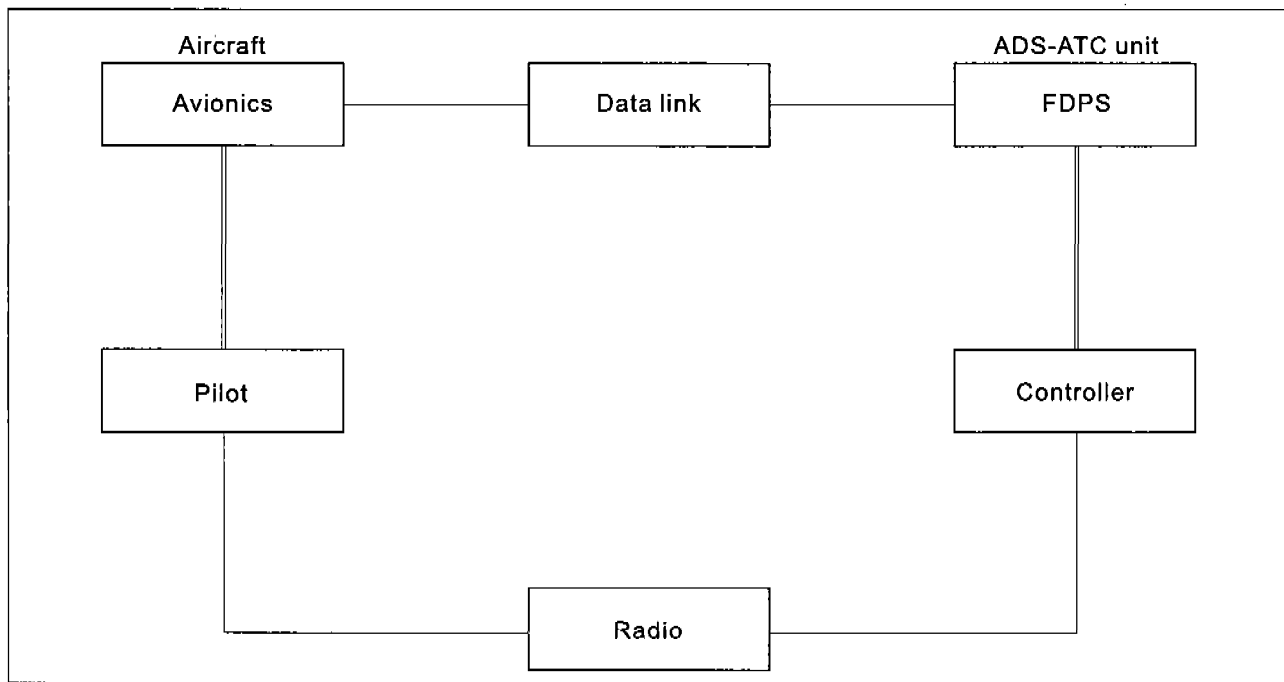
Figure A-8-1. Schematic representation of a possible ADS-ATC system

E_1: aircraft
E_2: data link subsystem
E_3: air situation generation subsystem
E_4: flight plan processing subsystem.

2.2 Data link and ground system

2.2.1   The elements E_3 and E_4 are subsystems within the FDPS. It is assumed that the flight plan processing subsystem E_4 is used for the comparison of the aircraft and system flight profile information as required with respect to ORs 4 and 9.

2.2.2   It is assumed further that each of the elements E_2 to E_4 can be in either of two states:

$$E_i = \begin{cases} 0 & \text{nominal performance} \\ & \\ 1 & \text{failure} \end{cases} \quad i = 2, 3, 4$$

2.2.3   For simplicity, it will be assumed that failure is equivalent to unavailability. As described in Reference 3 for ORs 4 and 9, the FDPS has to generate an alert when any delta (lateral, longitudinal, altitude, time) between aircraft and FDPS stored flight profile information exceeds the prevailing tolerances. The effect of the FDPS failing to

issue such an alert or the controller ignoring an alert issued is the same as if the FDPS were not available. Thus, the probability of failure or unavailability of the flight plan processing subsystem E_4 can be interpreted as the sum of the probabilities of real failure, failure to issue an alert when required and failure by the controller to respond to the alert.

2.2.4   The basic idea of References 1 and 2 is to consider various risk-initiating events resulting from the aircraft (E_1) and to consider the elements E_2, E_3 and E_4 as safety functions/systems or support functions/systems, which aim to mitigate the effects of the initiating events. This gives the general event tree of Figure A-8-2, where the branches going up at a node correspond to subsystem state 0 (nominal) and the branches going down correspond to subsystem state 1 (failure).

2.2.5   The next step is to calculate the probabilities associated with each path of the tree of Figure A-8-2. For this, the joint probability distribution of the system elements E_2, E_3 and E_4 needs to be specified. In fact, it is assumed that these elements are independent of E_1, the aircraft. It was also assumed in Reference 1 that the elements E_2, E_3 and E_4 were completely *independent*. The advantage of such an assumption is that the joint probability distribution can then be calculated as the product of the probability distributions of the individual elements.

Initiator	System			ADS-ATC system state	
An event related to E_1	E_2	E_3	E_4	Sequence number	State (E_2 E_3 E_4)
				1	000
				2	001
				3	010
				4	011
				5	100
				6	101
				7	110
				8	111
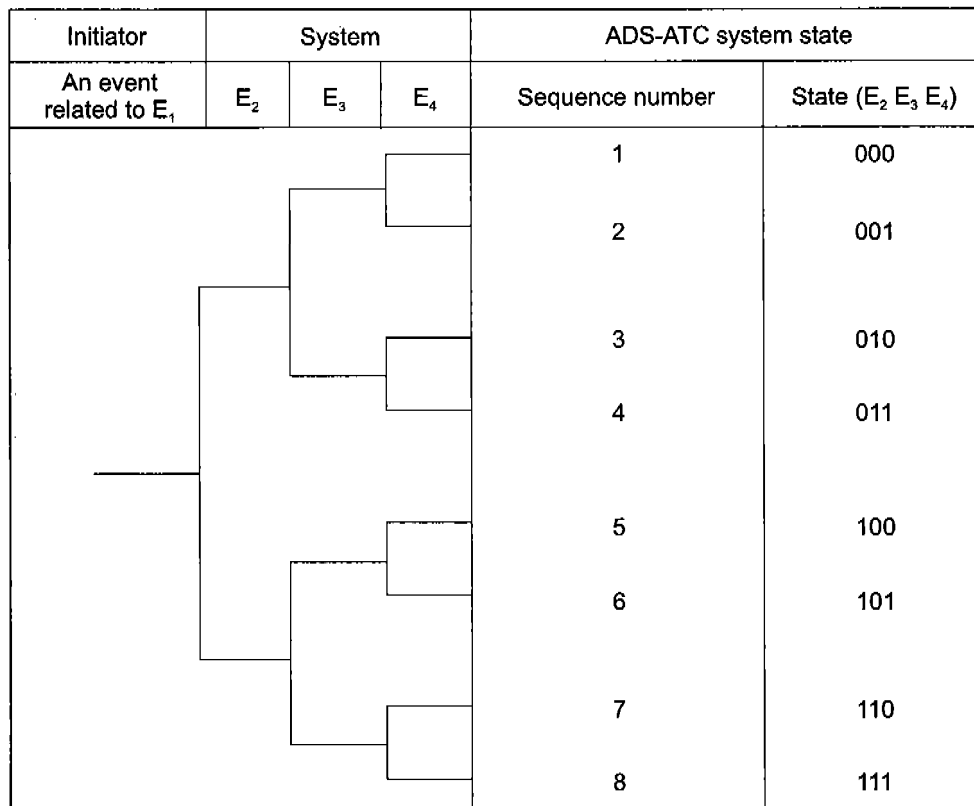
Figure A-8-2. General event tree for a risk-initiating event related to the aircraft E_1

2.2.6 This assumption, however, has been questioned with regard to E_3 and E_4, the air situation generation subsystem and the flight plan processing subsystem, because these systems are likely to share the same hardware. A hardware failure, therefore, may be expected to affect both E_3 and E_4. In addition to hardware failures, there may exist some software-related failures of E_3 and/or E_4. Depending on the degree of independent software development for E_3 and E_4, there may thus be some further dependence between these two system elements.

2.2.7 Although, in principle, it is possible to estimate failure rates of software, this is very difficult in practice. In the absence of such estimates, it is reasonable to assume that the software failure rates of the elements E_3 and E_4 are the same and that also the effect of hardware failures on E_3 and E_4 is the same. It then follows that their joint probability distribution is symmetrical in E_3 and E_4.

2.2.8 Reference 2 considered the other extreme case of complete *dependence* between E_3 and E_4, i.e. E_3 fails, if and only if E_4 fails, and E_3 gives nominal performance if and only if E_4 gives nominal performance. The independence of E_3 and E_4 to E_2 and E_1 was retained. It was shown that due to the assumed complete dependence the probabilities associated with the event Sequences 2, 3, 6 and 7 of Figure A-8-2 became zero and that, more importantly, those associated with the Sequences 4 and 8 became of the order of p_1 rather than $p_1 * p_1$ ($p_1 << 1$). The consequences of the risk-initiating events are likely to be the most serious for these two sequences, i.e. when both E_3 and E_4 are in failure mode. Consequently, only the event Sequences 1, 4, 5 and 8 need to be evaluated when complete dependence is assumed. This assumption will be retained in this paper.

2.3 The aircraft

2.3.1 The ADS-ATC system states of Figure A-8-2 hold for one particular risk-initiating event. Thus, all the risk-initiating events related to the aircraft need to be identified. To this end, the following three elements of an aircraft are distinguished:

A: pilot
B: navigation system } Flight Control System
C: data link interface

2.3.2 At the highest level, the navigation errors of an aircraft result from the flight control system (FCS). The FCS is dependent on correct programming by the pilot, the control algorithm and the position determination accuracy of the navigation system. The following states are distinguished for the pilot and the navigation system:

$$A = \begin{cases} 0 & \text{normal performance} \\ 1 & \text{way-point error} \\ 2 & \text{wrong-route error} \end{cases}$$

$$B = \begin{cases} 0 & \text{normal operation} \\ 1 & \text{degraded performance} \\ 2 & \text{failure} \end{cases}$$

2.3.3 For each of the nine possible combinations of states A and B, a separate navigation error distribution will be specified in Section 4. The effect of the control algorithm will be included in the navigation system state. In particular, the three pilot states combined with a navigation system failure, i.e. $B = 2$, may be interpreted as manual flying, whereas the remaining six cases may be interpreted as automatic flying.

2.3.4 Examples of navigation system degradation are gyro or accelerometer degradation in the case of an inertial navigation system or lane slip and sudden ionospheric disturbance for a system like Omega.

2.3.5 Finally, two states are distinguished for the aircraft data link interface, namely,

$$C = \begin{cases} 0 & \text{normal operation} \\ 1 & \text{failure} \end{cases}$$

2.3.6 In the same way as for the system elements E_2, E_3 and E_4, joint probabilities need to be defined for the states of the aircraft elements A, B and C. It is assumed that the aircraft elements A, B and C are completely independent. As element A refers to pilot errors and B and C refer to aircraft hardware, and given the way their states are defined, it can reasonably be assumed that element A is independent of elements B and C. Also, given the different type of the hardware elements B and C, it would seem reasonable to assume that B and C are independent. In this case it would hold that:

$$Prob\{A = j, B = k, C = l\} = Prob\{A = j\} *$$
$$Prob\{B = k\} * Prob\{C = l\}, \tag{1}$$
$$j, k = 0, 1, 2; \; l = 0, 1$$

$$Prob\{A = j\} = p_j^A, \; j = 0, 1, 2 \tag{2}$$
$$Prob\{B = k\} = p_k^B, \; k = 0, 1, 2 \tag{3}$$

$$Prob\{C = l\} = p_l^C, \; l = 0, 1 \tag{4}$$

with $p_1^A, p_2^A, p_1^B, p_2^B$ and p_1^C much smaller than 1.

2.3.7 It should be noted that a possible dependence between the states of the elements B and C can easily be modelled through conditional probabilities, for example see Reference 2. Given the very limited history of the data link interface technology for ADS applications, it is believed that it is currently not realistic to apply such a modelling for practical situations.

2.3.8 Table A-8-1[2] shows the full set of the eighteen combined states for the aircraft elements A, B and C in the four columns on the left (the other columns will be discussed in Section 4). Each of the eighteen combined aircraft states is a potential risk-initiating event in the sense that with each state is associated a probability of lateral overlap. Clearly, this probability of lateral overlap will be much smaller for the nominal aircraft state number 1 than for aircraft state number eighteen. On the other hand, as the probabilities other than p_0^A, p_0^B and p_0^C are much smaller than 1, some of the combined states of the elements A, B and C have a negligible probability of occurrence as compared to the others. The effect of a particular combination on system safety, however, depends on both the probability of that combination and its probability of lateral overlap as calculated from the distribution of the lateral navigation errors for that combination. These error distributions are considered in Section 4 and until then none of the above combinations will be neglected.

3. COLLISION RISK MODEL

3.1 Consider a typical pair of aircraft, i.e. both aircraft have the same probability densities $f_1(y_1)$ and $f_2(y_2)$ respectively. Let $f_{i,j,k}^{ABC}(y_1)$ denote the probability density of the lateral navigation errors of an aircraft labelled 1, which is in the combined state ijk with regard to its three elements A, B and C. It then holds that the overall probability density $f_1(y_1)$ of the lateral navigation errors of this aircraft can be expressed as

$$f_1(y_1) = \sum_{i=0}^{2} \sum_{j=0}^{2} \sum_{k=0}^{1} p_i^A \; p_j^B \; p_k^C \; f_{i,j,k}^{ABC}(y_1) \tag{5}$$

i.e. it is a mixture density containing (in this case) eighteen terms. It may happen that some of the component densities are identical for different sets of ijk values. The pertinent terms may then be combined.

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2. Tables A-8-1 to A-8-10 are listed at the end of this Appendix.

3.2 Similarly, for the other aircraft, labelled 2, the overall probability density of its lateral navigation errors is expressed as

$$f_2(y_2) = \sum_{p=0}^{2}\sum_{q=0}^{2}\sum_{r=0}^{1} p_p^{A}\ p_q^{B}\ p_r^{C}\ f_{p,q,r}^{ABC}(y_2) \qquad (6)$$

3.3 The number of parameters in equations (5) and (6) amounts to $3 + 3 + 2 = 8$. The law of total probability provides three normalization equations for these parameters, such that only five independent parameters need to be specified.

3.4 The lateral overlap probability for this pair of aircraft follows from

$$P_y(S_y) = Prob\Big\{\ \underline{d}\ \in\ \big[-\lambda_{y'}\ +\ \lambda_y\big]\Big\} =$$
$$\int_{-\lambda_y}^{\lambda_y} f_{\underline{d}}(x)\ dx\ \approx\ 2\lambda_y\ f_{\underline{d}}(0) \qquad (7)$$

where $\underline{d}$ denotes the lateral distance between the aircraft, i.e.

$$\underline{d} = S_y + \underline{y}_2 - \underline{y}_1 \qquad (8)$$

and

$$f_{\underline{d}}(d) = \int_{-\infty}^{\infty} f_1(y_1) f_2(d + y_1 - S_y) dy_1 \qquad (9)$$

Substitution of equations (5) and (6) into equation (9) gives

$$f_{\underline{d}}(d) = \sum_{i=0}^{2}\sum_{j=0}^{2}\sum_{k=0}^{1}\sum_{p=0}^{2}\sum_{q=0}^{2}\sum_{r=0}^{1} p_i^{A} p_j^{B} p_k^{C} p_p^{A} p_q^{B} p_r^{C}$$
$$\int_{-\infty}^{\infty} f_{i,j,k}^{ABC}(y_1) f_{p,q,r}^{ABC}(d + y_1 - S_y) dy_1 \qquad (10)$$

More details of the error densities $f_1(y_1)$ and $f_2(y_2)$ will be given in Section 4.

3.5 The above probability densities are dependent on the states of the elements E_2, E_3 and E_4 of the ADS-ATC system. This dependence can be made explicit by using the notation $f_1(y_1; E_2=\bullet, E_3=\bullet, E_4=\bullet)$ rather than $f_1(y_1)$ and similarly for $f_2(y_2)$. Consequently, the probability of lateral overlap $P_y(S_y)$ also depends on the states of the elements E_2, E_3 and E_4 which may be denoted more completely by $P_y(S_y; E_2=\bullet, E_3=\bullet, E_4=\bullet)$. It should be stressed that this dependence does *not* imply that additional parameters enter the model. All it means is that the state of $E_2E_3E_4$ determines *which* particular members of the set of basic densities $f_{i,j,k}^{ABC}(y_1)$ and $f_{i,j,k}^{ABC}(y_2)$ are present in the mixture densities equations (5) and (6).

3.6 It was tacitly assumed in the description above that only one type of navigation system was involved. The overall probability density of the lateral navigation errors of a typical aircraft can be derived in a similar manner for the case of M different types of navigation systems, each with their own navigation accuracy and failure and degradation characteristics. See Reference 1 for more details.

3.7 On the assumption that the traffic flows can be separated into same direction and opposite direction traffic, the collision risk model can be expressed as

$$N_{ay} = \begin{aligned}&[p_{20}p_{3040}\{P_y(S_y; E_2 = 0, E_3 = 0, E_4 = 0)\} +\\&p_{20}p_{3141}\{P_y(S_y; E_2 = 0, E_3 = 1, E_4 = 1)\} +\\&p_{21}p_{3040}\{P_y(S_y; E_2 = 1, E_3 = 0, E_4 = 0)\} +\\&p_{21}p_{3141}\{P_y(S_y; E_2 = 1, E_3 = 1, E_4 = 1)\}] +\end{aligned} \qquad (11)$$

$$\left[P_x(same)P_z(0)\left\{\frac{|\overline{\Delta V}|}{2\lambda_x} + \frac{|\overline{y}|}{2\lambda_y} + \frac{|\overline{z}|}{2\lambda_z}\right\} + P_x(opp)P_z(0)\left\{\frac{\overline{V}}{\lambda_x} + \frac{|\overline{y}|}{2\lambda_y} + \frac{|\overline{z}|}{2\lambda_z}\right\}\right]$$

where

$$p_{20} = Prob\{E_2 = 0\}; \quad p_{21} = Prob\{E_2 = 1\} \qquad (12)$$

$$\begin{aligned}p_{3040} &= Prob\{E_3 = 0\ ,\ E_4 = 0\} =\\ Prob\{E_3 &= 0\} = Prob\{E_4 = 0\} = p_0\end{aligned} \qquad (13)$$

$$\begin{aligned}p_{3141} &= Prob\{E_3 = 1\ ,\ E_4 = 1\} =\\ Prob\{E_3 &= 1\} = Prob\{E_4 = 1\} = p_1\end{aligned} \qquad (14)$$

3.8 The remaining symbols have their usual meaning as in the Reich collision risk model. In particular, $P_x(same)$ and $P_x(opp)$ denote the probabilities of longitudinal overlap of aircraft for same and opposite direction traffic respectively. These can be estimated by

$$P_x(same) = E_y(same)\frac{\lambda_x}{S_x}\ ;\quad P_x(opp) = E_y(opp)\frac{\lambda_x}{S_x} \qquad (15)$$

where $E_y(same)$ and $E_y(opp)$ denote the same and opposite direction lateral occupancy values respectively.

4. SYSTEM STATES AND LATERAL NAVIGATION ERRORS

4.1 . This section describes the types of navigation errors that are present under the different states of the ADS-ATC system. The effect of ADS on pilot-induced errors is examined first.

4.2 Effect of ADS on pilot-induced errors

4.2.1 As stated in Section 2, a consequence of the assumed complete dependence between the system elements E_3 and E_4 is that only the event Sequences 1, 4, 5 and 8 of

Figure A-8-2, have a non-zero probability. Therefore, only these four sequences are examined below.

4.2.2   The navigation errors of an aircraft are the combined effect of the navigation system and pilot induced errors. Table A-8-1 shows the possible combinations of these two component errors for an ATC system without ADS surveillance and intervention. A major function of ADS is to reduce or eliminate the effect of pilot-induced way-point and wrong-route errors. This is done by transmitting the aircraft projected flight profile to the ground and comparing it with the system flight profile and by regularly reporting the next and subsequent way-points to the ground elements of the ADS-ATC system. See ORs 4 and 9 of Reference 3. Thus, it is assumed that way-point errors and wrong-route errors are eliminated completely when the system elements E_2, E_3 and E_4 are in the normal operation state (state 0) and the aircraft itself is able to transmit the pertinent information. The latter depends on the state of the aircraft elements B and C.

4.2.3   The effect of the aircraft elements B (navigation system) and C (aircraft data link interface) on the capability of an aircraft to transmit its flight profile data is modelled as follows. It is clear that when the aircraft data link interface is not available, it will not be possible for an aircraft to transmit this data. Similarly, it is assumed (References 1 and 2) that the aircraft flight profile data are not accessible and cannot be transmitted when a navigation system failure has occurred. When a navigation system degradation exists, the flight profile data are normally accessible and available for transmission. In total, this results in four cases where the capability of transmitting flight profile data would not be available. In the absence of this capability, way-point insertion errors cannot be eliminated through a comparison with the FDPS stored data. This is likely to have only a very small effect on lateral collision risk, because it would require the simultaneous occurrence of at least two low probability events, i.e. way-point insertion error and data link interface and/or navigation system failure.

4.2.4   It then follows that state number 2 from Table A-8-1 will no longer exist in a correctly operating ADS-ATC system: i.e. the way-point errors will have been detected and eliminated. Table A-8-2 shows the possible combinations of navigation system and pilot-induced errors for the joint state 000 of $E_2E_3E_4$, i.e. event Sequence 1 of Figure A-8-2.

4.2.5   Consider now event Sequence 4 of Figure A-8-2, where both the systems E_3 and E_4 have failed. The joint state of $E_2E_3E_4$ is then 011. Due to the failure of the flight plan processing subsystem E_4, way-point and wrong-route errors cannot be detected and corrected. Thus, the resulting pilot-induced navigation errors present are the same as in an

environment without ADS. Table A-8-3, therefore, is identical to Table A-8-1.

4.2.6   When the data link subsystem fails, i.e. state 100 of $E_2E_3E_4$ and event Sequence 5 of Figure A-8-2, no corrective action is possible with regard to way-point and wrong-route errors and the result is the same as for state 011 (Table A-8-3).

4.2.7   Consider finally event Sequence 8 of Figure A-8-2, corresponding to state 111 of $E_2E_3E_4$. In this case, the way-point and wrong-route errors cannot be detected and corrected due to failure of both the data link subsystem E_2 and the flight plan processing subsystem E_4. The result is the same as for states 011 and 100 (Table A-8-3).

4.2.8   In summary, the three states 011, 100 and 111 of $E_2E_3E_4$ result in exactly the same lateral errors being present. Thus, the collision risk model equation (11) can be simplified by summing the last three groups of terms. The result is:

$$|N_{ay} = [p_{20}p_{3040}\{P_y(S_y; E_2= 0, E_3 = 0, E_4 = 0)\} + \qquad (16)$$

$$(p_{3141} + p_{21}p_{3040}) \{P_y(S_y; E_2 = 1, E_3 = 0, E_4 = 0)\}] *$$

$$\left[P_x(same)P_z(0)\left\{\frac{|\overline{\Delta V}|}{2\lambda_x} + \frac{|\bar{y}|}{2\lambda_y} + \frac{|\bar{z}|}{2\lambda_z}\right\} + P_x(opp)P_z(0)\left\{\frac{V}{\lambda_x} + \frac{|\bar{y}|}{2\lambda_y} + \frac{|\bar{z}|}{2\lambda_z}\right\}\right]$$

4.3   Effect of ADS on equipment-related errors

4.3.1   Consider now navigation system failures and degradations. It is clear that these remain as the main cause of lateral collision risk once way-point insertion errors (including wrong-route errors) have largely been eliminated. In fact, as noted in Reference 1, unauthorized users and equipment-related errors were responsible for the lateral collision risk in the analysis described by Davies (Reference 5) for a 100 per cent effective ADS system. Chénevier (Reference 8) also found that navigation system failures, represented by RNP-4 navigation system performance, rather than RNP-1 in his model, had an extremely adverse effect on system safety.

4.3.2   OR 18 of Reference 3 specifies that aircraft avionics will automatically report to the ground FDPS when an aircraft's navigation capability figure of merit (FOM) has changed. The controller will be alerted to this change. As a first step to model the effect of this OR it was assumed in Reference 1 that:

Aircraft that report a status change indicating navigation system failure will be treated by ATC in a special way such that they do not present any significant risk to any other aircraft. (This corresponds with the assumptions made by

Rome and Krishnan (Reference 10) and by Chénevier (Reference 8).)

4.3.3 As mentioned in Reference 2, this assumption has been questioned because it may not always be possible in a busy airspace to provide an alternative track or level with increased separation from other aircraft. Even if this were possible, it could take some time for ATC and the aircraft to achieve this new level/track. Thus, one approach would be to include in the model a small proportion of time for dealing with navigation system failures. A difficulty with this approach would be that this proportion of time might be variable, depending on the situation prevailing in the airspace, and, therefore, difficult to quantify. In addition, the actual way of implementing the FOM (and thus status reports of navigation system failures) is not yet well defined. Thus, a tentative, conservative approach would be to assume that navigation system failure situations will not be resolved.

4.3.4 Therefore, in this paper it is assumed that ADS has no impact on the consequences of navigation equipment degradations and failures.

4.4 Modelling individual lateral navigation error types

4.4.1 Probability distribution models are needed for the following types of lateral navigation errors:

navigation system related: normal operation degraded operation failure

pilot related: normal operation way-point insertion error wrong-route error

4.4.2 Based on a comparison of the models proposed in References 2 through 9, it was assumed by Reference 1 that each of the above types of navigation errors could be modelled by a Gaussian distribution with mean and standard deviation dependent on the type. Navigation system operation was further characterized by a zero mean value and a standard deviation dependent on the type of operation.

4.4.3 The assumption of Gaussianity is not critical to the modelling of the current paper or its predecessor Reference 1. It is because the convolution of two Gaussian densities conveniently produces another Gaussian density, thus eliminating the need for numerical convolution processes. Another argument put forward is that ADS will be likely to contribute to removing or cutting down the tails of mixture distributions describing conventional navigation error distributions.

4.4.4 Although the core distribution might well be Gaussian for some types of errors, this view does not seem to be generally shared by all of the experts. In this paper, therefore, it will be assumed that all of the individual navigation system error distributions are double exponential with a zero mean and a standard deviation dependent on the type of operation.

4.4.5 Normal pilot operation is also characterized by a zero mean value. The wrong-route error density is a mixture of two double exponential densities with respective mean values equal to plus and minus the prevailing lateral separation standard. The way-point error density consists essentially of two parts, one describing a way-point error to the left, the other describing a way-point error to the right. Each part is a double exponential mixture distribution with the means varying linearly between zero and plus or minus the separation standard S_y. This model is based on the well-known "triangular" shape of a way-point error. The longitudinal interval $2*L$ between three successive way-points is subdivided into $2*N_1^A$ sub-intervals and each sub-interval is assigned one of the terms of the mixture.

4.5 Modelling overall navigation errors per system state

4.5.1 The models for the individual lateral navigation error types described in Section 4 can be used to construct models for each of the 18 combinations of navigation system and pilot navigation errors in Tables A-8-2 and A-8-3. In this process, normal pilot performance means that no error needs to be added to the navigation system error, i.e. the contribution by the pilot is already included in the standard deviation of the navigation system error distribution. The distributions for the combinations of navigation system error and pilot induced way-point insertion or wrong-route errors are obtained by using the standard deviation of the navigation system error in the error distributions of the pertinent pilot error.

4.5.2 The densities thus obtained are the densities $f_{i,j,k}^{ABC}(y_1)$ and $f_{p,q,r}^{ABC}(y_2)$ defined in Section 2.

5. Application

5.1 A number of examples of the use of the collision risk modelling approach described in Sections 2 to 4 have been given in References 1 and 2. These examples were based on sets of parameter values taken from various sources and did not, therefore, apply to any specific airspace. In the current section, the NAT system will be considered. In particular, the feasibility of reducing the

current 60 NM lateral separation minimum to 30 NM will be examined. This question has already been partly addressed in References 4 and 5. The standard model currently in use for the NAT, i.e. the so-called "revised weighted errors model 2" was used together with some global assumptions on the effect of ADS on the safety in the system. In this context, it should be emphasized that the current modelling is not intended to replace the weighted errors model. Rather it should be seen as an alternative model, supporting the weighted errors model under similar conditions and extending it in some detail with regard to a number of parameters characteristic to an ADS-based ATC system.

5.2   As in previous examples, only a single type of navigation system is assumed to be present and all aircraft are assumed to be ADS-equipped. As it is currently not clear how a mixed ADS-environment would be implemented, this seems to be a reasonable assumption. As it was conservatively assumed in Section 4 that ADS would have no impact on the risk associated with navigation system failures and degradations, the main objective is to analyse the impact of ADS on the elimination of pilot-related errors, particularly as a function of the ADS system parameters.

5.3   A reference set of parameter values is given in Table A-8-4. These can be subdivided into three groups. The upper part of the table contains the usual Reich-type collision risk model parameters and was compiled from References 6, 7 and 9. The following three parameters, p_{21}, p_{31} and p_{41} denote the failure probabilities of the ADS data link subsystem (E_2), the air situation subsystem (E_3) and the flight plan processing subsystem (E_4) respectively. Reference 8 suggested the values shown in Table A-8-4, but they should be considered as preliminary. A sensitivity analysis with respect to these parameters will be carried out. Note from equation (15) in Section 4 that the dependence of the collision risk on these failure probabilities ($p_{3141}=p_{31}=p_{41}$) is readily analysed once the associated probabilities of lateral overlap have been calculated. These, in turn, depend on the last group of parameters in Table A-8-4, namely p_1^A, p_2^A, p_1^B, p_2^B and p_1^C. As can be seen from equation (10), the latter dependence is rather complicated.

5.4   Data on gross lateral navigation errors for the NAT are available in References 7 and 9. These errors are classified into classes A, B, C1, C2, D, E and F. No errors of class A, non-MNPS certified aircraft, were found in 1992 and 1993. Class B concerns ATC system loop errors, called wrong-route errors in this appendix. None were found in 1993 and two in 1992, one of which was a 2° error (to be counted as two errors). The numbers of flights in the two years were 213 905 and 204 040 respectively. This produces error probabilities of zero and 1.4703×10^{-5}. Class C

concerns different types of way-point errors, resulting in error probability estimates of 5.1425×10^{-5} and 3.4307×10^{-5} for the years 1993 and 1992 respectively. The classes D, E and F consist of other navigation errors, including navigation system failures. One error was found in these classes in 1993 and none in 1992, producing error probabilities of 4.675×10^{-6} and zero respectively. These values may be used both for navigation system failures and degradations as defined in this paper.

5.5   First, a worst-case combination of the error probabilities p_1^A, p_2^A, p_1^B, p_2^B and p_1^C was taken, combined with the NAT occupancy values for 1993. Table A-8-5 shows the calculated probability of lateral overlap for the different system states of $E_2E_3E_4$. Both the unweighted (middle column) and the weighted (last column) values are shown for each state. With regard to the overall value and the collision risk, only the weighted values are relevant. The unweighted values show what the lateral overlap probability would be if the airspace were in the pertinent state for all time. As was shown in Section 4, the (unweighted) probability of lateral overlap is the same for states 011, 100 and 111.

5.6   The probability of lateral overlap that would exist without ADS can also be read from Table A-8-5. It is the same value as the (unweighted) probability of lateral overlap for state 100, i.e. when the ADS data link would not be available. Thus, the probability of lateral overlap without ADS would be equal to 17.8×10^{-8}.

5.7   The overall weighted probability of lateral overlap in an ADS environment with the ORs 4 and 9 is 0.147×10^{-8}. As can be seen from Table A-8-5, this is essentially equal to the probability of lateral overlap of the airspace system in state 000, because of the very small proportions of time during which the subsystems E_2, E_3 or E_4 may fail. The associated collision risk is 2.74×10^{-10}, which is well below a proposed future TLS of 5×10^{-9} (Reference 11).

5.8   The same set of parameter values was used in combination with a reduced lateral separation minimum of $S_y = 30$ NM. The results are shown in Table A-8-6. The overall weighted probability of lateral overlap is 0.583×10^{-8} with an associated collision risk of 1.09×10^{-9}, which is amply below the proposed TLS.

5.9   A comparison of Tables A-8-5 and A-8-6 shows that the change in the separation minimum from 60 to 30 NM has a significant effect on the lateral overlap probability for the joint system state 000. It has only a minor effect for the other system states 011, 100 and 111. The latter is a consequence of the wrong-route errors not being eliminated for those states. In the example, these errors can be shown

to dominate the overlap probability and this is effectively independent of the prevailing separation minimum.

5.10 Next, the way-point and wrong-route error probabilities $p_1{}^A$ and $p_2{}^A$ have been doubled. The results for this case are shown in Table A-8-7 for $S_y = 60$ NM and in Table A-8-8 for $S_y = 30$ NM. The corresponding collision risk values are 2.81×10^{-10} and 1.10×10^{-9} respectively. Compared to the first example, the results appear to be fairly robust. This is a consequence of two effects, the low failure probabilities for the ADS subsystems and the low navigation system degradation and failure probabilities.

5.11 Thus, retaining the latest values of the error probabilities $p_1{}^A$ and $p_2{}^A$, the navigation system error probabilities were doubled also. Results for this case are shown in Tables A-8-9 and A-8-10. The overall weighted probabilities of lateral overlap are 0.294×10^{-8} and 0.115×10^{-7} for $S_y = 60$ and $S_y = 30$ NM respectively. The associated collision risk estimates are 5.49×10^{-10} and 2.15×10^{-9}.

5.12 Now, given the probabilities $P_y(S_y; E_2=0, E_3=0, E_4=0)$ and $P_y(S_y; E_2=0, E_3=0, E_4=0)$ for the combination of the largest pilot and navigation system error probabilities, the sensitivity of the collision risk to the failure probabilities p_{21} and p_{3141} of the ADS subsystems E_2, E_3 and E_4 can be evaluated through equation (16). Neglecting terms which are second order in the error probabilities, equation (16) can be approximated by:

$$N_{ay} \approx [P_y(S_y;000) + (p_{21} + p_{3141}) \\ \{P_y(S_y;100) - P_y(S_y;000)\}] * 0.186934 \quad (17)$$

Substituting for $P_y(S_y;000)$ and $P_y(S_y;100)$ from Table A-8-10 gives

$$N_{ay} \approx [1.15180 \times 10^{-8} + (p_{21} + p_{3141}) \\ \{41.2468 - 1.15180\} \times 10^{-8}] \times 0.186934 \quad (18)$$

or

$$N_{ay} \approx [1.15180 + (p_{21} + p_{3141}) \times 40.0950] \\ \times 10^{-8} \times 0.186934 \quad (19)$$

5.13 By equating the above expression to the TLS, all combinations of the failure probabilities p_{21} and p_{3141} can be determined, which would jointly be allowed to meet the TLS. It thus follows that the sum of the failure probabilities of the data link subsystem (E_2) and the flight plan processing subsystem (E_4) (or the air situation generation subsystem E_3) should not exceed a value of 2.8×10^{-2} in order to meet the TLS of 5×10^{-9} with a 30 NM lateral separation minimum.

6. CONCLUSIONS

6.1 A model for estimating the collision risk due to the loss of lateral separation between aircraft in an ADS-based ATC system, developed in two previous working papers, has been summarized.

6.2 The model explicitly takes into account the risk due to pilot-induced navigation errors and navigation system failures. The main feature of the model is the calculation of the overall probability of lateral overlap by appropriately weighting the probabilities of lateral overlap pertaining to particular states of the different subsystems of an ADS-ATC system. The resulting probability of lateral overlap is used in a Reich-type collision risk model for a hypothetical airspace with same and opposite direction traffic.

6.3 Pilot-related navigation is subdivided into normal operation, way-point insertion errors and wrong-route errors. Navigation system performance is subdivided into normal performance, degraded performance and failure. Each of these individual types of navigation errors is modelled by a double exponential probability distribution with mean and standard deviation dependent on the type of error. The latter quantities appear as parameters of the model for calculating the probability of lateral overlap. Other important parameters of this model are: the probabilities of the occurrence of the different pilot and navigation system errors, the probability of failure of an aircraft's data link interface and the availabilities of ADS subsystems. With regard to navigation system failures, a conservative approach has been followed in that it is assumed that ADS would not have any effect on their consequences.

6.4 Some example calculations have been carried out to examine the effect of ADS on the elimination of pilot-induced errors. Using recent data on gross navigational errors in the NAT region together with assumed values for the ADS system parameters, lateral collision risk estimates have been calculated for ADS in the NAT with a reference lateral separation minimum of 60 NM. The estimated risk has been compared with a future proposed TLS of 5×10^{-9} and the prospects for reducing the current separation minimum to 30 NM have been examined.

6.5 The preliminary results presented in this paper indicate that a future proposed TLS of 5×10^{-9} could be met in the NAT using a lateral separation minimum of 30 NM. Further work on accurate model parameters is necessary to confirm this conclusion.

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TABLES FOR APPENDIX 8

Table A-8-1. Navigation errors present without ADS

State number	E_1 A	E_1 B	E_1 C	Navigation system Normal error	Navigation system Degraded error	Navigation system Failure	Pilot Way-point error	Pilot Wrong-route error	Pilot Normal error
1	0	0	0	v					v
2	1	0	0	v			v		
3	2	0	0	v				v	
4	0	1	0		v				v
5	1	1	0		v		v		
6	2	1	0		v			v	
7	0	2	0			v			v
8	1	2	0			v	v		
9	2	2	0			v		v	
10	0	0	1	v					v
11	1	0	1	v			v		
12	2	0	1	v				v	
13	0	1	1		v				v
14	1	1	1		v		v		
15	2	1	1		v			v	
16	0	2	1			v			v
17	1	2	1			v	v		
18	2	2	1			v		v	

Table A-8-2. Navigation errors present in an ADS-ATC system based on ORs 4 and 9; system elements $E_2E_3E_4$ in state 000

E_1				Navigation system			Pilot		
State number	A	B	C	Normal error	Degraded error	Failure	Way-point error	Wrong-route error	Normal error
1	0	0	0	v					v
2	1	0	0	v					v
3	2	0	0	v					v
4	0	1	0		v				v
5	1	1	0		v				v
6	2	1	0		v				v
7	0	2	0			v			v
8	1	2	0			v	v		
9	2	2	0			v		v	
10	0	0	1	v					v
11	1	0	1	v			v		
12	2	0	1	v				v	
13	0	1	1		v				v
14	1	1	1		v		v		
15	2	1	1		v			v	
16	0	2	1			v			v
17	1	2	1			v	v		
18	2	2	1			v		v	

Table A-8-3. Navigation errors present in an ADS-ATC system based on ORs 4 and 9 for states 011, 100 and 111 of $E_2E_3E_4$

E_1				Navigation system			Pilot		
State number	A	B	C	Normal error	Degraded error	Failure	Way-point error	Wrong-route error	Normal error
1	0	0	0	v					v
2	1	0	0	v			v		
3	2	0	0	v				v	

E_1				Navigation system			Pilot		
State number	A	B	C	Normal error	Degraded error	Failure	Way-point error	Wrong-route error	Normal error
4	0	1	0		v				v
5	1	1	0		v		v		
6	2	1	0		v			v	
7	0	2	0			v			v
8	1	2	0			v	v		
9	2	2	0			v		v	
10	0	0	1	v					v
11	1	0	1	v			v		
12	2	0	1	v				v	
13	0	1	1		v				v
14	1	1	1		v		v		
15	2	1	1		v			v	
16	0	2	1			v			v
17	1	2	1			v	v		
18	2	2	1			v		v	

Table A-8-4. Parameter values for application of Section 5

Parameter	Source of information				
	Reference 9 NAT 1992	Reference 7 NAT 1993	Reference 6 NAT 1990	Reference 8 Chénevier	Reference 10 Rome & K
E_y(same)	0.980	1.002			
E_y(opp)	0.004	0.005			
S_y			60 NM		
S_x			120 NM		
$P_z(0)$			0.39		
λ_x			0.033 NM		
λ_y			0.029 NM		
λ_z			0.009 NM		

Parameter	Source of information				
	Reference 9 *NAT 1992*	*Reference 7* *NAT 1993*	*Reference 6* *NAT 1990*	*Reference 8* *Chénevier*	*Reference 10* *Rome & K*
$|\overline{\Delta V}|$			13 kts		
$\overline{V}$			480 kts		
$|\overline{y}|$			80 kts		
$|\overline{z}|$			1.5 kts		
p_{21}				4.0×10^{-6}	
p_{31} $(=p_1)$				4.0×10^{-6}	
p_{41} $(=p_1)$				4.0×10^{-6}	
L					300 NM
p_1^{C}				2.0×10^{-4}	
p_1^{B}	0.0	4.675×10^{-6}			0.0035
p_2^{B}	0.0	4.675×10^{-6}			0.0027
p_1^{A}	3.4307×10^{-5}	5.1425×10^{5}			
p_2^{A}	1.4703×10^{-5}	0.0			
σ_0^{B}	2 NM				
σ_1^{B}					20 NM
σ_2^{B}					40 NM

Table A-8-5. Unweighted and weighted probabilities of lateral overlap for different system states

$p_1^{A} = 5.1425 \times 10^{-5}, p_2^{A} = 1.4703 \times 10^{-5}, p_1^{B} = 4.675 \times 10^{-6}, p_2^{B} = 4.675 \times 10^{-6}$
$S_y = 60$ NM

State of system elements $E_2E_3E_4$	$P_y\{60;E_2=\bullet,E_3=\bullet,E_4=\bullet\}\times10^{-8}$	$P_y\{60;E_2=\bullet,E_3=\bullet,E_4=\bullet\}*$ $*\mathrm{Prob}\{E_2=\bullet,E_3=\bullet,E_4=\bullet\}\times 10^{-8}$
000	0.147	0.147
011	17.8	0.710×10^{-4}
100	17.8	0.710×10^{-4}
111	17.8	0.710×10^{-4}

Table A-8-6. Unweighted and weighted probabilities of lateral overlap for different system states

$p_1^A = 5.1425 \times 10^{-5}$, $p_2^A = 1.4703 \times 10^{-5}$, $p_1^B = 4.675 \times 10^{-6}$, $p_2^B = 4.675 \times 10^{-6}$
$S_y = 30$ NM

State of system elements $E_2E_3E_4$	$P_y\{30;E_2=\bullet,E_3=\bullet,E_4=\bullet\} \times 10^{-8}$	$P_y\{30;E_2=\bullet,E_3=\bullet,E_4=\bullet\}*$ $*\text{Prob}\{E_2=\bullet,E_3=\bullet,E_4=\bullet\} \times 10^{-8}$
000	0.583	0.583
011	20.63	0.825×10^{-4}
100	20.63	0.825×10^{-4}
111	20.63	0.825×10^{-4}

Table A-8-7. Unweighted and weighted probabilities of lateral overlap for different system states

$p_1^A = 1.0285 \times 10^{-5}$, $p_2^A = 2.9407 \times 10^{-5}$, $p_1^B = 4.675 \times 10^{-6}$, $p_2^B = 4.675 \times 10^{-6}$
$S_y = 60$ NM

State of system elements $E_2E_3E_4$	$P_y\{60;E_2=\bullet,E_3=\bullet,E_4=\bullet\} \times 10^{-8}$	$P_y\{60;E_2=\bullet,E_3=\bullet,E_4=\bullet\}*$ $*\text{Prob}\{E_2=\bullet,E_3=\bullet,E_4=\bullet\} \times 10^{-8}$
000	0.150	0.150
011	35.4	0.141×10^{-3}
100	35.4	0.141×10^{-3}
111	35.4	0.141×10^{-3}

Table A-8-8. Unweighted and weighted probabilities of lateral overlap for different system states

$p_1^A = 1.0285 \times 10^{-5}$, $p_2^A = 2.9407 \times 10^{-5}$, $p_1^B = 4.675 \times 10^{-6}$, $p_2^B = 4.675 \times 10^{-6}$
$S_y = 30$ NM

State of system elements $E_2E_3E_4$	$P_y\{30;E_2=\bullet,E_3=\bullet,E_4=\bullet\} \times 10^{-8}$	$P_y\{30;E_2=\bullet,E_3=\bullet,E_4=\bullet\}*$ $*\text{Prob}\{E_2=\bullet,E_3=\bullet,E_4=\bullet\} \times 10^{-8}$
000	0.587	0.587
011	40.7	0.163×10^{-3}
100	40.7	0.163×10^{-3}
111	40.7	0.163×10^{-3}

**Table A-8-9. Unweighted and weighted probabilities of
lateral overlap for different system states**

$p_1{}^A = 1.0285 \times 10^{-5}, p_2{}^A = 2.9407 \times 10^{-5}, p_1{}^B = 9.350 \times 10^{-6}, p_2{}^B = 9.350 \times 10^{-6}$
$S_y = 60$ NM

State of system elements $E_2E_3E_4$	$P_y\{60;E_2=\bullet,E_3=\bullet,E_4=\bullet\} \times 10^{-8}$	$P_y\{60;E_2=\bullet,E_3=\bullet,E_4=\bullet\}*$ $*\text{Prob}\{E_2=\bullet,E_3=\bullet,E_4=\bullet\} \times 10^{-8}$
000	0.293	0.293
011	35.5	0.142×10^{-3}
100	35.5	0.142×10^{-3}
111	35.5	0.142×10^{-3}

**Table A-8-10. Unweighted and weighted probabilities of
lateral overlap for different system states**

$p_1{}^A = 1.0285 \times 10^{-5}, p_2{}^A = 2.9407 \times 10^{-5}, p_1{}^B = 9.350 \times 10^{-6}, p_2{}^B = 9.350 \times 10^{-6}$
$S_y = 30$ NM

State of system elements $E_2E_3E_4$	$P_y\{30;E_2=\bullet,E_3=\bullet,E_4=\bullet\} \times 10^{-8}$	$P_y\{30;E_2=\bullet,E_3=\bullet,E_4=\bullet\}*$ $*\text{Prob}\{E_2=\bullet,E_3=\bullet,E_4=\bullet\} \times 10^{-8}$
000	1.152	1.152
011	41.2	0.165×10^{-3}
100	41.2	0.165×10^{-3}
111	41.2	0.165×10^{-3}

Appendix 9

THE EUROCONTROL HAZARD/RISK ANALYSIS METHODOLOGY

1. BACKGROUND

1.1 In future, RNP 1 routes will become available allowing, in principal, a reduction in spacing from the present 16 NM between centre lines to something approaching 6-8 NM. The navigation performance of RNP-1 is defined as having a 95 per cent containment value of 1 NM. However, this value defines only the achieved track-keeping performance of the navigation system. ATC system loop errors (blunders) and navigation system failures outside of this core are potentially very much larger. Hence, the route spacing achievable without considering controller intervention and based solely on collision risk modelling may be little better than that for present day ATS routes.

1.2 Collision risk modelling, taking into account the track-keeping performance, traffic density and the target levels of safety, without ATC ability to intervene, has been applied to route spacing for North Atlantic operations. European airspace, for which route-spacing standards were developed prior to the establishment of collision risk modelling techniques, includes radar surveillance for deviation monitoring and direct pilot/controller VHF voice communications, thus permitting ATC intervention to avoid potential losses of separation.

1.3 An initial study, sponsored by EUROCONTROL, examined the feasibility of using hazard analysis as an input into the development of minimum spacing between RNP-1 ATS routes. This study showed that hazard/risk analysis is a promising technique and that further work in developing a complete collision risk model, including the reduction in risk associated with the availability of surveillance and direct voice communication, would be desirable.

1.4 A follow-up study has been commissioned and is currently under way.

2. AIMS OF THE HAZARD ANALYSIS PROGRAMME

2.1 Hazard analysis originated in the development of automatic landing systems but has been developed substantially as a result of its application in high-technology industries, e.g. off-shore drilling, nuclear energy, for the assessment of the risks associated with the role of the human in the control loop.

2.2 This study aims to integrate the more conventional collision risk modelling (statistical analysis) with a detailed knowledge of the types, mechanisms and frequency of occurrences of deviations caused by ATC system loop errors located throughout the air traffic system. In addition, the programme takes account of the ability to use radar surveillance and VHF voice communications to eliminate perceived deviations and reduce risk.

2.3 The eventual aim of this programme is the development of RNP-1 lateral route spacing standards for application in continental European airspace. The methodology used in the study is adaptable, and could be used in applications in other regions or States, as appropriate.

3. METHODOLOGY

3.1 The overall methodology for the application of hazard analysis to the problem of collision risk in the presence of an ATC capability is shown schematically in Figure A-9-1 below.

3.2 Elements of the collision risk model

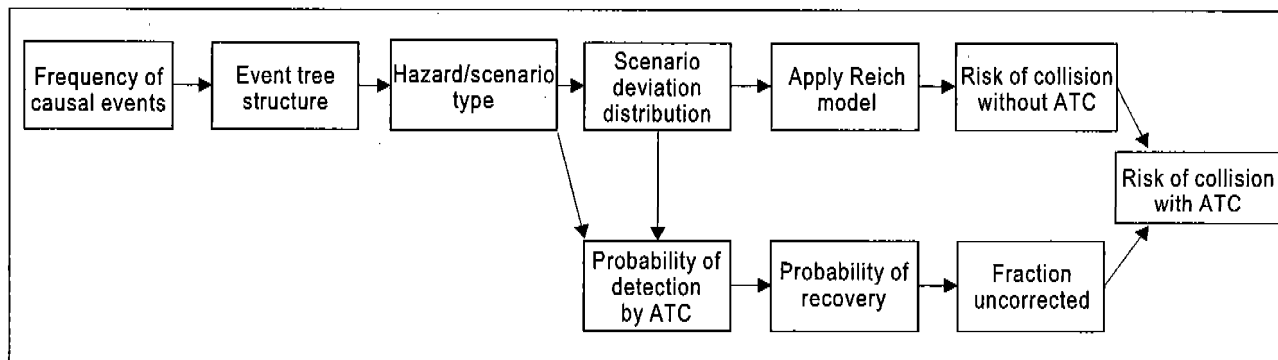3.2.1 The main elements of the collision risk model include the following:

Figure A-9-1. Collision risk model structure

a) the identification of scenarios which may lead to a loss of separation or a possible collision between two aircraft;

b) the identification of specific hazards and the frequency with which they occur. Each hazard leads individually, or in combination with other circumstances, to a deviation scenario with an attendant deviation distribution;

c) the calculation of the probable collision risk arising from the above scenarios assuming that the deviations are permitted to continue uncorrected (Reich model);

d) the calculation of the probability of deviation detection and correction by either the pilot or ATC. The different modes of ATC intervention, i.e. conflict avoidance or deviation correction modes, will affect the time dependence of different hazard types and their detection and recovery strategies; and

e) the deduction of the resultant overall probability of collision despite the surveillance capability. The probability of collision (in the absence of corrective action) is factored by the probability of non-detection and recovery before the collision occurs.

3.3 Notes on methodology

3.3.1 In the formulation of the model particular care has been taken to correctly represent the behaviour of both common mode failures (CMF), i.e. where a single hazard can cause the simultaneous malfunction or failure of several system elements, as well as events which might cause localized peaks in the tails of the deviation distributions of consequent importance when assessing the interaction/

convolution of the distributions. The current model takes account only of aircraft in level flight.

3.4 Model capability and output

3.4.1 The model will be capable of calculating the risk under given conditions and for various track spacings. The comparison between those risks and the applied TLS will help to determine the minimum track spacing. In addition, the model will determine the risk sensitivity to the various hazards and provide useful feedback with respect to the relationships between causal hazards and resultant risk. Finally, the model will predict other tangible events, which may be used for validation.

4. SCENARIOS

4.1 A number of scenarios are being examined and are shown below. These do not represent a complete set of possible scenarios but represent the types of resultant deviations induced by the identified hazards.

4.2 It should be noted that Figure A-9-2 could also represent a gentle wandering about the track centre line rather than a single drift off course. Also, the deviation may start, not from the centre line, but from an already offset track parallel to the centre line.

5. IDENTIFICATION OF HAZARDS AND DETERMINING THEIR FREQUENCY

5.1 Thus far, only hazards leading to lateral deviations have been considered.
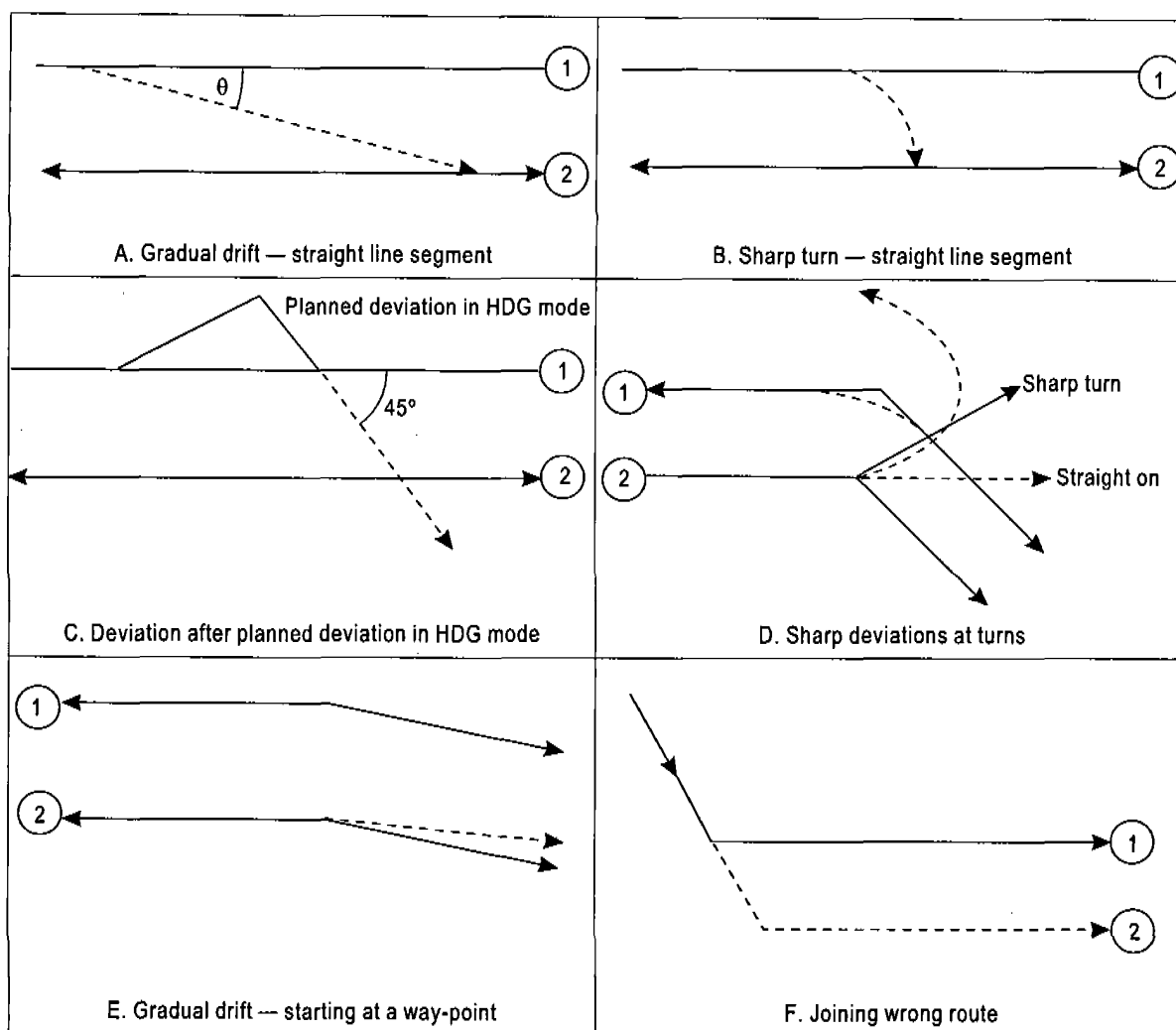
Figure A-9-2. Deviation scenarios

5.2 Hazard identification

5.2.1 Examples of the types of errors that are known to cause lateral deviations include the following:

a) general navigation capability and variability including navaid quality, database errors and the carriage of navigation equipment inadequate for RNP-1 routes;

b) flight crew error including incorrect data entry, way-point entry, cycling failure and general distraction;

c) ATC error including incorrect sector handover and controller distraction;

d) miscommunication between ATC and pilot, including call-sign confusion and the wrong aircraft responding to ATC instructions; and

e) administrative and system errors, including flight plan errors, misleading NOTAMs, aircraft equipment failures and software errors.

5.2.2 Even when these errors are noticed during a flight, many are considered to be of no consequence and are not reported. Ad hoc cockpit/control room measures tend to be unofficially developed for minor problems that are encountered on a regular basis. However, it is often not an individual fault that causes a problem, but when two or more occur in tandem with other minor problems, a significant deviation can result. Relatively minor problems will potentially become more important with the application of RNP-1.

5.2.3   The hazards were identified by a combination of the following techniques:

a)   searches of incident reports, databases, etc.; and

b)   formal hazard identification and analysis sessions.

5.3 Searches of incident reports and incident databases

5.3.1   Many hazards have been extracted from the statutory reporting schemes and studies compiled by national or international authorities.

5.3.2   Among other sources, the EUROCONTROL study has relied heavily on information from operators in the European region. This data includes events reported voluntarily by flight crews, operators and ATS providers (controllers) in addition to the mandatory event reports by the same groups.

5.3.3   Lower-risk events can also be significant, but considerable amounts of data must be available to provide a representative sample of statistics. The problem associated with data gathering systems has been that normally only significant events have been recorded, which produces too small a sample of data to produce meaningful statistics. This study has been particularly interested in the potential hazards noted in the interaction between the flight crew and the controllers.

5.4 Formal hazard identification and analysis sessions (HAZOP)

5.4.1   HAZOP is a technique used to determine the likely hazards and consequences within a high-technology environment in which humans form a major link in the decision processes. A team of four or five experienced personnel draft a checklist on which the hazard identification sessions themselves are based. Each session is attended by some ten specialist personnel representing flight crews, controllers, and equipment manufacturers who are guided through the checklist of potential risk-inducing situations. The specialists are then invited to offer their opinions on:

a)   likely causes;

b)   possible safeguards; and

c)   possible consequences.

5.4.2   At the hazard identification sessions no attempt is made to quantify the risks associated with the hazards or the frequency with which the initiating hazards occur.

5.5 Hazard frequency and ranking

5.5.1   The relative importance of the various initiating hazards is determined by estimating the frequency of occurrence and the potential resultant risk.

5.5.2   Hazard frequency estimation is carried out by consideration of the various data sources with additional information being derived from other sources (radar recordings, etc.) where available. Finally, a panel of experts is convened to judge the validity of these estimates. Views expressed at the hazard frequency estimation session are incorporated into a questionnaire concerning specific hazards, their consequences, detection and correction. This is then sent to a wider selection of cooperating pilots and controllers.

5.5.3   During the course of estimating the frequency, and potential resultant risk, some hazards stand out as being major sources of risk in terms of both likelihood and severity. It is necessary to rank these key hazards in order of their importance and to try to estimate their frequency of occurrence with greater accuracy, since they have a relatively large effect on the final system risk.

6. EVENTS, DETECTION AND RECOVERY

6.1   It is evident from Figure A-9-1 that a combination of hazards interact to cause a particular type of deviation. The consequences of this deviation, and the possibility of it developing into an incident, is determined by a similar set of interactions, which are most easily assessed as a detection and recovery tree.

6.2 Important factors in detection

6.2.1   Factors that are integral to the detection of a deviation include the following:

a)   deviation type, traffic levels, ATC and pilot workload;

b)   whether the deviation occurs when a turn is expected;

c)   whether the deviation occurs during a sector handover; and

d) the surveillance capability, including radar separation minima, basic radar accuracy, filtering and resolution.

6.2.2 The availability of alerting systems, the nature of the displays, the communication system, etc., all contribute to the variation in detection time.

6.3 Important factors in recovery

6.3.1 The following are important factors in the recovery from a deviation:

a) delays due to misidentification of aircraft;

b) misdirected corrective instruction or poor corrective manoeuvre; and

c) the time remaining in which to take corrective action.

6.4 Simple deviation and recovery model

6.4.1 A simple model has been developed to simulate an aircraft deviating within the scenarios described in section 4. The possibility of flight crew and ATC detection, the correction reaction times and eventual recovery to the appropriate separation minimum have been included. The aim is to determine the probability of the deviating aircraft infringing on the adjoining track.

6.5 Conflict detection and resolution for a given scenario

6.5.1 Event trees associated with each deviation scenario have been determined and parameters (probabilities and time-scales) applied to:

a) the detection of a deviating aircraft by ATC;

b) the ability to communicate that fact to the deviating aircraft; and

c) the ability of that aircraft to successfully complete a corrective (avoiding) manoeuvre.

6.5.2 The resultant structures are extremely complicated, and not all of the potential breakdowns of the tree can be readily analysed. However, the complexity has been reduced by assuming a more limited set of corrective time-scales, and deducing the likelihood of correction

before the application of short term conflict alert or reaching the closest point of approach, as shown in Figure A-9-3.

6.5.3 The capability of ATC to detect deviations is dependent on a number of circumstances described previously but particular attention should be paid to the likelihood of a CMF occurring.

6.6 Overall system collision risk under ATC

6.6.1 Figure A-9-4 is a further expansion of the structure of Figure A-9-3 but accumulated over N possible scenarios. The probability of a collision arising from a given scenario (in the absence of ATC) is given by P_N, but with ATC surveillance the deviation may be detected and corrected with varying probability right up to the time of collision. If the cumulative probability of this corrective action being applied successfully is PC_N per cent then the resultant probability of the collision occurring will be $P_N *$ $(100 - PC_N)$ per cent. The overall probability of a collision arising will now be the sum over all of the scenarios.

6.7 Future factors affecting hazard detection and correction

6.7.1 Other factors, which have not been included at this stage, but which may be increasingly important in future, are those automated features, which enable the ATS system to predict and detect specific hazards and to suggest optimum corrective strategies. These include:

a) on-board equipment that can detect drift from track or potential collision risk, (e.g. receiver autonomous integrity monitoring (RAIM), aircraft autonomous integrity monitoring (AAIM), traffic collision alert and avoidance system (TCAS)); and

b) automated ATC capabilities including automatic intruder alerts, which can highlight poor handovers and monitor manoeuvres close to a boundary.

7. MODEL SENSITIVITY

7.1 Part of the hazard/risk analysis study carries out a sensitivity analysis to assess the effect of various estimated values and model simplifications on the calculated system risk. It is important that key factors/parameters are determined at an early stage to ensure that accurate assessments

Deviation conflict detected		Conflict detected at STCA	Conflict resolved before CPA	Outcome	Probability
Deviation on collision course			Yes	No collision	
	Early		No	Collision	A %
			Yes	No collision	
	Later		No	Collision	B %
			Yes	No collision	
			No	Collision	C %
		Yes	Yes	No collision	
	Very late		No	Collision	D %
		No		Collision	E %
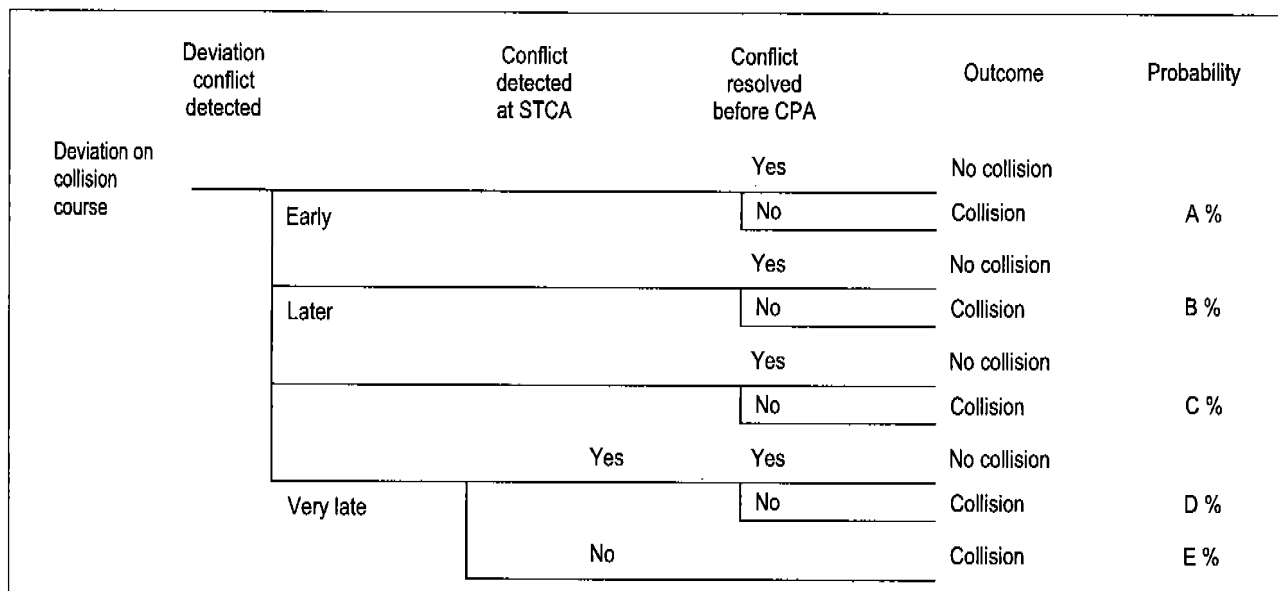
Figure A-9-3. Detection and correction tree

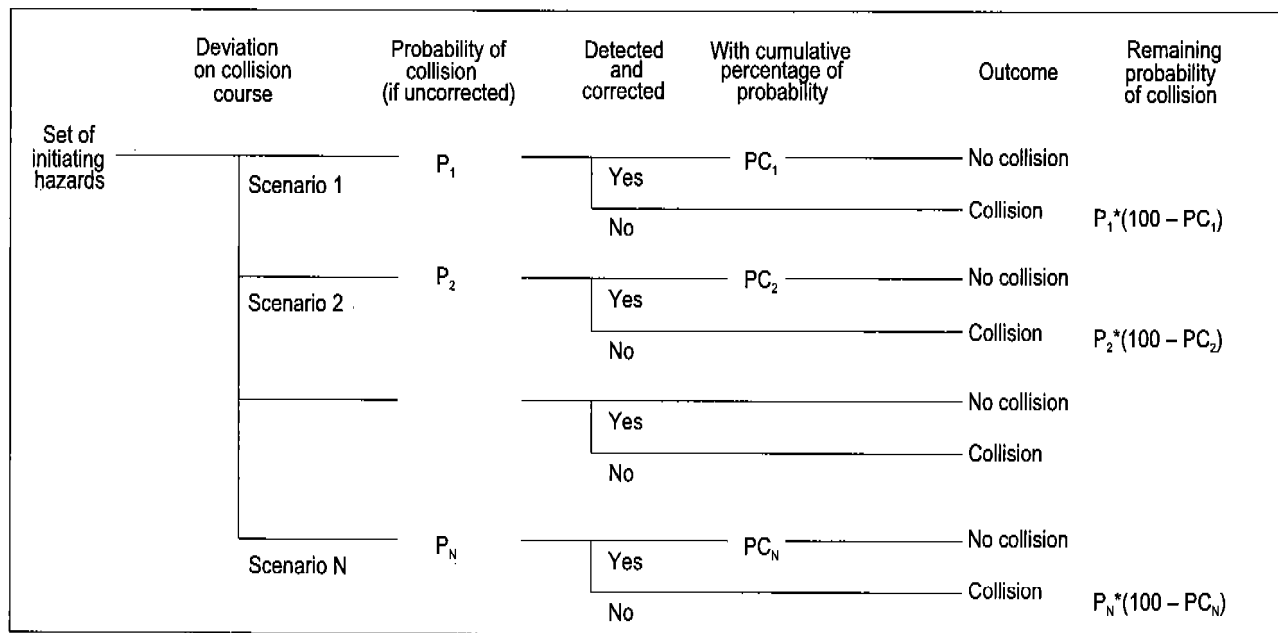| Deviation on collision course		Probability of collision (if uncorrected)	Detected and corrected	With cumulative percentage of probability	Outcome	Remaining probability of collision
Set of initiating hazards						
	Scenario 1	P_1	Yes	PC_1	No collision	
			No		Collision	$P_1{}^*(100 - PC_1)$
	Scenario 2	P_2	Yes	PC_2	No collision	
			No		Collision	$P_2{}^*(100 - PC_2)$
			Yes		No collision	
			No		Collision	
	Scenario N	P_N	Yes	PC_N	No collision	
			No		Collision	$P_N{}^*(100 - PC_N)$

Figure A-9-4. Collision risk with ATC detection and correction

are made of those parameters having a major influence on the result. The assessment methodology will also allow an evaluation of the impact of different environments on the TLS.

8. VALIDATION

8.1 Various practical data analyses have been carried out in the past, but it is very difficult to identify deviation statistics arising from specific causes. Those carried out by ICAO (1976) and EUROCONTROL (1982-84) give some results in relatively simple scenarios where attempts have been made to isolate any effects of ATC intervention. Even in these cases the tails of the deviation distributions are open to speculation.

8.2 In this case, the methodology predicts the distributions arising from specific combinations of hazards, different operating areas with different traffic levels, etc., and then applies the risk calculation to predict the resultant rates of infringement of separation. These rates of infringement are then compared with radar recordings to ensure that the recorded and predicted rates are in agreement over a wide range of infringement radii and circumstances.

9. APPLICATION

9.1 The collision risk model (CRM) is applied to suitable routes selected by the evaluation authority. Data on traffic densities, the route network environment and passing frequencies are made available for input to the model, which determines the separation required to achieve the required TLS.

10. CONTINUING PROGRAMME

10.1 In the European study, the immediate task is to continue the development of the deviation, detection and correction model with the inclusion of more complete scenarios and the validation of the early results.

10.2 Sensitivity analysis, which should be applied in the last stages of the model development, indicates those parts of the model that require future work through modification of the complexity or by developing more precise estimates of frequency of occurrence. The completed model can be used to calculate the risk associated with varying spacing minima and to determine what spacings will permit operations in accordance with the required TLS.

10.3 Further research should be concentrated on extending the understanding of ATC system loop error mechanisms and ATC intervention success rates, using both detailed data collections and additional real-time simulation studies. The impact of automatic deviation and other alerts or ATC tools need to be assessed as they are brought on line.

Appendix 10

APPLICATION OF RISK ANALYSIS TO AIRSPACE PLANNING IN AUSTRALIA

1. INTRODUCTION

1.1   The airspace planning methodology for the determination of separation minima outlines two basic methods of determining whether a system is acceptably safe. Firstly, comparison with a reference system can be made, provided there is an extensive history of system safety in terms of system flight hours. It is noted that this method is impractical for low traffic density.

1.2   The second method is to evaluate system risk against a threshold TLS. Safety critical parameters have to be identified and their effect on collision risk modelled. The risk analysis process follows the traditional steps of system definition, setting evaluation criteria, hazard identification, frequency estimation, consequence modelling, risk estimation, risk evaluation and risk reduction measures, if required. A TLS of 1.5×10^{-8} fatal accidents due to collisions per system flight hour is recommended.

1.3   The guidance material recommends techniques such as mathematical modelling, expert judgement and comparison with other similar operations. It also suggests using panels of operational experts guided by a trained facilitator, who will also use the large amount of accumulated knowledge on likely error rates in other industries.

1.4   This paper describes the use of just such an approach by Airservices Australia to address the risk of various airspace classification and technology options for the sparsely settled interior of continental Australia. A base case has been established for existing risk levels and an airspace risk model (ARM) developed.

1.5   The major part of Australia, outside the eastern seaboard, capital cities and major towns, is uncontrolled airspace, and will eventually be categorized as ICAO Class G. There are concerns about the collision risk that regular public transport (RPT) may be exposed to at certain uncontrolled rural aerodromes such as Dubbo, Ayers Rock

and Kununurra. These aerodromes have a mandatory broadcast zone (MBZ) and may require ICAO Class E airspace en route.

2. DUE DILIGENCE

2.1   The application of engineering techniques to the preservation of life and the protection of property assets has been well established in the process industries (Reference 1)[1]. Until recently, however, these techniques have seen little application in other industries. A multidisciplinary approach to risk management needs to consider both technology and human factors.

2.2   A common law duty of care exists for a safe work place and systems of work. The obligation to ensure that risk is "... as low as reasonably practical (ALARP)" is also enshrined in Australian Occupational Health and Safety (OH&S) legislation. To be found guilty of negligence, the answer to all four of the following questions needs to be "yes", on a balance of probability basis.

2.2.1   Question 1: Causation

Did the injury occur because of the "unsafe" matter on which the claim of negligence is based?

2.2.2   Question 2: Foreseeability

Is it possible to foresee that this injury could happen?

2.2.3   Question 3: Preventability

Is there a practical alternative to doing this job this way or with equipment within the employer control?

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1. All References are listed at the end of the appendix.

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2.2.4 Question 4: Reasonableness

What is the balance of the significance of the risk versus the effort required to reduce it?

2.3 Probability criteria are often used to judge risk for critical or catastrophic outcomes (single or multiple fatalities). Where risks are close to acceptable, one must demonstrate that the " . . . cost of reduction would exceed the improvement gained", while in the higher risk band, risk is tolerable " . . . only if reduction is impractical or if cost is grossly disproportionate to the improvement gained".

3. METHODOLOGY

3.1 Hazard and risk analysis techniques originated in the aerospace industry in the 1960s. They have significantly improved safety levels in the so-called high hazard chemical, petrochemical and nuclear industries. At the same time, principles of highly protected risk have been developed for fire protection in the manufacturing, paper and power industries.

3.2 A "cause-consequence" approach to modelling risk (Reference 1) was adopted to develop the airspace risk model. The approach combines fault tree and event tree techniques focused on a central event; the point in time at which control over potentially damaging energy is lost. These techniques have been applied to hazardous problems in a range of industries, both in Australia and overseas.

3.3 The concept of risk always has two elements, namely the frequency with which a hazard occurs and the consequence(s) of the hazardous event (Reference 2). An energy-damage approach is used in developing models to quantify the consequences of unwanted events and a time-sequence approach to identify the cause and quantify the likelihood.

4. APPLICATION

4.1 An airspace risk model has been developed to objectively determine risk levels associated with current and proposed methods of operating Australian airspace. The work began with a review of uncontrolled terminal airspace in relation to ICAO classifications. By inspection, it initially focused on aircraft transiting from the en-route to the circuit environments, as this was considered to be the highest risk area.

4.2 The ICAO model provides for a range of airspace types (A-G) with differing levels of service. The Australian

Airspace Classification Scheme (AACS) initially proposes minimal changes by keeping existing airspace boundaries and services, but revising nomenclature in accordance with ICAO recommendations. Due to safety concerns for IFR operations and pending further risk analysis, Class G airspace will require mandatory notification of IFR operations.

4.3 Aircraft have traditionally relied on radio calls to provide the "alert" component of the "alerted see-and-avoid" principle. At MBZ aerodromes, carriage of radio is mandatory while at common traffic advisory frequency (CTAF) aerodromes, it is not. The mandatory carriage and use of radio at MBZ aerodromes is confined to a volume of airspace usually of 15 NM radius and up to 5 000 ft above ground level (AGL).

4.4 Reference 2 defines a hazard as a physical situation with a potential for human injury. The term is taken to include danger to persons in a mid-air collision. In the terminal area just outside the circuit, arriving aircraft are both descending and manoeuvring from a variety of tracks, while aircraft departing are climbing and also manoeuvring.

4.5 The most likely type of collision pairs will depend on location and weather. They can be categorized according to whether aircraft are flying according to visual flight rules (VFR) or instrument flight rules (IFR). The latter range from low-capacity private/charter aircraft generally with only one pilot (IFR1), to high-capacity RPT aircraft operated by two pilots (IFR2).

4.6 Nine types of collision pairs are possible:

a) VFR/VFR in VMC (1 case);

b) IFR1/VFR and IFR2/VFR in VMC (2 cases); and

c) IFR1/IFR1, IFR1/IFR2 and IFR2/IFR2 in both VMC and IMC (6 cases).

4.7 The collision pair analysis considers both visual and instrument meteorological conditions (VMC and IMC).

5. AIRSPACE RISK MODEL (ARM)

5.1 The ARM focuses on the "near miss" as the critical event. A near miss is considered to occur when two (or more) aircraft come within defined horizontal and vertical limits, without being aware of each other's presence. For modelling purposes, a critical pair of aircraft is one where they come within 1.0 NM horizontally and 500 ft vertically. If they hit they become a "collision pair".

5.2　Cause-consequence modelling (Reference 1) combines traditional fault tree and event tree techniques by focusing on a central event: the point in time at which control over potentially damaging energy is lost.

5.3　Statistical averages are used to estimate the possible consequences of particular collision pairs. A societal risk approach considers the cumulative frequency of N or more fatalities occurring.

5.4　The ARM, which has thus far been applied mainly to the terminal area of an uncontrolled aerodrome, proposes that three phases *all* have to fail for a potentially conflicting pair of aircraft to become a critical pair:

a)　there is a breakdown in ATC separation procedures or, as in the case of uncontrolled aerodromes, an ATC separation service is not provided;

b)　the considered action phase fails. This phase includes ATS alerts when relevant. It is based on pilot coordination by radio and separation by procedural means such as separate altitudes or specific tracking details. Typically, it covers a four-minute period, between five minutes and 60 seconds from potential impact. Considered action by either aircraft will avoid a critical pair; and

c)　the evasive action phase fails. This phase is any situation where visual acquisition and avoidance is necessary, but typically between 60 seconds and 12 seconds from potential impact. It is affected by the

geometry of the critical pair, pre-warning (radio, other knowledge of aircraft), aircraft size, colour, visibility, crew vigilance and workload. Evasive action by either aircraft will avoid a critical pair.

5.5　In focusing on the near miss as the critical event, the loss of control of the situation is identified as the point at which movement of the control surfaces of an aircraft at risk would not have any significant effect by the time the collision point was passed: no matter what the pilot does, luck will rule the result. This is about 12 seconds before any collision/near miss.

5.6　The cause-consequence diagram is centred on the critical event from which consequences flow and towards which there are causal events, with time depicted as flowing from left to right across the page. The elements of the model are:

a)　loss of control — 12 seconds before mid-air collision or near miss;

b)　contributing events — considered action and evasive action phases; and

c)　range of outcomes — event tree analysis questions.

5.7　Figure A-10-1 represents the "AND" logic that ALL five identified causes have to fail in order for loss of control to occur. On the right side of the model, the balance of probabilities between outcomes is estimated.
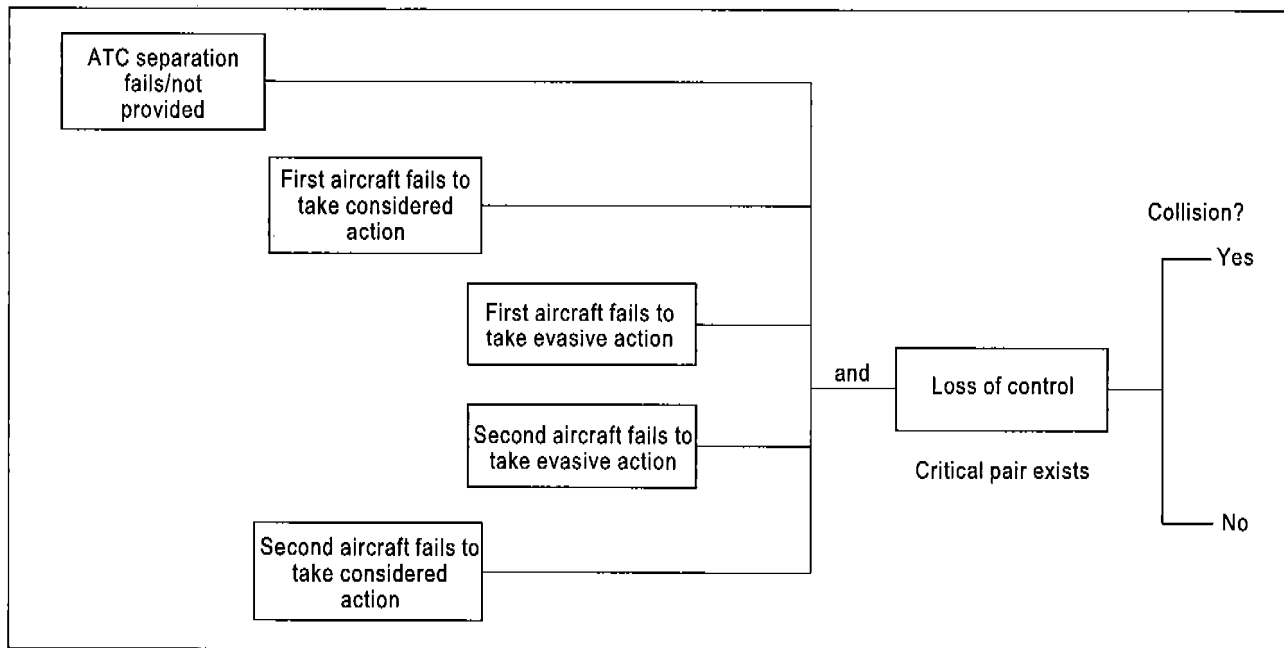

Figure A-10-1

5.8   The formulae used in the model are derived from the normal rules of combining probabilities.

Where an event can occur if either of the contributing events occur, i.e. there is more than one cause or failure mode, this is called an "OR" gate:

$$A \text{ OR } B = A + B - A*B$$

Note that the subtraction term is necessary so as not to double count the intersection of the two events. If both events have a failure probability of 1 in 10:

$$A \text{ OR } B = 0.1 + 0.1 - 0.1 * 0.1 = 0.19$$
$$(= 19\% = 1.9 \times 10^{-1} = 1.9 \text{ E-1 in scientific notation})$$

Conversely, where a control measure D is proposed to guard against an unwanted event C, this is logically an "AND" gate as both C must trigger and D must fail for loss of control to occur:

$$C \text{ AND } D = C*D$$

e.g. if both events have a failure probability of 0.1:

$$C \text{ AND } D = 0.1*0.1 = 0.01$$
$$(= 1\% = 1.0 \times 10^{-2} = /1 \text{ E-2 in scientific notation})$$

5.9   A traffic alert process will obviously fail if an aircraft cannot receive a call OR if no traffic alert is provided. Further, the provision of a traffic alert can come from ATS AND from the second aircraft, i.e. both must fail for there to be no alert. This is shown in Figure A-10-2.

5.10   An aircraft cannot receive a call if it has no receiver capability — receiver not installed OR receiver fails. The pilot can also fail, either by selecting the wrong frequency OR failing to listen. This part of the model is shown in Figure A-10-3.
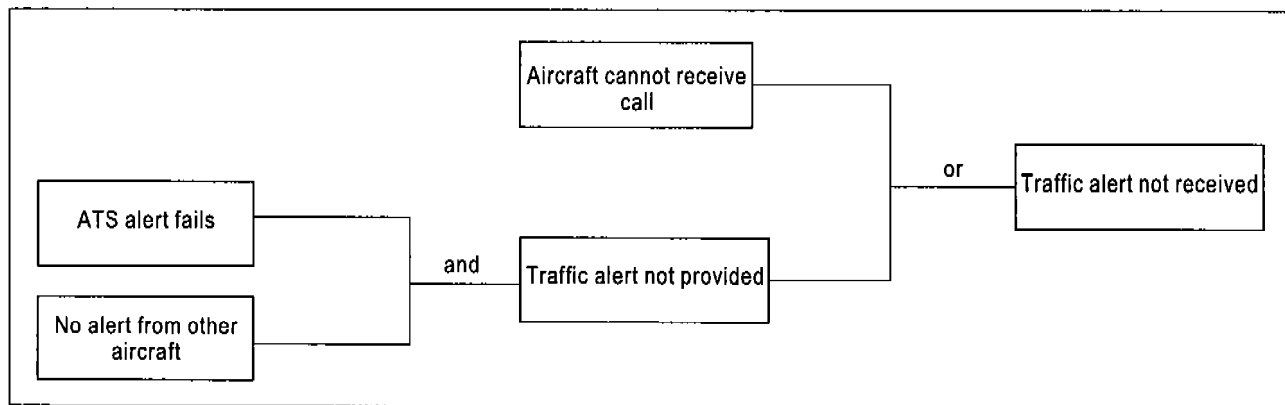
Figure A-10-2
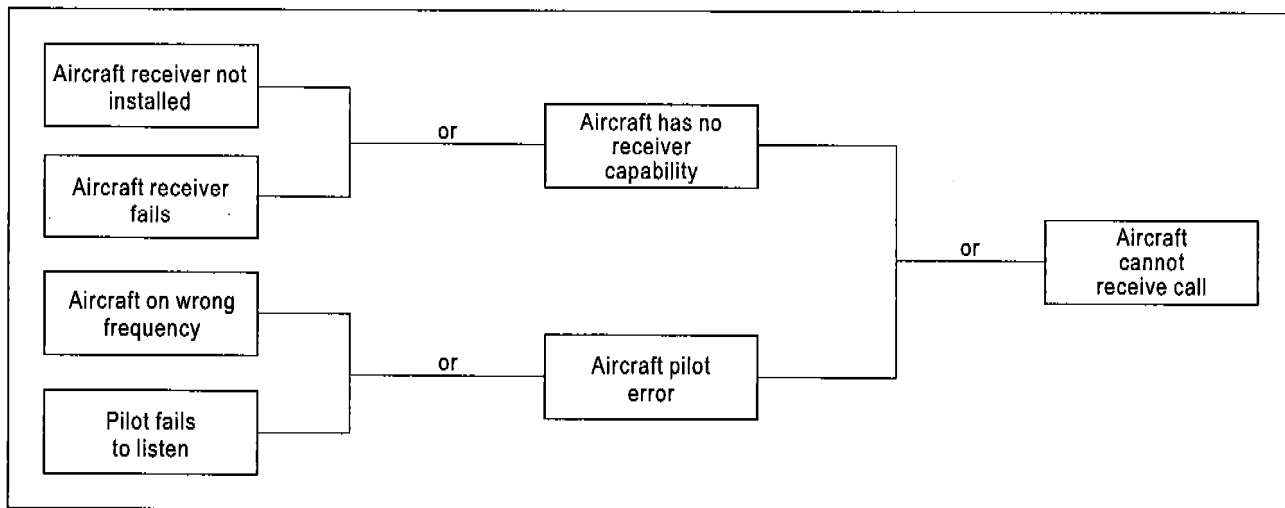
Figure A-10-3

5.11   Considered action fails if both aircraft fail to see each other OR if an aircraft is aware but makes an error. In this context "see" can have a broader meaning: the pilot of one aircraft "sees" the other in the "mind's eye", i.e. forms a mental picture of the other aircraft's location. The only option for an unalerted aircraft is visual acquisition, which is unlikely in the considered action phase. Even if one aircraft is aware of the other's position, it can still make an error, either by failing to respond to a potential threat, or by responding incorrectly (see Figure A-10-4).

5.12   Evasive action fails if both aircraft fail to see each other OR an aircraft is aware but makes an error. In this context "see" has only one meaning — unalerted visual acquisition. Other possibilities have already been taken into account in the considered action time-frame. By way of giving an example of where, in the model, a particular control technology would be considered, the role of ACAS is also depicted (Figure A-10-5).
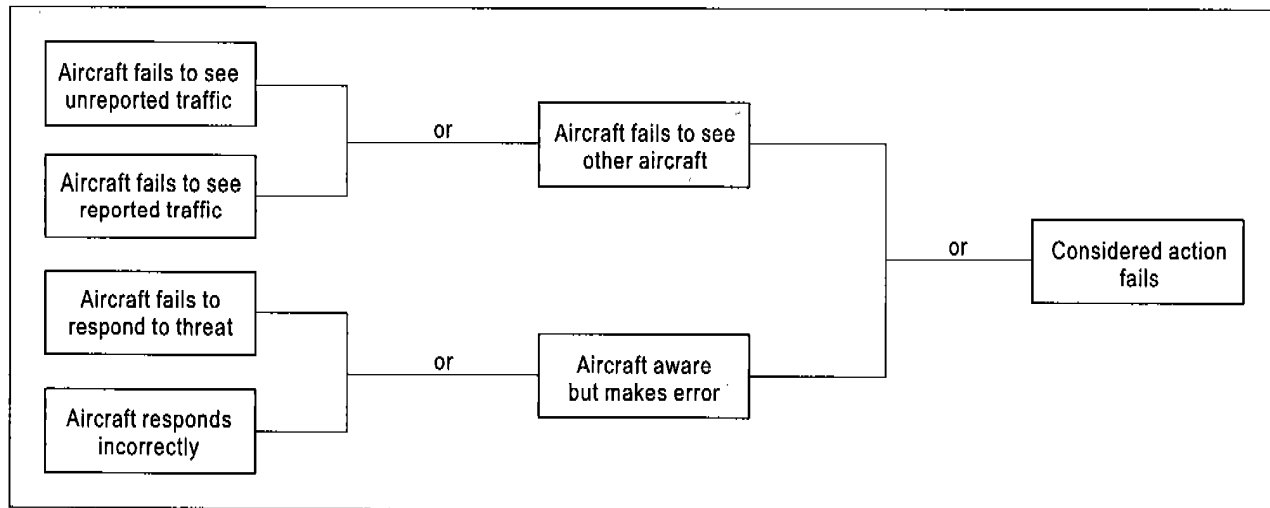
Figure A-10-4
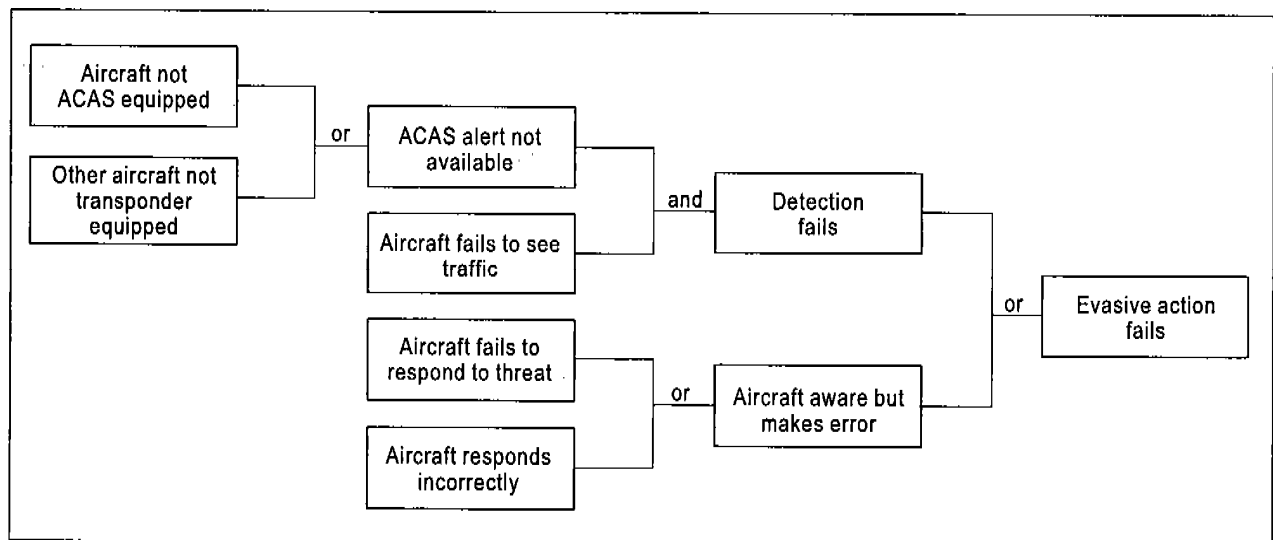
Figure A-10-5

6. QUANTIFICATION OF THE MODEL

6.1   The ARM was developed and quantified by a study team of operations and research personnel and consulting risk engineers. Probabilities for some components of the model were based directly on empirical data (e.g. equipment fit), some on indirect or extrapolated data (e.g. visual acquisition) and some on subjective data (e.g. human factors). The model and its probabilities were then scrutinized by a safety panel made up of a cross-section of industry representatives with current operational experience. While the panel accepted the model and some of the empirically derived probabilities, it derived probabilities for some components of the model by an iterative voting process.

6.2   An aircraft equipment survey conducted in 1994 (Reference 3) indicates that 5 per cent of VFR aircraft do not have radio installed, but that all IFR aircraft have radio installed.

6.3   Several incidents of aircraft receiver failure are reported by the Bureau of Air Safety Investigation (BASI) each week out of roughly 150 000 movements in Australia. Allowing for under-reporting, the failure rate of 1×10^{-4} (1 E-4) for electronic equipment typically adopted in process industry risk analysis (e.g. Reference 1) is regarded as realistic.

6.4   BASI reports suggest a failure probability of 8 in 100 000 for aircraft on the wrong frequency. The safety panel felt this figure would be highly dependent on experience. For VFR pilots it was considered ridiculously low. The panel voted for figures of 9 E-4 for IFR2 aircraft and 8.7 E-3 for VFR aircraft, i.e. the idea of the factor of 10 applying to VFR was agreed. By interpolation, a figure of 2.5 E-3 is used in the model for IFR1.

6.5   A mean probability of 1.3 E-2 was adopted for VFR pilots failing to listen, 1.2 E-3 for IFR2 pilots, with the interpolated value for IFR1 then being 4.03 E-3.

6.6   Failure rates of ATS alert are likely to be very low, say 1 in 1 million for radar or notification failures and 1 in 100 000 for processing or communications errors.

6.7   The study team suggested figures of 1 E-3 for an IFR pilot and 1 E-2 for a VFR pilot for failure to make calls. This reflects the textbook difference between an experienced competent operator and one who is merely trained. The safety panel decided on a figure of 1.4 E-3 for IFR2 and 6.2 E-2 for VFR pilots, giving 9.2 E-3 for IFR1 pilots.

6.8   The panel adopted 2.7 E-3 for an IFR2 pilot failure to respond to an identified threat, because they are trained

to organize and initiate separation. An estimate of 5.4 E-2 was adopted for VFR failure to respond to an identified threat, the corresponding figure for IFR1 being 1.22 E-2.

6.9   A typical IFR pilot who responded incorrectly once in 1 000 times would therefore do so about once every three years. The safety panel agreed this was close to the mark, adopting a mean of 1.1 E-3. For VFR the figure was 1.72 E-2 and for IFR1, 4.43 E-3.

6.10   Failure to act under clear and present danger was equated to personal experience of one mistake in 1 000 flights (a professional pilot would typically make 1 000 flights in two years). The safety panel adopted a mean of 1.15 E-3 for IFR2 pilots, 1.31 E-3 for VFR pilots and for IFR1 pilots, 1.23 E-3.

6.11   As to aircraft responding incorrectly, note that pilots have no practice or testing in conducting evasive manoeuvres in potential conflict situations. Information on two recent near misses was considered by the panel, which adopted a mean of 2.34 E-3 for IFR2. For VFR, the figure was 4.75 E-3 and for IFR1, 3.34 E-3.

7. MODEL RESULTS

7.1   The failure rate for IFR/IFR conflict pairs not being alerted ranged from 0.22 per cent to 1.89 per cent. In MBZs, the failure rate was 9.12 per cent for VFR/VFR pairs, and ranged from 2.44 per cent to 7.71 per cent for IFR/VFR pairs. In CTAFs, failure rates for VFR/VFR and IFR/VFR pairs were found to be very sensitive to VFR radio-participation rates, ranging from 15.45 per cent to 59.65 per cent.

7.2   Failure to detect an aircraft more than one minute away in the considered action phase was considered to range from 78 per cent to 94 per cent probability for pilots who were mentally alert but not "alerted". A figure of 5.7 E-3 was adopted by the panel for an IFR2 pilot's failure to realize the need for considered action when alerted. A mean failure rate of 8.9 E-2 was adopted for VFR pilots because their training does not emphasize enough situational awareness or thinking ahead. By interpolation, 2.36 E-2 was adopted for IFR1 pilots.

7.3   The evasive action figures were based on tables of cumulative probability up to the loss of control point of 12 seconds from potential impact. The critical factor was identified to be the size of the target aircraft. The failure probabilities adopted ranged from 24.8 per cent for an IFR2 looking for a VFR aircraft to 11.3 per cent for a VFR looking for an IFR2 aircraft.

7.4 The ratio between collision pairs and critical (near miss) pairs was considered to be about 1:300. Modelling of Dubbo aerodrome in New South Wales has been used to estimate the likelihood of a critical pair (near miss) existing on a given trip. Overall, with 25 000 movements in a year, 250 critical pairs were found, i.e. 1 per cent of total movements.

7.5   Applying the 1:300 ratio gives the conditional collision probabilities shown in Table A-10-1.

7.6 The likely average consequences for each collision pair are 20 fatalities for IFR2/IFR2, 11-12 if there is one IFR2 involved and 3.0-4.5 for collisions involving VFR/IFR1.

7.7 The relative risk results show that for IFR collision pairs, likelihood decreases sharply as consequence increases. This reflects the societal risk concept that society has a much greater aversion to high-consequence events. The model is sensitive to IMC where risk increased by an order of magnitude compared with VMC because the see-and-avoid contribution is not possible in IMC. However, risk is not significantly greater in either case if ATS is not provided. This is due to very high radio participation rates by IFR pilots.

7.8 For IFR/VFR pairs in MBZs, likelihood decreases by an order of magnitude as consequence increases, i.e. the risk (which is the product of likelihood times consequence) remains constant. There is a factor of 3 increase in risk from an MBZ up to a CTAF 90 per cent and a further factor of 2.5 increase in risk between the most optimistic and the most pessimistic assumptions about CTAF participation rates. Figure A-10-6 shows some of the relative risk results.

7.9 The results for pilots being completely unalerted are 1-2 orders of magnitude greater even than CTAF 70 per cent. A key conclusion therefore relates to the issue of *alert*. The probability of collision is high when both aircraft are unaware of the other. Any option, such as MBZ, that enables the aircraft to be aware of each other is a major benefit. With ACAS, the probability of loss of control is further reduced for each aircraft pair so equipped.

7.10 The "risk triangle" concept initially promoted by the UK Health and Safety Executive (Reference 4) and VRF Handbook (Reference 5) places the ALARP range between 10^{-4} per year (100 chances per million) and 10^{-6} (1 chance per million) per year for individual risk criteria for a critical exposed group. By comparison with tables of risks to individuals (Reference 6), this is saying that risks which are

Table A-10-1

Collision pair	VFR/VFR	IFR1/VFR	IFR2/VFR	IFR1/IFR1	IFR1/IFR2	IFR2/IFR2
Configuration						
Unalerted	7.84 E-5	1.61 E-4	7.07 E-5	2.76 E-4	2.34 E-4	6.28 E-5
CTAF 70%*	3.31 E-5	2.93 E-5	1.27 E-5			
CTAF 80%*	2.32 E-5	1.96 E-5	8.24 E-6			
CTAF 90%*	1.43 E-5	1.15 E-5	4.67 E-6			
MBZ	4.93 E-6	3.85 E-6	1.29 E-6			
IMC no ATS				1.10 E-5	3.80 E-6	6.54 E-7
IMC ATS				7.12 E-6	1.80 E-6	4.55 E-7
VMC no ATS				1.19 E-6	2.02 E-7	1.69 E-8
VMC ATS				7.92 E-7	1.00 E-7	1.23 E-8

* Refers to the percentage of radio-equipped aircraft that make radio calls.

more dangerous than driving a car are likely to be unacceptable, whereas those that are about as likely as being struck by lightning are trivial.

7.11   Individual risk focuses attention on critical exposed groups such as crew members, who may be involved in 500 movements per annum. The Dubbo collision pair analysis shows individual risk of being in a mid-air collision is 23 chances per million per year for an IFR1 at an MBZ. This risk rises to 40 chances per million in CTAF 90 per cent, 57 chances per million in CTAF 80 per cent and 79 chances per million in CTAF 70 per cent.

7.12   The results plotted in Figure A-10-7 show MBZ risk as "tolerable" (less than 25 chances per million for IFR1 crew) and CTAF 80 per cent risk as "barely tolerable" (up to 57 chances per million for IFR1 crew) depending on radio participation rates. In this ALARP region, the obligation remains to reduce risk "as low as reasonably practicable".

7.13   Risks of different consequences are often compared on the basis that risks are similar if a tenfold increase in severity is accompanied by a tenfold decrease in likelihood. However, it appears that once the death threshold has passed, the community has a much greater aversion to multiple fatality incidents. Figure A-10-8 shows the cumulative probability of N or more fatalities compared to tentative societal risk criteria.

7.14   Further work is needed across the aviation industry as a whole regarding the risk parameters: passengers killed, safe passenger kilometres flown, safe passenger seat kilometres flown, etc.

8. DISCUSSION

8.1   Risk analysis techniques are advocated as an essential ingredient in determining safety policy.

8.2   Unalerted see-and-avoid constitutes an unacceptable risk. The size of the target aircraft is the critical factor. Probabilities of failure to acquire the target vary from 11.3 per cent for VFR aircraft looking for a two-pilot IFR aircraft, to 33 per cent for a single IFR pilot acquiring a small VFR aircraft in the evasive action phase.

8.3   The model is extremely sensitive to the CTAF VFR radio participation rate and surveys are in hand to further explore this issue.

8.4   The risk analysis clearly indicates that the current IFR to IFR separation procedures for uncontrolled airspace provide a high degree of safety, placing the risk of mid-air collisions between IFR aircraft in the trivial régime. However, when considering the effect of operating in an area with no mandatory radio requirement (outside MBZs), the risk of a two-pilot IFR aircraft coming into conflict with VFR aircraft becomes a higher but tolerable risk. The risk of conflict between pairs of single-pilot IFR aircraft and single-pilot IFR/VFR pairs is significantly higher again, and enters the area of barely tolerable risk.

8.5   Further work has commenced on where to set limits/criteria for establishing an MBZ over a CTAF, and what risk reductions might be achieved for what dollars spent on implementing new technologies such as ACAS.

8.6   The cause-consequence modelling approach can be calibrated to give an assessment of the existing risk of the particular system under study. By testing such models against both the available data and the experiences of senior management and technical personnel in the industry concerned, one can ensure that the model accurately reflects the best available information and knowledge at the time when it is used to make decisions regarding risk acceptance and risk reduction, if required.

9. NEXT STEPS

9.1   The first step is to recognize that although the basic structure of the ARM is unlikely to change, the quantitative results presented here are preliminary. As further research is conducted, particularly in the critical areas of probability of seeing other aircraft, and the near miss to collision ratio, then the results will change.

9.2   Secondly, the ARM needs to be refined to consider several classes of IFR aircraft, notably the low (less than 10 passenger seats), medium (10-38 seats) and high (greater than 38 seats) capacity RPT aircraft. This is important because the ARM is sensitive to the number of likely fatalities in a mid-air collision.

9.3   Thirdly, the relationship between absolute risk and risk acceptance criteria needs to be addressed. In particular, should risk criteria be based on some critically exposed group, such as RPT pilots or frequent flyers, or on some concept of overall risk? This will influence such things as the criteria for upgrading an aerodrome from a CTAF to an MBZ, or from an MBZ to a control tower.

AIR SERVICES AIRSPACE RISK MODEL

IFR/VFR MTAF compared to various
participation rates in CTAF

RELATIVE LIKELIHOOD
expressed as probability of collision if a
conflict pair exists

Model run results involving
VFR/VFR, IFR1/VFR & IFR2/VFR
are numbered respectively as follows:

— 13, 14, 15 CTAF 70% alerted
— 10, 11, 12 CTAF 80% alerted
— 7, 8, 9 CTAF 90% alerted

— 1, 2, 3 MTAF

Note risk for a given degree of alerting
remains constant (likelihood decreases as
consequence increases). However, relative
risk is very sensitive to the actual degree of
alerting with CTAF risk much higher than
MTAF risk.

Consequence assumptions   VFR/VFR 3.0
(average fatalities):
                           IFR1/VFR 3.8   IFR2/VFR 11.7
                           IFR1/IFR1 4.5   IFR1/IFR2 12.4 IFR2/IFR2 20.3

Figure A-10-6. Relative risk results

Levels of risk and ALARP

Unacceptable region

Risk cannot be justified except in extraordinary circumstances.

Limit for H.S.E.U.K., Royal Soc. U.K. * 10^{-4} per year

10 VFR CTAF 80%

11 IFR1 CTAF 80%

Tolerable only if further risk reduction is impractical or if cost is grossly disproportionate to the improvement gained.

12 IFR2 CTAF 80%

1 VFR MBZ
2 IFR1 MBZ

The ALARP or

Tolerability region
(risk is undertaken only if a benefit is desired)

Limit for W.A.E.P.A. * 10^{-5} per year

As the risk is reduced, the less proportionately it is necessary to spend to reduce it. This concept of diminishing proportion is shown by the triangle.

Objective for N.S.W. DoP, H.S.E.U.K., 10^{-6} per year
Royal Soc. U.K. *

Broadly acceptable region
(no need for detailed working to demonstrate ALARP)

3 IFR2 MTAF

Necessary to maintain assurance that risk remains at this level.

Negligible risk

Trivial risk 3×10^{-7} per year

Individual Risk Criteria for Critical Exposed Group compared to Airspace Risk Model
Results for aircrew for Dubbo terminal area ICAO Class G uncontrolled airspace.

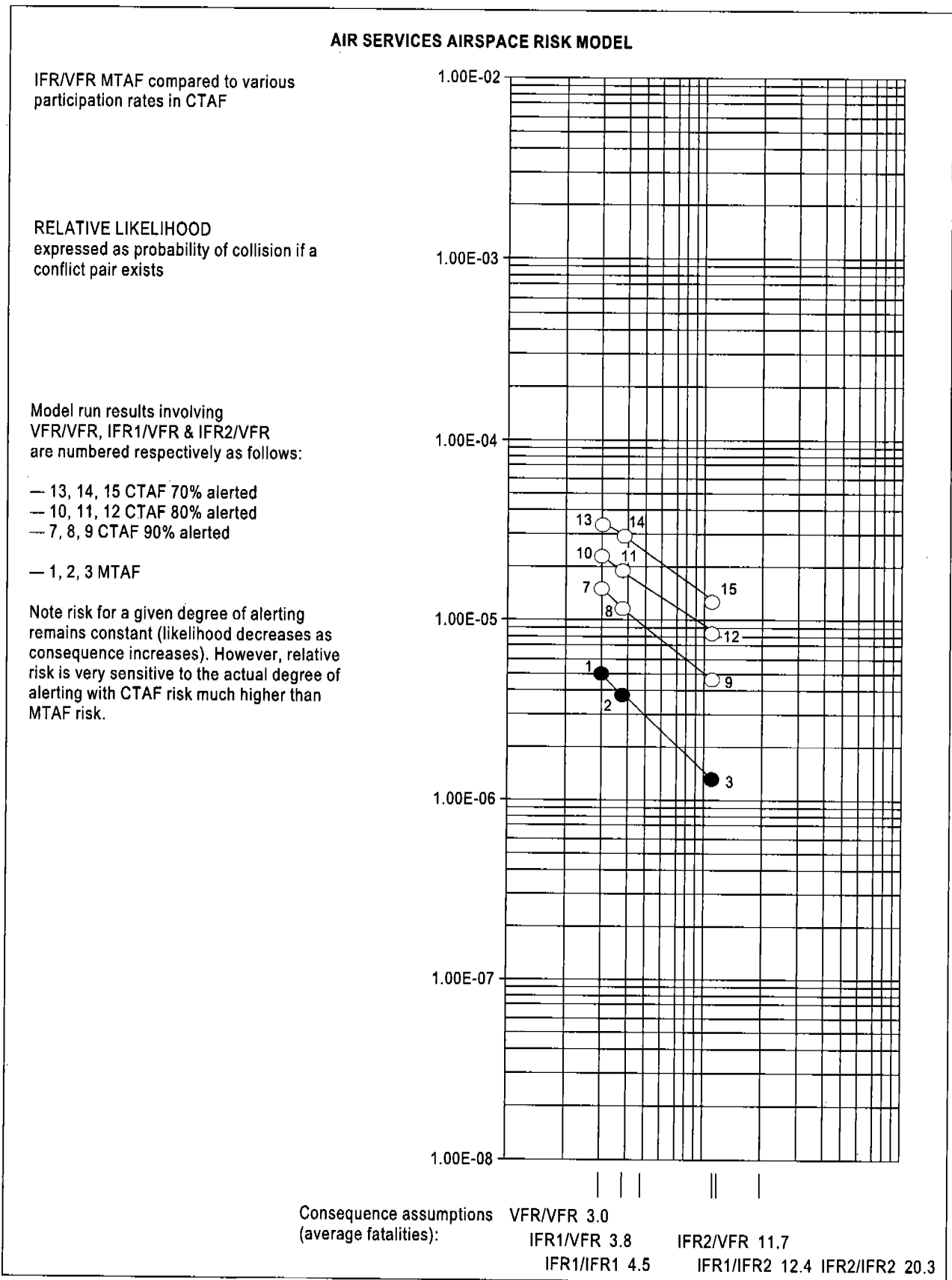*Note.— This diagram (without the quantification) appears in IEC 1508 as Figure B1.*

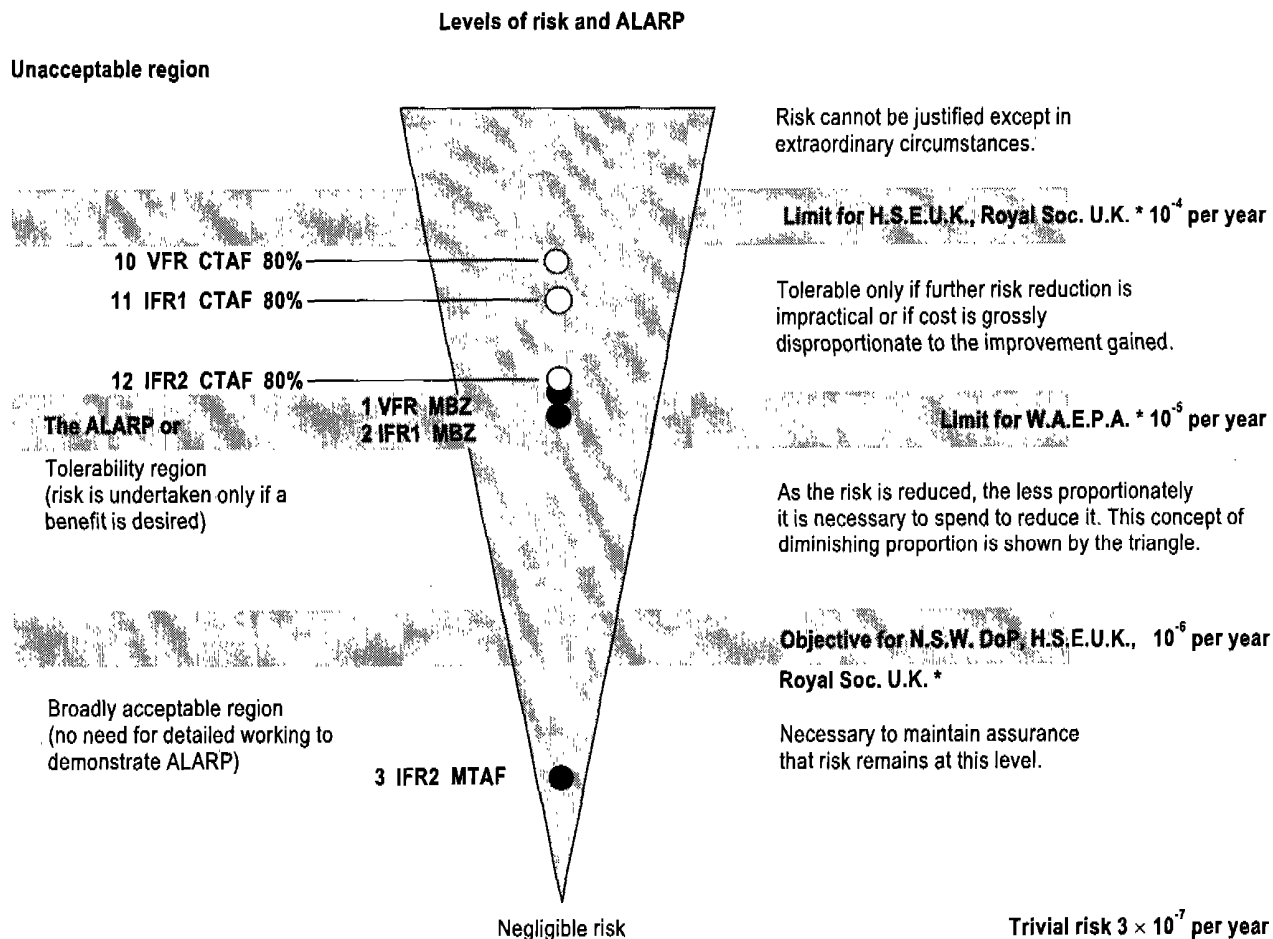**Figure A-10-7. Individual risk criteria for critical exposed group
compared to Airspace Risk Model results for aircrew for Dubbo terminal area —
ICAO Class G uncontrolled airspace**

AIR SERVICES AIRSPACE RISK MODEL

Dubbo MTAF
25 000 movements pa

Frequency of a collision
involving N or more
fatalities per year

◈	MTAF
□	CTAF 90%
△	CTAF 80%
○	CTAF 70%

Societal Risk Tolerability
Lines are taken from UK Health & Safety
Commission report on "Major hazard aspects of
the transport of dangerous substances".
These criteria relate to criteria for
ports and for road and rail risk expressed in
one locality. Application to airspace planning
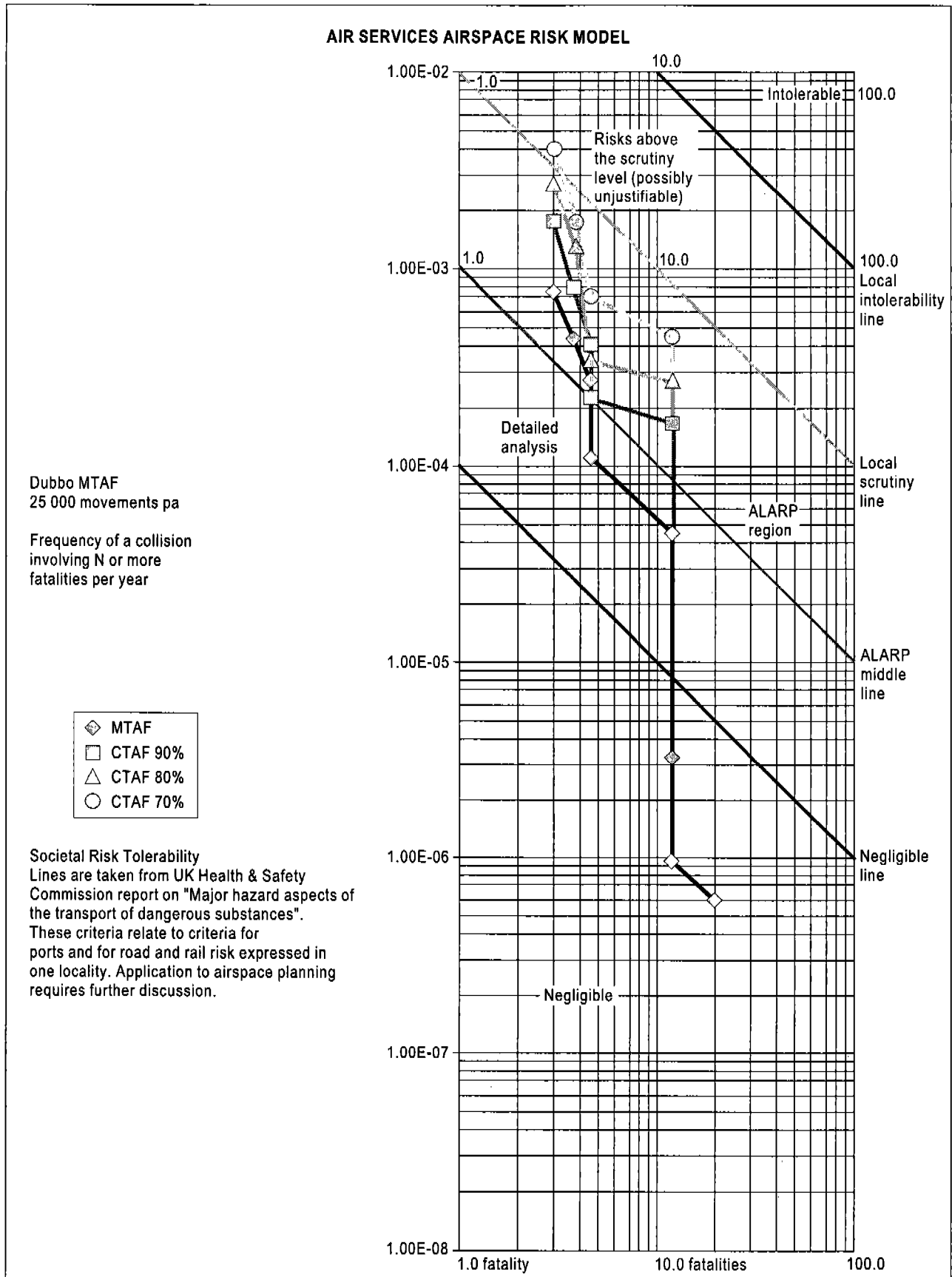requires further discussion.

Figure A-10-8. Societal risk results

9.4 The fourth step is based on the recognition that the results so far are based on only one of Australia's larger uncontrolled hub aerodromes. Are these results applicable to all uncontrolled aerodromes? Traffic data surveys have been conducted at several other locations, to which the study team is now applying its modelling techniques to test the general applicability of the model.

9.5 The fifth step is to extend the application of the model to cover the en-route phase of flight, comparing the risk with the levels of service in Classes G and C. This requires completing the hitherto undeveloped part of the model on ATC separation services. When this is done, the ARM will be almost complete, and can then be applied to all classes of airspace.

9.6 The subsequent steps will therefore be to progressively apply the ARM to the higher classes of airspace, from D to A, and also to factor in such technological developments as the use of TCAS.

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Appendix 11

AIR NAVIGATION SYSTEMS ANALYSIS
AND PLANNING METHODOLOGY

1. INTRODUCTION

1.1 The methodology for assessing collision risk in the air navigation system currently in use in the Russian Federation, is based on the event tree method suggested in this manual. However, in view of the extreme complexity of the problem in developing this methodology, use was made of simplifications which significantly eased the process of reducing the safety analysis to numerical values.

1.2 The methodology detailed in this appendix takes into account ATC errors, which lead to serious flight accidents. These errors can either be due to ATC operational errors or due to technical failures of the ATC system. No account is taken of aircraft technical errors or pilot operational errors.

1.3 An iterative approach is used in which four different system states are considered, three of which affect the controller's ability to control the airspace. Initially, functional channels are considered to be structural system units, which differ from each other in the degree to which a failure can affect the system. It should be noted that functional channels refer to the equipment used to realize ATC functions. Once the basic characteristics required from the functional channels (to perform ATC functions) have been determined, a more detailed analysis can be performed taking into account the various components of the ATC system.

1.4 The methodology takes account of the system risk caused by the nature of the uncontrolled aircraft flow, the parameters of the preplanning subsystem, the air navigation characteristics of the aircraft traffic, the controller's actions to prevent loss of separation and the parameters of the collision avoidance system (CAS). It should be noted that although airborne CAS are factored into this methodology, current ACAS technology is not recognized by ICAO as being suitable to provide a contribution to the determination of a separation standard or route spacing.

1.5 By applying the methodology, it is possible to determine the required performance of a limited group of system parameters, which guarantee the maintenance of the required air traffic safety at a given capacity. These same parameters are indirect criteria for assessing safety if the performance of the parameters are already defined, i.e. it is possible to use the methodology to both assess the safety of a system currently in use and to determine the performance required from a system to ensure that it is safe.

2. STATEMENT OF THE PROBLEM

2.1 The methodology considers the following air navigation system characteristics:

a) tactical ATC indices, namely: separation standards; dimensions of the potential conflict area $(S_x, S_y, S_z, L_y, L_x)$;

b) the airspace structure and the nature of the aircraft flows: weighting coefficients of the different types of relative motion between aircraft (Γ) and the frequency of potential conflicts in an uncontrolled aircraft flow (λ_{pc});

c) the tactical flight characteristics of the aircraft flows, namely: the mean aircraft flow speed; the accuracy in holding the aircraft flow to the assigned path and flight plan; the probability that an aircraft is not equipped with an airborne CAS, $(V, \sigma_x, \sigma_y, \sigma_z, C_{cas})$. It should be noted that within the methodology it is assumed that if an airborne CAS is on board the aircraft, the CAS will prevent a collision;

d) indices of the reliability of ergonomic ATC support and the ATCO workload associated with air traffic planning, $P_{er.1}$, and that associated with radar control functions, $P_{er.2}$. These two parameters are related to

the planner and tactical controllers' abilities to prevent a collision respectively. It is possible to take into account the communications, surveillance and automation facilities, amongst other things, by means of their effect on the values of $P_{er.1}$ and $P_{er.2}$. The numerical values of $P_{er.1}$ and $P_{er.2}$ used within the methodology represents the probability of the controller not preventing a collision;

e) indices of the reliability of the functional channels: ($P_1^{ch}, P_2^{ch}, P_3^{ch}$). These parameters represent the combined reliability of the various components of the ATC system.

Then the concept of the $\overline{GSP}$ (general system performance) vector can be introduced.

$$\overline{GSP} = (S_x, S_y, S_z, L_y, L_x, \Gamma, \lambda_{pc}, V, \sigma_x, \sigma_y, \sigma_z, C_{cas}, P_{er.1}, P_{er.2}, P_1^{ch}, P_2^{ch}, P_3^{ch}).$$

2.2 Satisfactory general system performances ($\overline{SGSP}$) parameters are those $\overline{GSP}$ parameters that ensure the attainment of the required air navigation system capacity and safety with the minimum economic expenditure.

2.3 An analytical correlation can be obtained, which links the mean collision risk (N) in an air navigation system with the values of the $\overline{GSP}$ vector components and parametrically with system capacity (c): $N = f(\overline{GSP} ; c)$.

This makes it possible to formally describe the $\overline{SGSP}$ vector as follows:

$$f(\overline{GSP} ; c) - TLS = 0; \tag{1}$$

with the criteria of the type $E \to min$, \hspace{1cm} (2)

where the TLS is the target level of safety and E is the cost of installing and operating an air navigation system, which satisfies equation (1).

2.4 It is possible to show that f($\overline{GSP}$; c) is a formula for the mathematical expectation of a discrete random variable, which expresses the collision risk for various established air navigation system states. Four system states are singled out:

a) healthy system state (state probability P_0, system collision risk N_0);

b) a system state involving a hardware failure that causes air traffic to become uncontrollable (state probability P_1, system collision risk N_1);

c) a system state involving hardware failures that give the controller an increased ATC workload: however, there is a probability (not greater than 0.1) that the system traffic will become completely uncontrollable (system state probability P_2, system collision risk N_2);

d) a system state involving hardware failures that do not lead to a loss of control: as a rule, however, they increase the controller's workload, which leads to an increased probability of controller errors of at least one order of magnitude (system state probability P_3, system collision risk N_3).

2.5 The formal notation for the mean risk in the air navigation system is the following expression:

$$N = f(\overline{GSP} ; c) = P_0 N_0 + P_1 N_1 + P_2 N_2 + P_3 N_3 \tag{3}$$

2.6 The proportion of collisions that are due to failures of the functional channels (ρ) or control personnel ($1 - \rho$) are known. It is therefore possible to break down the mean risk (3) into that associated with a healthy system state and one where hardware failures occur:

$$(1-\rho).TLS = P_0 N_0; \tag{4}$$

$$\rho.TLS = P_1 N_1 + P_2 N_2 + P_3 N_3 \tag{5}$$

2.7 Equation (4) makes it possible to either analyse the air navigation system according to $\overline{GSP}$ parameters, which are not related to the technical reliability of the functional channels or to obtain an assessment of the $\overline{SGSP}$ vector components, which characterize a healthy air navigation system hardware state (i.e. to determine the performance requirements of a healthy air navigation system).

2.8 Equation (5) makes it possible to perform similar operations with the $\overline{GSP}$ components, which characterize the technical reliability of the functional channels.

3. THEORETICAL BASIS OF ANALYSING THE $\overline{GSP}$ VECTOR AND DESIGNING AN ATS SYSTEM, WHICH MEETS THE SAFETY AND CAPACITY REQUIREMENTS

3.1 The basis for analysing the $\overline{GSP}$ vector in terms of meeting a target level of safety (TLS) and capacity (c) involves collision risk equations reflecting the different hardware states, i.e. N_0, N_1, N_2 and N_3. For the four different system states, the following analytical expressions are used.

As noted earlier, this methodology only considers the risk due to controller operational errors or technical faults within the ATC system:

$$N_0 = C_{cas} C \cdot P_{er.2} \cdot (P_{er.1} \cdot \lambda_{pc} \cdot \overline{P}_{coll./pc} + (1 - P_{er.1}) \cdot \overline{N}^{(c)}_{sect}) \quad (6)$$

$$N_1 = C_{cas} \cdot \lambda_{pc} \cdot \overline{P}_{coll./pc} \quad (7)$$

$$N_2 = 0.1 \cdot N_1 + 0.9 \cdot N_3 \quad (8)$$

$$N_3 = C_{cas} \cdot C_2 \cdot P_{er.2} \cdot (C_1 \cdot P_{er.1} \cdot \lambda_{pc} \cdot \overline{P}_{coll./pc} + (1 - C_2 \cdot P_{er.1}) \cdot \overline{N}^{(c)}_{sect}) \quad (9)$$

3.2   The right side of equations (6), (7), (8) and (9) contain either $\overline{GSP}$ vector components (C_{cas}, $P_{er.1}$, $P_{er.2}$, λ_{pc}) or functions of the components ($\overline{P}_{coll./pc}$, $\overline{N}^{(c)}_{sect}$). $\overline{P}_{coll./pc}$ is the mean collision probability given that an aircraft pair are in a potential conflict situation in an uncontrolled situation, i.e. this is the probability that the aircraft actually collide given that they are in a potential conflict situation. $\overline{N}^{(c)}_{sect}$ is the mean sectoral collision risk in an ATC sector with a planned flow. These components are calculated from equations given in Reference 1[1].

3.3 C_1 and C_2 are the coefficients which show the increase in the probability of controller preplanning and radar control errors respectively.

3.4   In order to perform the analysis, it is necessary to define some additional information:

System state probabilities

3.4.1   The probability of the ATS system being in a certain state involving hardware failures is assumed to be inversely proportional to the collision risk in that state. If safety data is available to allow the quantitatively weightings (δ_i) of the overall risk due to the risk of the ith state ($i = 1, 2, 3$) to be assigned, the system state probabilities are equal to:

$$P_i = \delta_i \frac{\rho \cdot TLS}{N_i} , \quad i = 1, 2, 3 \quad (10)$$

3.4.2   If it is not possible to statistically assign the weightings δ_i, an equal contribution to the risk balance by the different hardware failures states will be assumed:

$$P_i = \frac{1}{3} \frac{\rho \cdot TLS}{N_i} ; \quad (11)$$

Functional channel reliabilities

3.4.3   If a number of different failures can independently put the system into the ith state and the weightings are known for each of these types of failures, then the reliabilities of the functional channels (failures of which determine the ith system state) are calculated from:

$$P^{ch}_{ij} = 1 - \mu_{ij} \cdot P_i \quad \text{(given} \sum_{j=1}^{q_i} \mu_{ij} = 1 \text{);} \quad (12)$$

where q_i is the number of different failures which will put the system in the ith state and μ_{ij} is the weighting assigned to the jth failure (of state i).

3.4.4   If it is assumed that channel failures have an identical effect on the system transition to the ith state, the reliability of the channels can be assessed as:

$$P^{ch}_i = 1 - \frac{P_i}{q_i} . \quad (13)$$

3.4.5   In the absence of validated *a priori* information on the effect the failures have on the risk balance throughout the system as a whole, it is necessary to use equations (11) and (13) to solve the analysis problem for the future.

3.5   By examining the sign of the following expressions:

$$9.7 N_0 - \frac{(1 - \rho) TLS}{P_0} \quad (14)$$

$$\sum_{i=1}^{3} P_i \cdot N_i - \rho \cdot TLS ,$$

$$\text{where } P_0 = 1 - (P_1 + P_2 + P_3), \quad (15)$$

it is possible to determine if the general system requirements meet the safety criteria. In other words, if the sign of the values calculated from expressions (14) and (15) are negative, the ATS system meets the TLS and the capacity requirements adopted. If this is not the case, it will be necessary to change the design of the system and hence some of the $\overline{GSP}$ vector components so that the TLS and capacity requirements are met.

3.6   The task of planning a system that meets the target level of safety and capacity requirements may not have a single solution since the result may be achieved by influencing various $\overline{GSP}$ vector parameters. Nevertheless, the selection of concrete planning activities must be preceded by solving an equation of the type:

$$N_0(x_i) - \frac{(1 - \rho) TLS}{P_o} = 0 \quad (17)$$

where the unknown x_i represents capacity (c) or a $\overline{GSP}$ vector component that is not related to functional reliability. In other words, prior to specifying the functional reliability it is necessary to determine the parameter values needed to satisfy the safety criteria for a healthy system state.

3.7 As a rule, a convenient way to make a system satisfactory in terms of air traffic safety is by changing the following general system performance:

a) parameters relating to the aircraft flow performance characteristics, i.e. C_{cas}, σ_x, σ_y and σ_z;

b) parameters characterizing the airspace structure and the nature of the aircraft flow in the ATS system's area of responsibility, i.e. Γ and λ_{pc};

c) parameters characterizing the level of ATC automation in the ATS system being considered, i.e. $P_{er.1}$ and $P_{er.2}$.

3.8 The functional channel reliability requirements follow from the $\overline{SGSP}$ vector component values obtained by solving equations (11) and (13).

4. EXAMPLE OF THE USE OF THE PROPOSED METHODOLOGY FOR ANALYSING AND PLANNING AN EN-ROUTE AIR NAVIGATION SYSTEM

4.1 A hypothetical upper airspace ATC sector with a sufficiently complex topography is considered: the sector contains elements with parallel relative traffic, longitudinal relative traffic, lateral relative traffic, parallel traffic in the vertical plane and traffic relative to airway intercepts and convergences. Traffic density along intersecting and converging airways reaches 60 per cent of the sector's capacity. For each airspace element in the hypothetical sector, the proportion of time spent within each type of relative movement is known. Let us assume that a target level of safety (TLS) of 2.0×10^{-8} (fatal accidents per flight hour) and a capacity (c) of 30 (aircraft/hour) have been established.

4.2 The $\overline{GSP}$ vector components in the case being examined have the following values:

$S_x = 30$ km; $S_y = 30$ km; $S_z = 500$ m;

$L_x = 45$ km;

$\lambda_{pc} = 0.6$;

$V = 850$ km/hour; $\sigma_x = 4$ km; $\sigma_y = 2$ km;

$P_z(0) = 0.25$; $C_{cas} = 1$; $P_{er.1} = P_{er.2} = 0.5 \times 10^{-2}$.

4.3 From these parameters $\overline{P}_{coll/pc}$ was calculated to be 1.13×10^{-3} and $N_{sect}^{(c)}$ to be 2.02×10^{-6}. These values are calculated using equations that can be found in Reference 1.

4.4 In addition, it is known that the proportion of risk due to hardware failures, ρ, is equal to 0.3. Therefore, the TLS for the different system states is split as follows: a TLS of 1.4×10^{-8} for a healthy system state and a TLS of 0.6×10^{-8} for a system state involving hardware failures.

4.5 The number of channels in the first, second and third states are 3, 4, and 5, respectively (i.e. $q_1 = 3$, $q_2 = 4$, $q_3 = 5$). In addition, it is considered that the probabilities of the system being in the first, second and third states, respectively, are inversely proportional to collision risks N_1, N_2 and N_3, respectively. We shall consider that in overload situations the probability of controller preplanning and radar control errors increases by a factor of ten (i.e. $C_1 = C_2 = 10$).

4.6 The problem of analysing the system in a hardware non-failure state comes down to an assessment of risk N_0. For the case being considered, N_0 is assessed to be 2.7×10^{-8} (fatal accidents per flight hour), which reveals an unsatisfactory air traffic safety state and a need to change the system performance.

4.7 It is possible to reduce the risk by altering a number of parameters. In this example, regulating effects were selected by altering the parameters λ_{pc}, $P_{er.1}$, and $P_{er.2}$. The satisfactory risk N_0 (i.e. $\leq 1.4 \times 10^{-8}$) can be obtained with:

$\lambda_{pc} = 0.23$;

$P_{er.1} = P_{er.2} = 4.43 \times 10^{-3}$ if $P_{er.1}$ and $P_{er.2}$ are equal to each other and change simultaneously;

$P_{er.1} = 3.06 \times 10^{-3}$ if only $P_{er.1}$ changes;

$P_{er.2} = 4.24 \times 10^{-3}$ if only $P_{er.2}$ changes;

The other parameters remain as above.

4.8 In order to show the functional channel reliability requirements, it is necessary to select a concrete method for regularizing the task in a hardware non-failure state. We shall consider that the system will be planned by changing the parameters $P_{er.1} = P_{er.2}$.

4.9 Then, taking account of the q_i and C_j values adopted (see above), a satisfactory system can be achieved with the following values:

$P_{er.1} = P_{er.2} = 1.11 \times 10^{-3}$;

$N_1 = 2.60 \times 10^{-4}$ (fatal accidents per flight hour);

$N_2 = 2.60 \times 10^{-5}$ (fatal accidents per flight hour);

$N_3 = 5.42 \times 10^{-8}$ (fatal accidents per flight hour);

$P_1 = 7.70 \times 10^{-6}$;

$P_2 = 7.68 \times 10^{-5}$;

$P_3 = 3.69 \times 10^{-2}$;

$P_1^{ch} = 0.9999974$;

$P_2^{ch} = 0.9999808$;

$P_3^{ch} = 0.9926193$.

REFERENCE

1. Mathematical support for the methodology for the analysis and planning of air navigation systems; RGSCP 9 WP/5, May 1996; (In Russian).

Appendix 12

REGIONAL AIRSPACE
PLANNING METHODOLOGY

1. INTRODUCTION

1.1 The main problem in regional airspace planning is the rational use of the available material and equipment resources to enhance air traffic safety and efficiency. The solution to this problem is directly linked to reducing the existing aircraft separation standards, which makes the problem of ensuring air traffic safety in a region a high-priority item.

1.2 The distinction between regional airspace planning and ATS system development planning in a specific region consists of the following:

— regional planning comes up against the problem of limited material and equipment resources when the system cannot be fully implemented in specific areas;

— in solving problems at the local level, the task boils down to determining the required system performance, and at the regional level, to optimizing the strategy for equipping the region with specific systems. In the first case, use is made of a methodology for optimizing system construction and performance, and in the second a methodology for optimizing system disposition in a region when resources are limited.

1.3 This appendix provides a regional airspace planning methodology aimed at ensuring air traffic safety and efficiency by defining a rational strategy for equipping the ATS system with ground-based flight support facilities and systems. This methodology has been used in ATS system development planning in the Russian Federation for a number of years.

2. AIRSPACE PLANNING PHASES

2.1 It is advisable to carry out airspace planning related to determining the development of the technical equipment of ground-based flight support facilities and systems in three phases.

2.2 *Phase I.* Elaboration of a long-term 10 to 15 year forecast.

Purposes:

1. To study the possibilities of meeting the future technical equipment requirements together with the further development of the equipment in accordance with emerging trends (study forecast).

2. To determine the problems anticipated and define long-term goals.

3. To determine possible options and select the most preferable option for the development of the technical equipment with a view to meeting the future requirements, if its traditional development does not meet these requirements (normative forecast).

Main problems to be resolved:

1. Determining the anticipated ATS system operating conditions.

   — anticipated ATS requirements;

   — future requirements with regard to ensuring air traffic safety and efficiency;

   — trends in the development of ground-based and airborne aircraft equipment; and

   — anticipated financing sources and volumes.

2. Assessing the possibility of meeting the future air traffic safety and efficiency requirements in the anticipated ATS system operating conditions, together with the further development of the system in accordance with emerging trends.

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3. Synthesizing the possible options for developing ATS system technical equipment:

— development of the technical equipment provision concept; and

— development of possible options for implementing the concept.

4. Analysing the options from a cost/benefit point of view:

— determination of the values of the indices of the effectiveness of the options;

— determination of the resources needed to implement the options;

— assessment of the risk in implementing the options; and

— determination of the most preferable option.

2.3   Phase II. Elaboration of a 5 to 10 year development programme.

Purposes:

1. To determine the most effective means of implementing the preferred technical equipment development option.

2. To elaborate the equipment provision strategy.

Main problems to be solved:

1. Defining goal-oriented indices.

2. Developing possible means of implementing the preferred option for developing the technical equipment at sites (ATC zones and areas) in the region.

3. Calculating the indices of the effectiveness and cost of the means of implementing the preferred technical equipment development option at sites in the region.

4. Defining the composition of the measures (in time and in space) to implement the preferred option for achieving the goal-oriented indices with the minimum amounts of financing.

2.4   Phase III. Elaboration of a 3 to 5 year technical equipment plan.

Purpose:

To develop the composition of the measures to equip specific ATC areas and zones with flight support facilities, taking account of the equipment they have at present, the measures provided for in the development programme and the resource support possibilities for the period under consideration.

Main problems to be solved:

1. Defining the air traffic service requirements.

2. Determining the ATS system sites at which the technical equipment must be developed during the period under study on the basis of:

— an analysis of the equipment the sites have at present;

— the air traffic requirements as elucidated;

— the measures stipulated in the development programme; and

— other reasons.

3. Specifying the amounts of financing.

4. Determining the composition of the technical facilities required for each site whose technical equipment is to be developed.

5. Calculating the effectiveness indices to be achieved.

3. COMPUTER METHODS AND PROCEDURES USED IN IMPLEMENTING THE AIRSPACE PLANNING METHODOLOGY

3.1   The main factors characterizing the development of the ATS system and their interrelationship are presented in Figure A-12-1.

3.1.1   For a formal description of the processes in airspace planning, the factors W, R, S, E and P are the most suitable (in view of the possibility of formalizing them). Formally, the impact of each factor can be described by the following indices:

$W_{ij}(t)$ — indices characterizing air traffic density,

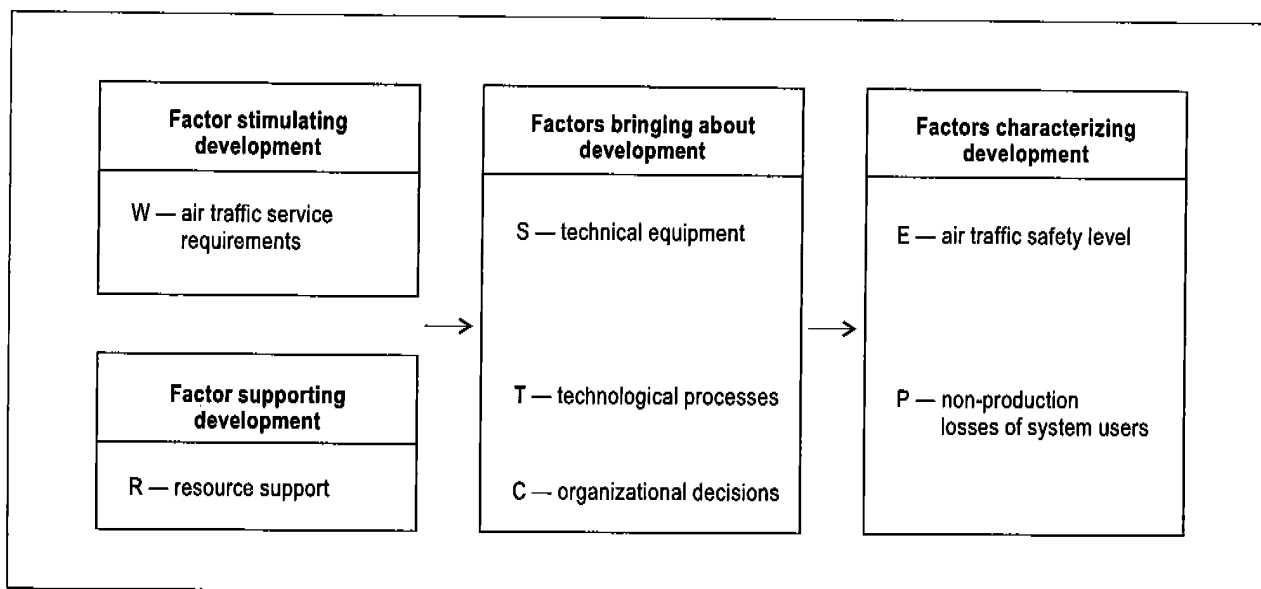i — number (title) of the index,

Figure A-12-1

j — number ("name") of the site characterized by the ith index,

t — the moment in time being considered;

$R_{ij}(t)$ — indices characterizing the resource support;

$S_{ij}(t)$ — indices of the technical equipment — number of technical facilities (systems) of the ith designation at the jth site at time t.

$E_{ij}(t)$ — indices characterizing the air traffic safety level,

$P_{ij}(t)$ — indices characterizing the non-production losses of system users.

3.2 The main methods for solving the formal problems stipulated by the airspace planning methodology are:

a) applied statistical analysis methods (ASAM), which are used in constructing standard economic and mathematical models of sets of simultaneous econometric equations or production functions; and

b) optimization methods (OM) based on mathematical (linear and non-linear) programming models, which are used to find the best solutions on the basis of a given criterion under conditions involving a number of limitations.

3.3 The ASAM are mainly used in phase I of airspace planning in elaborating the long-term forecast.

3.3.1 The use of the ASAM makes it possible to construct a basic model (set of models) describing the interrelationship between the factors in the system in the illustrative form:

$$f(W,R,S,E,P,t) = 0$$

3.3.2 The solution of phase I, problem 1 allows the anticipated air traffic service requirements W^o, the requirements for the air traffic safety level E^o and the effectiveness (non-production losses of system users) P^o of the system to be determined. The basic model is then used to calculate the required financial support.

$$R^T = R^T(W^o,E^o,P^o\,|\,S=S^o)$$

under traditional technical equipment development conditions.

3.3.3 If R^T significantly exceeds the financing possibilities R^o, it is necessary to solve phase I, problem 3, as a result of which some possible non-traditional technical equipment options S_i are worked out using non-formal methods, among others.

3.3.4 Afterwards, phase I, problem 4 can be solved, i.e. option S^{opt} found, on the basis of the conditions

$$E = E(W^o,R^o,P^o\,|\,S=S^{opt}) \geq E^o$$
$$P = P(W^o,R^o,E^o\,|\,S=S^{opt}) \leq P^o$$

3.3.5 If the conditions indicated above cannot be met simultaneously, and it is not possible to expand the total number of options S_i being considered, the requirements for the effectiveness index P^O and/or the resource limitation R^T may be relaxed.

3.4 The use of the OM makes it possible to solve basic problem 4 in phase II. To solve it, the results of the solution of phase II, problems 1-3 are used as the reference data:

a) the goal-oriented values of the effectiveness indices E^* and P^*;

b) the possible means S_{ij}^{opt} of implementing the preferred option S^{opt} for developing the technical equipment at the sites of the region (i - option number, j - site name); and

c) the indices of effectiveness E_{ij}^{opt}, P_{ij}^{opt} and cost R_{ij}^{opt} of the ith means of implementing S^{opt} at the jth site.

Problem 4 can then be formulated as:

find $\overline{S_{ij}^{opt}}$ from the condition:

$$\sum_{ij} R_{ij}^{opt} S_{ij}^{opt} \to \min$$

under the limitations:

$$\sum_{ij} \alpha_j E_{ij}^{opt} \geq E^*; \quad \alpha_j \quad = \quad \text{a weighting factor}$$
representing the proportion of total system flight hours which were spent in sector j

$$\sum P_{ij}^{opt} \leq P^*$$

$$\sum_i S_{ij}^{opt} = 1; \quad S_{ij}^{opt} \in \{0, 1\}$$

3.4.1 The OM is used in a similar way in solving phase III, problem 4. Here, however, the aggregate value of the cost of implementation and the value of one of the indices of effectiveness (or a linear combination of the two indices), which must attain the extreme value in the optimum selection of the composition of the technical facilities for each system site, are used as the goal-oriented function.

3.4.2 One substantial limitation is a limitation on the volume of financing, which is fully defined at the technical equipment plan elaboration stage.

Appendix 13

ANALYSIS TO DERIVE LATERAL NAVIGATION PERFORMANCE REQUIREMENTS FOR THE NORTH PACIFIC (NOPAC) ROUTE SYSTEM FOR 50 NM SPACING BETWEEN PARALLEL ROUTES

1.   In assessing the navigational performance required to achieve reduced lateral separations between adjacent North Pacific (NOPAC) routes, the United States Federal Aviation Administration (FAA) applied the well-known Reich collision risk model, described in the *Air Traffic Services Planning Manual* (Doc 9426), Part II, Section 2. The mathematical notation used in this appendix follows that of the manual.

2.   The initial step in applying the model was to determine relevant occupancies. North Pacific (NOPAC) estimates from 1985 gave $P_x(same) = 7.73 \times 10^{-6}$ and $P_x(opp) = 9.59 \times 10^{-6}$. Applying the relation $P_x = \dfrac{\lambda_x}{2\,S_x}\,E_y$, with the longitudinal interval $S_x = 120$ NM and aircraft length $\lambda_x = 0.0382$ NM, resulted in $E_y(same) = .049$ and $E_y(opp) = .060$. The relevant speed parameters (all measured in kt) had the values

$$\overline{|\Delta V|} = 29,\ \overline{|\dot{y}(S)|} = 42.22,\ \overline{|\dot{z}(0)|} = 1.5,\ \text{and}\ V = 480.$$

Average aircraft wingspan, λ_y, was 0.0351 NM, and average height, λ_z, was 0.0105 NM. The resulting value of K_{same} was then 1 052.43, and that of K_{opp} was 13 238.3. The relation

$$\frac{E_y(same) \cdot K_{same} + E_y(opp) \cdot K_{opp}}{K_{same}} = E_y(same)_{equivalent}$$

yielded the value $E_y(same)_{equivalent} = 0.806$. To account for growth from 1985 to 1995, analysts assumed an annual growth rate of 6 per cent. Over a ten-year period, that rate gave rise to an inflator of $(1.06)^{10} = 1.79$. Since there was a slight downturn in traffic growth in 1990 and 1991, and since growth in occupancy lags somewhat behind traffic growth, the inflator probably overstated the expected 1995 level of occupancy and thus allowed for conservative risk estimates during the following few years of NOPAC operation. Multiplying the 1985 value of $E_y(same)_{equivalent}$ by the inflator yielded a 1995 value of $E_y(same)_{equivalent}$ equal to 1.44. To account for the possibility of a small amount of opposite-direction traffic, $E_y(opp)_{equivalent}$ was taken to be 1 per cent of $E_y(same)_{equivalent}$, i.e. 0.014.

3.   As written in the *ATS Planning Manual*, Part II, Section 2, Chapter 4, Appendix C, the number of accidents due to loss of planned lateral separation, in 10 million flying hours, is

$$N_{ay} = 10^7\, P_y(S)\, P_z(0)\, \frac{\lambda_x}{S_x}\left[E_y(same) \cdot K(same) \;+\; E_y(opp) \cdot K(opp)\right].$$

FAA analysts used the 1995 values of $E_y(same)_{equivalent}$ and $E_y(opp)_{equivalent}$ to respectively represent $E_y(same)$ and $E_y(opp)$ in this basic equation. They took the value of $P_z(0)$ to be 0.38, and the values of all other parameters on the right side of the equation, except $P_y(S)$, were those given above. Taking a maximum tolerable value of 0.2 for N_{ay}, they then solved to obtain a maximum tolerable lateral overlap probability $P_y(S) = 9.66 \times 10^{-8}$.

4.   For many years aircraft lateral errors have been modelled as double double exponential (DDE) random variables. The probability density function of a DDE random variable has the form

$$f(y) \;=\; \frac{1-\alpha}{2\lambda_1}\, e^{\frac{-|y|}{\lambda_1}} \;+\; \frac{\alpha}{2\lambda_2}\, e^{\frac{-|y|}{\lambda_2}}$$

in which the parameters α, λ_1 and λ_2 satisfy the conditions $0 < \alpha < 1$, and $0 < \lambda_1 < \lambda_2$. When two parallel routes are separated by a distance S, the probability that aeroplanes assigned to different routes have laterally overlapping positions is

$$P_y(S) \;=\; 2\lambda_y\left[\left(\frac{1-\alpha}{2\lambda_1}\right)^2(\lambda_1+S)e^{-\frac{S}{\lambda_1}} \;+\; \left(\frac{\alpha}{2\lambda_2}\right)^2(\lambda_2+S)e^{-\frac{S}{\lambda_2}}\right.$$

$$\left. +\; \frac{\alpha(1-\alpha)}{2}\left(\frac{e^{-\frac{S}{\lambda_1}} + e^{-\frac{S}{\lambda_2}}}{\lambda_1 + \lambda_2} \;+\; \frac{e^{-\frac{S}{\lambda_2}} - e^{-\frac{S}{\lambda_1}}}{\lambda_2 - \lambda_1}\right)\right].$$

The sum enclosed in rectangular brackets on the right side of this equation consists of three terms of which the second, $\left(\dfrac{\alpha}{2\lambda_2}\right)^2(\lambda_2+S)e^{-\frac{S}{\lambda_2}}$, contributes very little. The first term, $\left(\dfrac{1-\alpha}{2\lambda_1}\right)^2(\lambda_1+S)e^{-\frac{S}{\lambda_1}}$, dominates the sum when λ_1 is more than (approximately) $S/15$. However, in September 1992, the Review of the General Concept of Separation Panel (RGCSP) Working Group A determined that required navigation performance (RNP) for oceanic flight should be set small enough to keep λ_1 well below that value (preferably no more than $S/18$) and thus reduce the influence of the first term to a negligible amount. The result of this approach, therefore, is to render the third term dominant. That is, when RNP is less than $S/6$, the three-term sum in brackets consists almost entirely of the term $\dfrac{\alpha(1-\alpha)}{2}\left(\dfrac{e^{-\frac{S}{\lambda_1}} + e^{-\frac{S}{\lambda_2}}}{\lambda_1 + \lambda_2} \;+\; \dfrac{e^{-\frac{S}{\lambda_2}} - e^{-\frac{S}{\lambda_1}}}{\lambda_2 - \lambda_1}\right)$. Furthermore, since α is normally quite small, whenever λ_1 is small, the third term can be approximated quite accurately by the simpler expression

$\dfrac{\alpha}{\lambda_2} e^{-\frac{S}{\lambda_2}}$. Under those circumstances, $P_y(S) \approx 2\lambda_y \dfrac{\alpha}{\lambda_2} e^{-\frac{S}{\lambda_2}}$. As a function of λ_2, $\dfrac{e^{-\frac{S}{\lambda_2}}}{\lambda_2}$ has one local maximum,

which occurs at $\lambda_2 = S$. Its value there is $\dfrac{1}{e \cdot S}$. By choosing $\lambda_2 = S$, α can conservatively be taken to have the

value $\dfrac{P_y(S) \cdot e \cdot S}{2\lambda_y} = .000187$.

5.   As was mentioned above, aircraft using parallel routes separated by 50 NM should meet an RNP of 50/6 NM. Since the RGCSP preferred integer values of RNP, FAA analysts assumed that the RNP of such a route system would be the next smallest integer, i.e. 8 NM. The corresponding value of λ_1 is $\dfrac{8}{-\ln(.05)} = 2.6705$. Thus if the lateral errors of the fleet using the route system are characterized by a DDE density, its parameters must be no worse than $\alpha = 0.000187$, $\lambda_1 = 2.6705$, and $\lambda_2 = 50$.

6.   It is possible to express the navigational performance required for operation in the planned route system in somewhat simpler terms. The value of λ_1 corresponds to a standard deviation of typical lateral errors of $\sqrt{2}\,\lambda_1$, or 3.78 NM. The gross error performance can also be described more simply. As is done in North Atlantic minimum navigation performance specification (MNPS) airspace, aircraft could be constrained to spend all but a certain proportion, η, of their flying time within one-half separation standard (i.e. within $S/2$) of their route centre lines, and they could be constrained to spend all but another proportion of their flying time, ζ, more than some established distance (e.g. 10 NM) away from either adjacent route's centre line. By integrating the DDE density function specified above, between appropriate limits (i.e. by evaluating the DDE distribution function at appropriate points), FAA analysts found the values of η and ζ to be $\eta = 1.994 \cdot 10^{-4} \approx 2.0 \cdot 10^{-4}$ and $\zeta = 2.802 \cdot 10^{-5} \approx 2.8 \cdot 10^{-5}$. Table A-13-1 summarizes the two (equivalent) means of describing the navigational performance required of the NOPAC fleet in order for N_{ay}, the number of accidents expected in 10 million NOPAC flight hours, to remain less than the maximum tolerable value of 0.2.

**Table A-13-1.   Two (equivalent) means of describing
the navigational performance required of the NOPAC fleet**

DDE distribution	$\alpha = 0.000187$	$\lambda_1 = 2.67$ NM	$\lambda_2 = 50$ NM
Navigation constraint	RNP = 8 NM	$\eta = 2.0 \times 10^{-4}$	$\zeta = 2.8 \times 10^{-5}$

Appendix 14

ESTIMATING OCCUPANCY AND THE RATE OF ACCIDENTS DUE TO THE LOSS OF PLANNED LATERAL SEPARATION FOR THE INDIVIDUAL COMPONENTS OF A SYSTEM OF PARALLEL ROUTES

1. INTRODUCTION

The North Atlantic Organized Track System (NAT OTS) consists of two sets of one-way routes, adjusted each day so that eastbound traffic takes advantage of the jet stream, and westbound traffic avoids it. In practice the NAT OTS functions as two systems, each with about a half-dozen parallel routes. When computing occupancy for each set of routes, analysts examine traffic records, typically for the fourth and fifteenth days of each month. They note the number of flights that passed some chosen longitude (e.g. 30°W or 40°W) and the time at which each flight passed that longitude. If two aircraft were assigned to adjacent routes at the same flight level and passed the chosen longitude within 15 minutes of each other, they make up a "proximate pair". Ignoring the effect of "random" traffic — which is not relevant to the present discussion — we note that a route system's occupancy is computed as twice the number of proximate pairs, divided by the number of flights that passed the chosen longitude. We especially note that occupancy is computed for an *entire* one-way route system. Though there does not seem to be any impediment to sorting the data so as to compute occupancies for *specific pairs of adjacent flight paths*, such computations are not routinely carried out by the air traffic authorities that produce occupancy statistics. This appendix first suggests a method of estimating occupancies for pairs of paths by using the occupancy of the entire system and the relative distribution of traffic over the system's flight paths. The path-pair occupancies are then used to derive expressions for the accident rates of the path pairs and for the accident rates of individual paths, flight levels and routes. This appendix shows the weighting factors that must be applied to flight hourly accident rates in order for them to be added to each other, and it demonstrates that the rates derived for individual paths, path pairs, flight levels and routes are consistent with the accident rate computed (by the Reich model) for the entire system.

2. NOTATION AND BASIC RELATIONSHIPS

2.1   We assume that a one-way system of parallel oceanic air routes consists of n routes, $R_1, ..., R_n$, each operating on the same m flight levels, $L_1, ..., L_m$. We let $P_{i,j}$ denote the flight path which is the restriction of route R_j to flight level L_i. We assume that the average time needed for an aeroplane to cross the ocean does not vary with the aeroplane's assigned route, and we let T (hours) denote that time.

2.2   We let O denote the occupancy of the entire system, f the number of flights that passed the chosen longitude, and p the number of proximate pairs. Thus (ignoring "random" traffic) the occupancy computation

applies the equation $O = \dfrac{2p}{f}$. Reports of occupancy computations normally indicate the observed values of f and p, as well as the value of O derived from them.

2.3 We parenthetically note that Appendix 13 derives another method of computing occupancy, viz. the method given in the *Air Traffic Services Planning Manual* (Doc 9426). Doc 9426 defines occupancy to mean twice the ratio of route-system proximity time to route-system flight time, but it is easy to see that this definition is essentially equivalent to the definition in 1.1 and 2.2. By virtue of the "snapshot" principle (the uniformity assumption that underlies much of collision risk theory for systems of parallel routes) the number of observed pairs, p, is the same at any observation point along the route system (though the specific pairs observed at one point are not necessarily the ones observed at another point). Since T denotes the number of hours needed to traverse the ocean on any of the system's routes, we see that there are pT hours of proximity time and fT hours of flight time during the observation period. Thus twice the ratio of proximity time to flight time is simply $\dfrac{2pT}{fT}$, or $\dfrac{2p}{f}$ (QED).

2.4 Given a pair of adjacent paths at the same altitude, $P_{i,j}$ and $P_{i,j+1}$ ($1 \le i \le m$, $1 \le j \le n-1$), we let $O_{i,j}$ denote their occupancy. We do not know the number of flights counted on each path, but it can nonetheless be given a name. We let $f_{i,j}$ be the number of flights that passed the chosen longitude while assigned to path $P_{i,j}$ ($1 \le i \le m$, $1 \le j \le n$), and we let $p_{i,j}$ be the number of proximate pairs that consisted of one aeroplane from $P_{i,j}$ and one aeroplane from $P_{i,j+1}$ ($1 \le i \le m$, $1 \le j \le n-1$). Though we do not know the numerical values of the $f_{i,j}$ and $p_{i,j}$, we do know that every flight is on exactly one path so that $f = \sum_{i=1}^{m} \sum_{j=1}^{n} f_{i,j}$, and every proximate pair belongs to exactly one pair of adjacent paths so that $p = \sum_{i=1}^{m} \sum_{j=1}^{n-1} p_{i,j}$.

3. OCCUPANCY OF A SPECIFIC PAIR OF PATHS

3.1 Restricting our attention to a particular pair of adjacent paths at the same flight level, $P_{k,l}$ and $P_{k,l+1}$, we compute the pair's occupancy as twice the number of proximate (aircraft) pairs having one member from each path, divided by the total number of flights on the two paths. That is:

$$O_{k,l} = \frac{2p_{k,l}}{f_{k,l} + f_{k,l+1}} = \frac{2p_{k,l}}{f} \cdot \frac{f}{f_{k,l} + f_{k,l+1}},$$

provided that $f_{k,l}$ and $f_{k,l+1}$ are not both 0. If they are both 0, we take $O_{k,l}$ to be 0, since it would not make sense to imagine non-zero occupancy for a pair of paths that do not have any traffic.

3.2 Lacking empirical knowledge of the numbers $p_{i,j}$, we can nevertheless make a reasonable assumption as to their distribution. We expect $p_{k,l}$, the number of proximate aircraft pairs associated with paths $P_{k,l}$ and $P_{k,l+1}$, to be proportional to $f_{k,l} \cdot f_{k,l+1}$, the product of the traffic levels realized on those two paths. In particular, we

assume that $p_{k,l} = \dfrac{f_{k,l} \cdot f_{k,l+1}}{\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} f_{i,j} \cdot f_{i,j+1}} \cdot p$. If $f_{k,l} = 0$ or $f_{k,l+1} = 0$, then $p_{k,l}$ must be 0, and in that case $O_{k,l}$ must also

be 0, which agrees with our intuitive understanding that a path pair exhibits zero occupancy when either of its members does not have any traffic. In the rest of this appendix we assume that $f_{k,l}$ and $f_{k,l+1}$ are both non-zero.

3.3   The assumption of proportionality in the last paragraph is an approximation whose accuracy needs to be determined by empirical studies. In particular we note that in any data collection the numbers of proximate pairs, $p_{i,j}$, would be integers, yet we have no guarantee that the formula for $p_{k,l}$ will yield an integer. However, the formula does satisfy the conservation criterion stated as the last equation of the previous section:

$$\sum_{k=1}^{m}\sum_{l=1}^{n-1} p_{k,l} \;=\; \sum_{k=1}^{m}\sum_{l=1}^{n-1}\left(\frac{f_{k,l}\cdot f_{k,l+1}}{\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} f_{i,j}\cdot f_{i,j+1}} \cdot p \right) \;=\; p\cdot\left(\frac{\displaystyle\sum_{k=1}^{m}\sum_{l=1}^{n-1} f_{k,l}\cdot f_{k,l+1}}{\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} f_{i,j}\cdot f_{i,j+1}} \right) \;=\; p\cdot 1 \;=\; p.$$

3.4   In order to develop an expression for $O_{k,l}$ in terms of O, we adopt a simplifying notation. For each path $P_{i,j}$, let $r_{i,j}$ denote the quotient $\dfrac{f_{i,j}}{f}$. Each $r_{i,j}$ is then the fraction of the route system's traffic that travelled on path $P_{i,j}$, so

$$\sum_{i=1}^{m}\sum_{j=1}^{n} r_{i,j} = \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{f_{i,j}}{f} \;=\; \frac{1}{f}\sum_{i=1}^{m}\sum_{j=1}^{n} f_{i,j} = \frac{f}{f} \;=\; 1.$$

Recalling the expression for $O_{k,l}$ in 3.1, we apply this new notation to the factor $\dfrac{f}{f_{k,l}+f_{k,l+1}}$, rewriting it as

$\dfrac{1}{\dfrac{f_{k,l}}{f}+\dfrac{f_{k,l+1}}{f}} \;=\; \dfrac{1}{r_{k,l}+r_{k,l+1}}$. We also take advantage of this notation to rewrite the number of proximate pairs belonging to each path pair:

$$p_{k,l} = \frac{f_{k,l}\cdot f_{k,l+1}}{\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} f_{i,j}\cdot f_{i,j+1}} \cdot p = \frac{\dfrac{f_{k,l}}{f}\cdot\dfrac{f_{k,l+1}}{f}}{\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1}\frac{f_{i,j}}{f}\cdot\frac{f_{i,j+1}}{f}} \cdot p = \frac{r_{k,l}\cdot r_{k,l+1}}{\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} r_{i,j}\cdot r_{i,j+1}} \cdot p.$$

At least one pair of adjacent flight paths must have positive traffic loads on both of its paths (otherwise there is no occupancy to compute) so in the denominator at least one term, $r_{i,j} \cdot r_{i,j+1}$, is positive, and we can be sure that the denominator is non-zero.

3.5   We can now substitute the expressions derived in the last three paragraphs into the expression for $O_{k,l}$ that was given in 3.1. We find that

$$O_{k,l} = 2 \cdot \frac{r_{k,l} \cdot r_{k,l+1} \, p}{\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} r_{i,j} \cdot r_{i,j+1}} \cdot \frac{1}{f} \cdot \frac{1}{r_{k,l} + r_{k,l+1}} = \frac{r_{k,l} \cdot r_{k,l+1}}{\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} r_{i,j} \cdot r_{i,j+1}} \cdot \frac{1}{r_{k,l} + r_{k,l+1}} \cdot \frac{2p}{f} \cdot$$

3.6   Finally, recalling from 2.2 that $\dfrac{2p}{f}$ is O, the occupancy computed for the entire route system, we write

$$O_{k,l} = \frac{r_{k,l} \cdot r_{k,l+1}}{\left(\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} r_{i,j} \cdot r_{i,j+1}\right) \cdot \left(r_{k,l} + r_{k,l+1}\right)} \cdot O.$$

3.7   For the sake of notational convenience, we let

$$M_{k,l} = \frac{r_{k,l} \cdot r_{k,l+1}}{\left(\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} r_{i,j} \cdot r_{i,j+1}\right) \cdot \left(r_{k,l} + r_{k,l+1}\right)}$$

whenever $r_{k,l}$ and $r_{k,l+1}$ are both non-zero. We call $M_{k,l}$ the occupancy multiplier for the pair of paths, $P_{k,l}$ and $P_{k,l+1}$, and using this notation we write $O_{k,l} = M_{k,l} \cdot O$. That is, $M_{k,l}$ is the ratio of the occupancy of path pair ($P_{k,l}$, $P_{k,l+1}$) to the occupancy of the entire route system.

3.8   Expressing $O_{k,l}$ as $M_{k,l} \cdot O$ may be useful because having obtained O, the occupancy of the entire route system, we can then estimate the occupancy of any pair of adjacent paths, even without knowing the relevant number of proximate aircraft pairs or the number of flights on each of the paths. All that we need is an estimate of the fraction of traffic, $r_{i,j}$, on each path. For some studies it may be possible to estimate those fractions from flight plan data; in other cases it may suffice to posit hypothetical values.

4. NUMERICAL EXAMPLES

4.1   If mn, the number of paths, is not a relatively small number, calculating the multipliers $M_{k,l}$ may become a bit tedious — at least if the calculation has not been programmed into a computer. There are, however, a couple of "extreme" cases in which we can rapidly obtain results even without relying on automation.

4.2   Suppose, for example, that all traffic is concentrated on just two adjacent paths, $P_{k,l}$ and $P_{k,l+1}$. In that case the only non-zero $r_{i,j}$ are $r_{k,l}$ and $r_{k,l+1}$, and $r_{k,l} + r_{k,l+1} = 1$. For any pair (i,j) other than (k,l), $O_{i,j} = 0$. The sum $\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} r_{i,j} \cdot r_{i,j+1}$ has only one non-zero term, viz. $r_{k,l} \cdot r_{k,l+1}$, and so $M_{k,l} = \dfrac{r_{k,l} \cdot r_{k,l+1}}{\left(r_{k,l} \cdot r_{k,l+1}\right) \cdot 1} = 1$, which agrees with our intuitive understanding that when all of the traffic is assigned to just one pair of adjacent routes, the occupancy for that route pair must be the overall occupancy O.

4.3   Table A-14-1 shows a spreadsheet that calculates the multipliers $M_{k,l}$ for a route system having four routes at each of three flight levels. With two exceptions, the numbers inserted into column "r(i,j)", which shows the fraction of traffic on each path, are all zero. The two exceptions are for routes R_2 and R_3 on flight level L_2. Path $P_{2,2}$ has 30 per cent of the system's traffic, while $P_{2,3}$ carries the remaining 70 per cent. The spreadsheet calculates an occupancy multiplier of 1 for that pair of paths and shows the code "NA" ("not applicable") for all other multipliers.

4.4   A much more interesting case occurs when we consider the traffic to be uniformly distributed over all mn flight paths. In that case, each $r_{i,j} = \dfrac{1}{mn}$, and so

$$M_{k,l} \;=\; \frac{\dfrac{1}{mn}\cdot\dfrac{1}{mn}}{\left(\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1}\dfrac{1}{mn}\cdot\dfrac{1}{mn}\right)\cdot\left(\dfrac{1}{mn}+\dfrac{1}{mn}\right)} \;=\; \frac{\left(\dfrac{1}{mn}\right)^2}{m(n-1)\cdot\left(\dfrac{1}{mn}\right)^2\cdot\dfrac{2}{mn}} \;=\; \frac{1}{m(n-1)\cdot\dfrac{2}{mn}} \;=\; \frac{n}{2(n-1)}$$

for every pair of paths, $P_{k,l}$ and $P_{k,l+1}$, i.e. for every pair of adjacent paths at the same flight level. We note that this formula for $M_{k,l}$ is independent of m, the number of flight levels in the system, and we also observe that as n increases, $M_{k,l}$ approaches 1/2 from above.

4.5   Table A-14-2 shows a spreadsheet similar to that of Table A-14-1, but the numbers in column "r(i,j)" are all $1/12 = .0833$, which is the fraction of traffic borne by each flight path when the traffic is uniformly distributed over all 12 paths. Since n, the number of routes, is 4, the formula derived in the last paragraph tells us that the multipliers must all be $\dfrac{4}{2(4-1)} = \dfrac{2}{3}$, and indeed Table A-14-2 shows that the spreadsheet calculates all of the occupancy multipliers to be 0.6667.

4.6   Empirical studies may eventually show that for certain traffic distributions the assumption of proportionality (in 3.2) does not accurately describe the distribution of proximate pairs. However, the assumption *must* be accurate for the case in which the traffic is uniformly distributed. When all flight paths carry the same load, the assumption infers that every pair of adjacent paths produces the same number of proximate aircraft pairs. That is not surprising. As long as the theory treats the number of proximate pairs produced by a pair of paths as a function solely of the traffic loads on those paths, it follows that whenever the paths all carry the same load, every pair of paths must produce the same number of proximate aircraft pairs, regardless of the specific form of the function. That is, if g denotes the function, then $p_{i,j} = g(f_{i,j}, f_{i,j+1})$ for

$1 \le i \le m$, $1 \le j \le n-1$. If the traffic is uniformly distributed, then every $f_{i,j}$ and $f_{i,j+1}$ has the same value, viz. $\dfrac{f}{mn}$,

and so every $p_{i,j} = g\!\left(\dfrac{f}{mn}, \dfrac{f}{mn}\right)$. Therefore, $p = \displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} p_{i,j} = m(n-1)\cdot g\!\left(\dfrac{f}{mn}, \dfrac{f}{mn}\right)$, so $p_{i,j} = g\!\left(\dfrac{f}{mn}, \dfrac{f}{mn}\right)$

$= \dfrac{p}{m(n-1)}$, which clearly does not depend on either i or j.

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**Table A-14-1.   Calculation of occupancy multipliers
for a system of four routes on three flight levels,
with traffic distributed as in column "r(i,j)"**

Flight level i	Route j	r(i,j)	r(i,j) * r(i,j+1)	M(i,j)	Path pair
1	1	0.0000	0.0000	NA	L1, R1 & R2
1	2	0.0000	0.0000	NA	L1, R2 & R3
1	3	0.0000	0.0000	NA	L1, R3 & R4
1	4	0.0000			
2	1	0.0000	0.0000	NA	L2, R1 & R2
2	2	0.3000	0.2100	1.0000	L2, R2 & R3
2	3	0.7000	0.0000	NA	L2, R3 & R4
2	4	0.0000			
3	1	0.0000	0.0000	NA	L3, R1 & R2
3	2	0.0000	0.0000	NA	L3, R2 & R3
3	3	0.0000	0.0000	NA	L3, R3 & R4
3	4	0.0000			
	Sums:	1.0000	0.2100		

**Table A-14-2.   Calculation of occupancy multipliers
for a system of four routes on three flight levels,
with traffic distributed as in column "r(i,j)"**

Flight level i	Route j	r(i,j)	r(i,j)* r(i,j+1)	M(i,j)	Path pair
1	1	0.0833	0.0069	0.6667	L1, R1 & R2
1	2	0.0833	0.0069	0.6667	L1, R2 & R3
1	3	0.0833	0.0069	0.6667	L1, R3 & R4
1	4	0.0833			
2	1	0.0833	0.0069	0.6667	L2, R1 & R2
2	2	0.0833	0.0069	0.6667	L2, R2 & R3
2	3	0.0833	0.0069	0.6667	L2, R3 & R4
2	4	0.0833			
3	1	0.0833	0.0069	0.6667	L3, R1 & R2
3	2	0.0833	0.0069	0.6667	L3, R2 & R3
3	3	0.0833	0.0069	0.6667	L3, R3 & R4
3	4	0.0833			
	Sums:	1.0000	0.0625		

4.7   Table A-14-3 shows the same sort of spreadsheet shown in Tables A-14-1 and A-14-2, but with a typical traffic distribution. Flight level L_1 carries 1/4 of the traffic, L_2 has 5/12 of it, and L_3 has the remaining 1/3. On each flight level, route R_1 has 1/12 of the traffic, R_2 has 1/3 of it, R_3 has 5/12 of it, and R_4 has the remaining 1/6. The occupancy multipliers calculated by the spreadsheet range from a minimum value of 0.2033, for path pair $(P_{1,1}, P_{1,2})$, to a maximum of 0.9412 for path pair $(P_{2,2}, P_{2,3})$.

4.8   In the examples given so far, $M_{k,l}$ has been less than or equal to 1, but that does not hold true in general. Indeed, we see from the definition (in 3.7) that $M_{k,l} \geq 1$ if and only if $\dfrac{r_{k,l} \cdot r_{k,l+1}}{r_{k,l} + r_{k,l+1}} \geq \sum\limits_{i=1}^{m} \sum\limits_{j=1}^{n-1} r_{i,j} \cdot r_{i,j+1}.$

Table A-14-4 shows a traffic distribution derived from that of Table A-14-3 by shifting traffic from one path to another. In particular, the loads on paths $P_{1,2}$, $P_{1,4}$, $P_{3,1}$, and $P_{3,3}$ were respectively shifted onto paths $P_{1,1}$, $P_{1,3}$, $P_{3,2}$, and $P_{3,4}$, so that all six pairs of adjacent paths on flight levels L_1 and L_3 have zero occupancy. As can be seen in the column headed "r(i,j)*r(i,j+1)", the three products $r_{1,j} \cdot r_{1,j+1}$ are all zero, as are the three products

$r_{3,j} \cdot r_{3,j+1}$. The sum $\sum\limits_{i=1}^{m} \sum\limits_{j=1}^{n-1} r_{i,j} \cdot r_{i,j+1}$ is less than $\dfrac{r_{2,2} \cdot r_{2,3}}{r_{2,2} + r_{2,3}}$ and $\dfrac{r_{2,3} \cdot r_{2,4}}{r_{2,3} + r_{2,4}}$, and so $M_{2,2} = 1.8824$ and $M_{2,3} = 1.2101$ are both greater than 1.

5.   ACCIDENT RATES FOR PAIRS OF ADJACENT FLIGHT PATHS

5.1   Let $a_{k,l}$ denote the expected rate of accidents for the pair of paths $P_{k,l}$ and $P_{k,l+1}$. In the Reich model for the risk of collision due to the loss of planned lateral separation, we write $a_{k,l}$ as a product of 5 factors: 1) the probability that two aeroplanes which are assigned to adjacent paths actually have laterally overlapping positions; 2) the probability that two aeroplanes assigned to the same flight level have vertically overlapping positions; 3) the probability that two proximate aeroplanes (proximate in the sense of 1.1) have longitudinally overlapping positions; 4) the sum of the reciprocals of the times needed for aircraft to pass each other in each of the three physical dimensions; and 5) the occupancy, $O_{k,l}$, of the pair $P_{k,l}$ and $P_{k,l+1}$. The accident rate $a_{k,l}$ is expressed in units of accidents per flight hour.

5.2   In the present discussion we are not concerned with changes in any of the first four factors mentioned in 5.1 and can assume that they are invariant over all pairs of paths. Therefore, we can simplify the notation in the present section by expressing the product of the first four factors as a constant C. That is, for every pair of paths $P_{k,l}$ and $P_{k,l+1}$, $a_{k,l} = C \cdot O_{k,l}$. Using the results of section 3, we rewrite this equation as

$$a_{k,l} = C \cdot O \cdot M_{k,l} = C \cdot O \cdot \frac{r_{k,l} \cdot r_{k,l+1}}{\left(\sum\limits_{i=1}^{m} \sum\limits_{j=1}^{n-1} r_{i,j} \cdot r_{i,j+1} \right) \cdot \left(r_{k,l} + r_{k,l+1} \right)}.$$

Since $C \cdot O$ is the accident rate of the entire system (in accidents per flight hour), $M_{k,l}$ is also the ratio of the accident rate of path pair $(P_{k,l}, P_{k,l+1})$ to that of the entire system. We also find it useful to express the equation for $a_{k,l}$ in one more form, i.e. as

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**Table A-14-3.   Calculation of occupancy multipliers
for a system of four routes on three flight levels,
with traffic distributed as in column "r(i,j)"**

Flight level i	Route j	r(i,j)	r(i,j)* r(i,j+1)	M(i,j)	Path pair
1	1	0.0208	0.0017	0.2033	L1, R1 & R2
1	2	0.0833	0.0087	0.5647	L1, R2 & R3
1	3	0.1042	0.0043	0.3630	L1, R3 & R4
1	4	0.0417			
2	1	0.0347	0.0048	0.3388	L2, R1 & R2
2	2	0.1389	0.0241	0.9412	L2, R2 & R3
2	3	0.1736	0.0121	0.6050	L2, R3 & R4
2	4	0.0694			
3	1	0.0278	0.0031	0.2711	L3, R1 & R2
3	2	0.1111	0.0154	0.7529	L3, R2 & R3
3	3	0.1389	0.0077	0.4840	L3, R3 & R4
3	4	0.0556			
	Sums:	1.0000	0.0820		

**Table A-14-4.   Calculation of occupancy multipliers
for a system of four routes on three flight levels,
with traffic distributed as in column "r(i,j)"**

Flight level i	Route j	r(i,j)	r(i,j)* r(i,j+1)	M(i,j)	Path pair
1	1	0.1042	0.0000	NA	L1, R1 & R2
1	2	0.0000	0.0000	NA	L1, R2 & R3
1	3	0.1458	0.0000	NA	L1, R3 & R4
1	4	0.0000			
2	1	0.0347	0.0048	0.6776	L2, R1 & R2
2	2	0.1389	0.0241	1.8824	L2, R2 & R3
2	3	0.1736	0.0121	1.2101	L2, R3 & R4
2	4	0.0694			
3	1	0.0000	0.0000	NA	L3, R1 & R2
3	2	0.1389	0.0000	NA	L3, R2 & R3
3	3	0.0000	0.0000	NA	L3, R3 & R4
3	4	0.1944			
	Sums:	1.0000	0.0410		

$$a_{k,l} \cdot (r_{k,l} + r_{k,l+1}) \;=\; C \cdot O \cdot \frac{r_{k,l} \cdot r_{k,l+1}}{\displaystyle\sum_{i=1}^{m} \sum_{i=1}^{n-1} r_{i,j} \cdot r_{i,j+1}} \; .$$

5.3   Let T denote the time needed for a typical aeroplane to cross the ocean on the route system. During the 24-hour period used to determine occupancy, the $f_{k,l}$ aeroplanes on path $P_{k,l}$ contributed $T \cdot f_{k,l}$ flight hours. Thus path $P_{k,l}$ experienced a traffic load of $\dfrac{T \cdot f_{k,l}}{24}$ flight hours per hour. Likewise, path $P_{k,l+1}$ experienced a traffic load of $\dfrac{T \cdot f_{k,l+1}}{24}$ flight hours per hour, and so the two paths experienced a (combined) traffic load of $\dfrac{T}{24} \cdot (f_{k,l} + f_{k,l+1}) \; \dfrac{\text{flight hours}}{\text{hour}}$. We can therefore express the accident rate for the pair of paths, $P_{k,l}$ and $P_{k,l+1}$, as

$$a_{k,l} \; \frac{\text{accidents}}{\text{flight hour}} \cdot \frac{T}{24} \cdot (f_{k,l} + f_{k,l+1}) \; \frac{\text{flight hours}}{\text{hour}} \;=\; \frac{T}{24} \cdot a_{k,l} \cdot (f_{k,l} + f_{k,l+1}) \; \frac{\text{accidents}}{\text{hour}} \; .$$

5.4   When accident rates are expressed in units of accidents per hour, we can add the rates for any two path pairs in order to obtain the total accident rate (still in accidents per hour) for those two pairs. Thus, for example, we find the total accident rate for the path pairs $(P_{1,2}, P_{1,3})$ and $(P_{3,3}, P_{3,4})$ to be

$$\frac{T}{24} \cdot a_{1,2} \cdot (f_{1,2} + f_{1,3}) \; \frac{\text{accidents}}{\text{hour}} \;+\; \frac{T}{24} \cdot a_{3,3} \cdot (f_{3,3} + f_{3,4}) \; \frac{\text{accidents}}{\text{hour}}$$

$$=\; \frac{T}{24} \, [a_{1,2}(f_{1,2} + f_{1,3}) + a_{3,3}(f_{3,3} + f_{3,4})] \; \frac{\text{accidents}}{\text{hour}} \; .$$

More generally, we obtain the total accident rate for all of the pairs in a set of path pairs by simply adding the rates (in accidents per hour) for the pairs in that set.

5.5   We can then convert the total rate from units of accidents per hour to units of accidents per flight hour. In the example in the last paragraph, path pairs $(P_{1,2}, P_{1,3})$ and $(P_{3,3}, P_{3,4})$ experienced $T\big(f_{1,2} + f_{1,3} + f_{3,3} + f_{3,4}\big)$ flight hours during the 24-hour period used to determine occupancy, so their traffic load was at the rate of $\dfrac{T}{24}\big(f_{1,2} + f_{1,3} + f_{3,3} + f_{3,4}\big) \; \dfrac{\text{flight hours}}{\text{hour}}$. Expressed in traditional units, the two path pairs had a combined accident rate of

$$\frac{\dfrac{T}{24}\,[a_{1,2}(f_{1,2} + f_{1,3}) + a_{3,3}(f_{3,3} + f_{3,4})] \; \dfrac{\text{accidents}}{\text{hour}}}{\dfrac{T}{24}\big(f_{1,2} + f_{1,3} + f_{3,3} + f_{3,4}\big) \; \dfrac{\text{flight hours}}{\text{hour}}} \;=\; \frac{a_{1,2}(f_{1,2} + f_{1,3}) + a_{3,3}(f_{3,3} + f_{3,4})}{f_{1,2} + f_{1,3} + f_{3,3} + f_{3,4}} \; \frac{\text{accidents}}{\text{flight hour}} \, ,$$

and when the numerator and denominator are divided by f, we re-express this rate as

$$\frac{a_{1,2}(r_{1,2}+r_{1,3})+a_{3,3}(r_{3,3}+r_{3,4})}{r_{1,2}+r_{1,3}+r_{3,3}+r_{3,4}} \quad \frac{\text{accidents}}{\text{flight hour}}.$$

5.6   Similarly, the accident rate for all of flight level L_k is

$$\frac{\displaystyle\sum_{j=1}^{n-1} a_{k,j}(r_{k,j}+r_{k,j+1})}{\displaystyle\sum_{j=1}^{n} r_{k,j}} \quad \frac{\text{accidents}}{\text{flight hour}},$$

and the accident rate for the route pair (R_l,R_{l+1}) is

$$\frac{\displaystyle\sum_{i=1}^{m} a_{i,l}(r_{i,l}+r_{i,l+1})}{\displaystyle\sum_{i=1}^{m} (r_{i,l}+r_{i,l+1})} \quad \frac{\text{accidents}}{\text{flight hour}}.$$

Note that in the formula for L_k, the denominator shows each path's flow counted once, even though all but the outermost paths belong to two path pairs.

5.7   Finally, we write the accident rate of the entire system, in accidents per hour, as

$$\frac{T}{24}\sum_{i=1}^{m}\sum_{j=1}^{n-1} a_{i,j}(f_{i,j}+f_{i,j+1}) \quad \frac{\text{accidents}}{\text{hour}}.$$

During the 24-hour period used to determine occupancy, the system experienced Tf flight hours, giving it a traffic rate of $\dfrac{Tf}{24} \dfrac{\text{flight hours}}{\text{hour}}$. Expressed in traditional units, the entire route system had an accident rate of

$$\frac{\dfrac{T}{24}\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} a_{i,j}(f_{i,j}+f_{i,j+1}) \dfrac{\text{accidents}}{\text{hour}}}{\dfrac{Tf}{24} \dfrac{\text{flight hours}}{\text{hour}}} = \frac{\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} a_{i,j}(f_{i,j}+f_{i,j+1})}{f} \quad \frac{\text{accidents}}{\text{flight hour}}$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{n-1} a_{i,j}\left(\frac{f_{i,j}}{f}+\frac{f_{i,j+1}}{f}\right) \frac{\text{accidents}}{\text{flight hour}} = \sum_{i=1}^{m}\sum_{j=1}^{n-1} a_{i,j}(r_{i,j}+r_{i,j+1}) \frac{\text{accidents}}{\text{flight hour}}$$

$$= \sum_{k=1}^{m}\sum_{l=1}^{n-1} a_{k,l}(r_{k,l}+r_{k,l+1}) \frac{\text{accidents}}{\text{flight hour}}.$$

However, we also know from 5.2 that each $a_{k,l} \cdot (r_{k,l} + r_{k,l+1}) = C \cdot O \cdot \dfrac{r_{k,l} \cdot r_{k,l+1}}{\sum\limits_{i=1}^{m} \sum\limits_{j=1}^{n-1} r_{i,j} \cdot r_{i,j+1}}$. Substituting this

expression into the sum, we find the accident rate of the entire route system to be

$$\sum_{k=1}^{m} \sum_{l=1}^{n-1} C \cdot O \cdot \frac{r_{k,l} \cdot r_{k,l+1}}{\sum\limits_{i=1}^{m} \sum\limits_{j=1}^{n-1} r_{i,j} \cdot r_{i,j+1}} = C \cdot O \cdot \frac{\sum\limits_{k=1}^{m} \sum\limits_{l=1}^{n-1} r_{k,l} \cdot r_{k,l+1}}{\sum\limits_{i=1}^{m} \sum\limits_{j=1}^{n-1} r_{i,j} \cdot r_{i,j+1}} = C \cdot O \cdot 1 = C \cdot O,$$

which is exactly the result that we expected from the Reich model. Thus we find that the formulae derived above for the accident rates of path pairs and collections of path pairs are consistent with the accepted model.

6.   ACCIDENT RATES FOR INDIVIDUAL FLIGHT PATHS

6.1   Paths $P_{k,1}$ and $P_{k,n}$ are the outer paths on flight level L_k, so each of them has only one adjacent path on that flight level. The aeroplanes on $P_{k,1}$ are exposed to risk from those on $P_{k,2}$, while the aeroplanes on $P_{k,n}$ are exposed to risk from those on $P_{k,n-1}$. If $2 \leq l \leq n-1$, the aeroplanes on path $P_{k,l}$ are exposed to the risk of collision with aeroplanes on two adjacent paths, $P_{k,l-1}$ and $P_{k,l+1}$.

6.2   The rate of accidents for the path pair $(P_{k,1} , P_{k,2})$ is $a_{k,1}$ $\dfrac{\text{accidents}}{\text{flight hour}}$. As was shown in 5.3, we can re-express this rate as $\dfrac{T}{24} \cdot a_{k,1} \cdot (f_{k,1} + f_{k,2})$ $\dfrac{\text{accidents}}{\text{hour}}$. Every collision due to the loss of planned lateral separation is counted as two accidents, each involving an aeroplane assigned to one of the relevant paths. Therefore, path $P_{k,1}$ has an accident rate of $\dfrac{1}{2} \cdot \dfrac{T}{24} \cdot a_{k,1} \cdot (f_{k,1} + f_{k,2})$ $\dfrac{\text{accidents}}{\text{hour}}$. Path $P_{k,1}$ experiences $T \cdot f_{k,1}$ flight hours in 24 hours, or $\dfrac{T f_{k,1}}{24}$ $\dfrac{\text{flight hours}}{\text{hour}}$, so its accident rate, expressed in traditional units, is

$$\frac{\dfrac{1}{2} \cdot \dfrac{T}{24} \cdot a_{k,1} \cdot (f_{k,1} + f_{k,2}) \ \dfrac{\text{accidents}}{\text{hour}}}{\dfrac{T}{24} \cdot f_{k,1} \ \dfrac{\text{flight hours}}{\text{hour}}} = \frac{a_{k,1} \cdot (f_{k,1} + f_{k,2})}{2 f_{k,1}} \ \frac{\text{accidents}}{\text{flight hour}}.$$

Dividing the numerator and denominator by f, we re-write $P_{k,1}$'s accident rate as

$$\frac{a_{k,1} \cdot (r_{k,1} + r_{k,2})}{2 r_{k,1}} \ \frac{\text{accidents}}{\text{flight hour}}.$$

Applying entirely analogous reasoning, we find that the hourly rate for path $P_{k,n}$ is

$$\frac{1}{2} \cdot \frac{T}{24} \cdot a_{k,n-1} \cdot (f_{k,n-1} + f_{k,n}) \ \frac{\text{accidents}}{\text{hour}},$$

while its traditional (flight hourly) rate is

$$\frac{a_{k,n-1} \cdot (r_{k,n-1} + r_{k,n})}{2 \, r_{k,n}} \ \frac{\text{accidents}}{\text{flight hour}}.$$

6.3   If $2 \le l \le n-1$, $P_{k,l}$'s accidents (due to the loss of planned lateral separation) arise from its aeroplanes' collisions with aeroplanes assigned to adjacent paths $P_{k,l-1}$ and $P_{k,l+1}$. The path pairs $(P_{k,l-1}, P_{k,l})$ and $(P_{k,l}, P_{k,l+1})$ have respective flight hourly accident rates $a_{k,l-1}$ and $a_{k,l}$, which can be converted (as in 5.3) to the respective hourly accident rates $\frac{T}{24} \cdot a_{k,l-1} (f_{k,l-1} + f_{k,l}) \ \frac{\text{accidents}}{\text{hour}}$ and $\frac{T}{24} \cdot a_{k,l} (f_{k,l} + f_{k,l+1}) \ \frac{\text{accidents}}{\text{hour}}$. Every collision involving aeroplanes assigned to either of these path pairs results in an accident involving an aeroplane assigned to path $P_{k,l}$, so $P_{k,l}$'s hourly accident rate is

$$\frac{1}{2} \cdot \frac{T}{24} \cdot a_{k,l-1} (f_{k,l-1} + f_{k,l}) \ \frac{\text{accidents}}{\text{hour}} \ + \ \frac{1}{2} \cdot \frac{T}{24} \cdot a_{k,l} (f_{k,l} + f_{k,l+1}) \ \frac{\text{accidents}}{\text{hour}}$$

$$= \frac{1}{2} \cdot \frac{T}{24} \left[a_{k,l-1} (f_{k,l-1} + f_{k,l}) + a_{k,l} (f_{k,l} + f_{k,l+1}) \right] \frac{\text{accidents}}{\text{hour}}.$$

Since $P_{k,l}$ experiences $\dfrac{T f_{k,l}}{24}$ flight hours per hour, its accident rate, expressed in units of accidents per flight hour, is

$$\frac{\dfrac{1}{2} \cdot \dfrac{T}{24} \left[a_{k,l-1} (f_{k,l-1} + f_{k,l}) + a_{k,l} (f_{k,l} + f_{k,l+1}) \right] \dfrac{\text{accidents}}{\text{hour}}}{\dfrac{T f_{k,l}}{24} \ \dfrac{\text{flight hours}}{\text{hour}}},$$

which immediately simplifies to

$$\frac{\left[a_{k,l-1} (f_{k,l-1} + f_{k,l}) + a_{k,l} (f_{k,l} + f_{k,l+1}) \right]}{2 f_{k,l}} \ \frac{\text{accidents}}{\text{flight hour}}.$$

Dividing the numerator and denominator by f, the total number of flights, finally yields $P_{k,l}$'s accident rate in the form

$$\frac{\left[a_{k,l-1} (r_{k,l-1} + r_{k,l}) + a_{k,l} (r_{k,l} + r_{k,l+1}) \right]}{2 \, r_{k,l}} \ \frac{\text{accidents}}{\text{flight hour}}.$$

6.4   In 5.6 we found the accident rate of flight level L_k by adding appropriately weighted accident rates of the pairs of adjacent paths on that flight level. We expect to obtain the same result by adding appropriately weighted accident rates of the individual paths.

6.5   In order to add the weighted rates of the individual paths, we express them in units of accidents per hour. From 6.2 and 6.3 we recall that on flight level L_k, $P_{k,1}$ has $\frac{1}{2} \cdot \frac{T}{24} \cdot a_{k,1} \cdot (f_{k,1} + f_{k,2})$ accidents per hour, while $P_{k,l}$ (for $2 \leq l \leq n-1$) has $\frac{1}{2} \cdot \frac{T}{24} \left[a_{k,l-1}(f_{k,l-1} + f_{k,l}) + a_{k,l}(f_{k,l} + f_{k,l+1}) \right]$ accidents per hour, and $P_{k,n}$ has $\frac{1}{2} \cdot \frac{T}{24} \cdot a_{k,n-1} \cdot (f_{k,n-1} + f_{k,n})$ accidents per hour. Thus, in each hour, the number of accidents experienced by the flight level is

$$\frac{1}{2} \cdot \frac{T}{24} \left(a_{k,1} \cdot (f_{k,1} + f_{k,2}) + \sum_{l=2}^{n-1} \left[a_{k,l-1}(f_{k,l-1} + f_{k,l}) + a_{k,l}(f_{k,l} + f_{k,l+1}) \right] + a_{k,n-1} \cdot (f_{k,n-1} + f_{k,n}) \right) .$$

Regrouping the terms of the sum within the large parentheses, we get:

$$a_{k,1}(f_{k,1} + f_{k,2}) + \sum_{l=2}^{n-1} \left[a_{k,l-1}(f_{k,l-1} + f_{k,l}) + a_{k,l}(f_{k,l} + f_{k,l+1}) \right] + a_{k,n-1}(f_{k,n-1} + f_{k,n})$$

$$= a_{k,1}(f_{k,1} + f_{k,2}) + [a_{k,1}(f_{k,1} + f_{k,2}) + a_{k,2}(f_{k,2} + f_{k,3})] + [a_{k,2}(f_{k,2} + f_{k,3}) + a_{k,3}(f_{k,3} + f_{k,4})] + \dots$$

$$+ [a_{k,n-3}(f_{k,n-3} + f_{k,n-2}) + a_{k,n-2}(f_{k,n-2} + f_{k,n-1})] + [a_{k,n-2}(f_{k,n-2} + f_{k,n-1}) + a_{k,n-1}(f_{k,n-1} + f_{k,n})]$$

$$+ a_{k,n-1}(f_{k,n-1} + f_{k,n}) = [a_{k,1}(f_{k,1} + f_{k,2}) + a_{k,1}(f_{k,1} + f_{k,2})] + [a_{k,2}(f_{k,2} + f_{k,3}) + a_{k,2}(f_{k,2} + f_{k,3})] + \dots$$

$$+ [a_{k,n-2}(f_{k,n-2} + f_{k,n-1}) + a_{k,n-2}(f_{k,n-2} + f_{k,n-1})] + [a_{k,n-1}(f_{k,n-1} + f_{k,n}) + a_{k,n-1}(f_{k,n-1} + f_{k,n})]$$

$$= \sum_{l=1}^{n-1} 2 \cdot a_{k,l}(f_{k,l} + f_{k,l+1}).$$

Thus, flight level L_k's hourly accident rate is

$$\frac{1}{2} \cdot \frac{T}{24} \cdot \sum_{l=1}^{n-1} 2 \cdot a_{k,l}(f_{k,l} + f_{k,l+1}) \ \frac{\text{accidents}}{\text{hour}} \ = \ \frac{T}{24} \cdot \sum_{l=1}^{n-1} a_{k,l}(f_{k,l} + f_{k,l+1}) \ \frac{\text{accidents}}{\text{hour}} .$$

The flight level experiences $\dfrac{T(f_{k,1} + f_{k,2} + \dots + f_{k,n})}{24}$ flight hours per hour, so its accident rate, expressed in traditional units, is

$$\frac{\dfrac{T}{24}\cdot\displaystyle\sum_{l=1}^{n-1}a_{k,l}(f_{k,l}+f_{k,l+1})\;\dfrac{\text{accidents}}{\text{hour}}}{\dfrac{T}{24}\displaystyle\sum_{l=1}^{n}f_{k,l}\;\dfrac{\text{flight hours}}{\text{hour}}}\;=\;\frac{\dfrac{1}{f}}{\dfrac{1}{f}}\cdot\frac{\displaystyle\sum_{l=1}^{n-1}a_{k,l}(f_{k,l}+f_{k,l+1})}{\displaystyle\sum_{l=1}^{n}f_{k,l}}\;\;\frac{\text{accidents}}{\text{flight hour}}$$

$$=\;\frac{\displaystyle\sum_{l=1}^{n-1}a_{k,l}(r_{k,l}+r_{k,l+1})}{\displaystyle\sum_{l=1}^{n}r_{k,l}}\;\;\frac{\text{accidents}}{\text{flight hour}},$$

which agrees with the result stated at the beginning of 5.6.

6.6   We use a similar technique to find the accident rate for any of the n routes in the system. Route R_1 consists of paths $P_{i,1}$ for $i=1,2,...,$ m, and in 6.2 we noted that $P_{i,1}$ experiences $\dfrac{1}{2}\cdot\dfrac{T}{24}\cdot a_{i,1}\cdot(f_{i,1}+f_{i,2})$ accidents per hour. Adding the hourly rates we find that R_1 has $\dfrac{1}{2}\cdot\dfrac{T}{24}\cdot\displaystyle\sum_{i=1}^{m}a_{i,1}(f_{i,1}+f_{i,2})$ accidents per hour. Since it experiences $T\cdot\displaystyle\sum_{i=1}^{m}f_{i,1}$ flight hours of traffic in 24 hours, its accident rate in traditional units is

$$\frac{\dfrac{1}{2}\cdot\dfrac{T}{24}\cdot\displaystyle\sum_{i=1}^{m}a_{i,1}(f_{i,1}+f_{i,2})\;\dfrac{\text{accidents}}{\text{hour}}}{\dfrac{T}{24}\displaystyle\sum_{i=1}^{m}f_{i,1}\;\dfrac{\text{flight hours}}{\text{hour}}}\;=\;\frac{\dfrac{1}{f}}{\dfrac{1}{f}}\cdot\frac{\displaystyle\sum_{i=1}^{m}a_{i,1}(f_{i,1}+f_{i,2})}{2\cdot\displaystyle\sum_{i=1}^{m}f_{i,1}}\;\;\frac{\text{accidents}}{\text{flight hour}}$$

$$=\;\frac{\displaystyle\sum_{i=1}^{m}a_{i,1}(r_{i,1}+r_{i,2})}{2\cdot\displaystyle\sum_{i=1}^{m}r_{i,1}}\;\;\frac{\text{accidents}}{\text{flight hour}}.$$

Analogous reasoning shows that R_n's accident rate is $\dfrac{\displaystyle\sum_{i=1}^{m}a_{i,n-1}(r_{i,n-1}+r_{i,n})}{2\cdot\displaystyle\sum_{i=1}^{m}r_{i,n}}\;\;\dfrac{\text{accidents}}{\text{flight hour}}.$

6.7   If $2\le j\le n-1$, then (as noted in 6.3) each path $P_{i,j}$ has $\dfrac{1}{2}\cdot\dfrac{T}{24}\left[a_{i,j-1}(f_{i,j-1}+f_{i,j})+a_{i,j}(f_{i,j}+f_{i,j+1})\right]$ accidents per hour, so that route R_j has $\dfrac{1}{2}\cdot\dfrac{T}{24}\displaystyle\sum_{i=1}^{m}\left[a_{i,j-1}(f_{i,j-1}+f_{i,j})+a_{i,j}(f_{i,j}+f_{i,j+1})\right]\dfrac{\text{accidents}}{\text{hour}}.$ Since R_j

experiences $\dfrac{T}{24}\sum\limits_{i=1}^{m}f_{i,j}$ flight hours per hour, its accident rate, expressed in traditional units, is

$$\frac{\dfrac{1}{2}\cdot\dfrac{T}{24}\sum\limits_{i=1}^{m}\left[a_{i,j-1}\,(f_{i,j-1}+f_{i,j})\,+\,a_{i,j}\,(f_{i,j}+f_{i,j+1})\right]\dfrac{\text{accidents}}{\text{hour}}}{\dfrac{T}{24}\sum\limits_{i=1}^{m}f_{i,j}\dfrac{\text{flight hours}}{\text{hour}}},$$

which immediately simplifies to

$$\frac{\sum\limits_{i=1}^{m}\left[a_{i,j-1}\,(f_{i,j-1}+f_{i,j})\,+\,a_{i,j}\,(f_{i,j}+f_{i,j+1})\right]}{2\sum\limits_{i=1}^{m}f_{i,j}}\quad\frac{\text{accidents}}{\text{flight hour}}.$$

Dividing the numerator and denominator by f, we re-express the accident rate of route R_j in terms of traffic ratios as

$$\frac{\sum\limits_{i=1}^{m}\left[a_{i,j-1}\,(r_{i,j-1}+r_{i,j})\,+\,a_{i,j}\,(r_{i,j}+r_{i,j+1})\right]}{2\sum\limits_{i=1}^{m}r_{i,j}}\quad\frac{\text{accidents}}{\text{flight hour}}.$$

6.8   In 6.5 we showed that flight level L_i experiences $\dfrac{T}{24}\cdot\sum\limits_{j=1}^{n-1}a_{i,j}\,(f_{i,j}+f_{i,j+1})$ accidents per hour. Adding the hourly rates for all the flight levels, we see that the entire route system experiences $\dfrac{T}{24}\cdot\sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n-1}a_{i,j}\,(f_{i,j}+f_{i,j+1})\dfrac{\text{accidents}}{\text{hour}}$. This is the same hourly rate cited at the beginning of 5.7, where the sum was derived from the hourly accident rates of path pairs, rather than those of individual paths. Without repeating the argument in that paragraph, we see that the flight hourly accident rate for the entire route system, computed from the rates of individual paths, remains consistent with the rate computed by the Reich model.

7.   OCCUPANCIES FOR PAIRS OF NON-ADJACENT ROUTES

7.1   The traditional definition of lateral occupancy applies only to adjacent routes, and so occupancy computations generally count two aeroplanes as proximate only if they are assigned to adjacent co-altitude flight paths. Since the risk of collision (due to the loss of planned lateral separation) between aeroplanes assigned to adjacent routes is far greater than the risk of collision between aeroplanes assigned to non-adjacent routes, traditional occupancy estimates generally account for most of the exposure to risk. However, as airspace management authorities decrease route separations, it is reasonable to expect an increase in the risk of collision between aeroplanes assigned to non-adjacent routes. While that risk is likely to remain much lower than the risk

from aeroplanes assigned to adjacent routes, it may eventually constitute a significant part of the total risk. Indeed, the simplest way to determine whether it is significant is to compute both components of the risk and compare them. The present section develops formulae that can be used to estimate the lateral occupancy of pairs of non-adjacent flight paths. Those estimates can be used to compute the rate of collisions between aeroplanes assigned to non-adjacent paths, by applying a formula similar to the one given in 5.1 and 5.2.

7.2   We begin by deriving occupancies for paths whose lateral separation is twice the minimum distance. Thus we are dealing with pairs of paths $(P_{i,j}, P_{i,j+2})$, for $1 \le i \le m$ and $1 \le j \le n-2$. Each path $P_{i,j}$ still carries $f_{i,j}$ aeroplanes over a 24-hour period, so the route system carries a total of $f = \sum_{i=1}^{m} \sum_{j=1}^{n} f_{i,j}$ aeroplanes during that time. We continue to let $r_{i,j}$ denote $\dfrac{f_{i,j}}{f}$, the fraction of the route system's traffic that travels on path $P_{i,j}$. On the other hand, we need to extend some of the notation used in sections 1 through 6. We let $p_{i,j}^{(2)}$ denote the number of proximate pairs consisting of an aeroplane assigned to $P_{i,j}$ and an aeroplane assigned to $P_{i,j+2}$, and, as in sections 1 through 6, we consider a pair to be proximate if its members pass the chosen longitude (or any other line having an analogous function) within 15 minutes of each other. We let $p^{(2)} = \sum_{i=1}^{m} \sum_{j=1}^{n-2} p_{i,j}^{(2)}$ denote the total number of proximate pairs obtained from routes separated by twice the lateral separation minimum. In section 2 we could rely on having an empirically derived value for p, but in the present section we do not have any data to justify a value for its analogue, $p^{(2)}$, and so another method to estimate its value will need to be found. Note that we have no particular reason to expect p and $p^{(2)}$ to have the same value. Indeed, since there are m fewer path pairs separated by two separation standards than by one separation standard (i.e. $m(n-2)$ pairs rather than $m(n-1)$ pairs) if traffic is uniformly distributed over all of the route system's paths, we expect $p^{(2)}$ to be less than p. Finally, we let $O^{(2)}$ denote the occupancy that we would compute if we counted only proximate pairs from co-altitude paths separated by two separation standards. Since the route system still carries a total of f aeroplanes, we compute $O^{(2)}$ by a formula analogous to that used for computing O, i.e. $O^{(2)} = \dfrac{2p^{(2)}}{f}$. (In a system having exactly three routes, only flights assigned to the outer routes can be members of proximate pairs that are counted by $p^{(2)}$. Thus it might be thought that in defining $O^{(2)}$ for such a system, we should take the denominator to be the number of flights on the outer routes, i.e. $\sum_{i=1}^{m} (f_{i,1} + f_{i,3})$, rather than f, the total number of flights in the system. Such a definition might be useful if the rate of collisions between aeroplanes assigned to non-adjacent routes were of some interest in itself. However, the collision rate for aeroplanes assigned to non-adjacent routes is of interest only as a component of the total collision rate, and thus the denominator needs to account for all flights in the system.)

7.3   In section 1 we observed that occupancy statistics are not normally produced for individual pairs of adjacent flight paths. In section 3 we overcame this difficulty by assuming that the number of proximate pairs of aircraft for a given pair of adjacent co-altitude flight paths was proportional to the product of the traffic levels on those paths. In the present section we observe that since air traffic controllers do not attempt to coordinate the longitudinal positions of aeroplanes assigned to different paths, the assumption of proportionality is just as valid for pairs of non-adjacent flight paths as it is for pairs of adjacent paths. Thus, if $1 \le k \le m$ and $1 \le l \le n-2$,

we expect $\dfrac{p_{k,l}^{(2)}}{p_{k,l}}$ to equal $\dfrac{f_{k,l} \cdot f_{k,l+2}}{f_{k,l} \cdot f_{k,l+1}}$, which we simplify to $\dfrac{f_{k,l+2}}{f_{k,l+1}}$, and it then follows that $p_{k,l}^{(2)} = \dfrac{f_{k,l+2}}{f_{k,l+1}} p_{k,l}$.

Substituting the expression for $p_{k,l}$ from 3.1, we find that

$$p_{k,l}^{(2)} = \frac{f_{k,l+2}}{f_{k,l+1}} \cdot \frac{f_{k,l} \cdot f_{k,l+1}}{\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} f_{i,j} \cdot f_{i,j+1}} \cdot p = \frac{f_{k,l} \cdot f_{k,l+2}}{\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} f_{i,j} \cdot f_{i,j+1}} \cdot p,$$

and dividing the numerator and denominator of the quotient by f^2, we can rewrite this equation as

$$p_{k,l}^{(2)} = \frac{r_{k,l} \cdot r_{k,l+2}}{\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} r_{i,j} \cdot r_{i,j+1}} \cdot p.$$

7.4   Adding the numbers of proximate pairs (of aircraft) attributable to each pair of co-altitude flight paths separated by two separation standards, we find that

$$p_{k,l}^{(2)} = \sum_{k=1}^{m}\sum_{l=1}^{n-2} p_{k,l}^{(2)} = \frac{\displaystyle\sum_{k=1}^{m}\sum_{l=1}^{n-2} r_{k,l} \cdot r_{k,l+2}}{\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} r_{i,j} \cdot r_{i,j+1}} \cdot p.$$

For the sake of notational convenience, we let

$$R^2 = \frac{\displaystyle\sum_{k=1}^{m}\sum_{l=1}^{n-2} r_{k,l} \cdot r_{k,l+2}}{\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} r_{i,j} \cdot r_{i,j+1}}$$

so that $p^{(2)} = R^{(2)}p$. Since $O^2 = \dfrac{2p^{(2)}}{f}$, we can write $O^2 = \dfrac{2R^{(2)}p}{f}$, and since $O = \dfrac{2p}{f}$, we finally write $O^{(2)} = R^{(2)}O$. Thus $R^{(2)}$ is the ratio of occupancy attributable to paths separated by two separation standards, to occupancy attributable to paths separated by one separation standard.

7.5   In section 3, we derived $O_{k,l}$, the occupancy for the adjacent (co-altitude) flight paths $P_{k,l}$ and $P_{k,l+1}$. Likewise, we now derive $O_{k,l}^{(2)}$, the occupancy for $P_{k,l}$ and $P_{k,l+2}$, a pair of flight paths separated by two separation standards. As in section 3, we take $O_{k,l}^{(2)}$ to be 0 whenever either of $f_{k,l}$ or $f_{k,l+2}$ is 0, and when both of them are non-zero, we take $O_{k,l}^{(2)}$ to be $\dfrac{2p_{k,l}^{(2)}}{f_{k,l} + f_{k,l+2}}$, i.e. twice the number of proximate pairs on the paths, divided by the

number of flights on the paths. We then rewrite the occupancy using the formula for $p_{k,l}^{(2)}$ derived in 7.3:

$$O_{k,l}^{(2)} = \frac{2p_{k,l}^{(2)}}{f_{k,l}+f_{k,l+2}} = \frac{2}{f} \cdot p_{k,l}^{(2)} \cdot \frac{f}{f_{k,l}+f_{k,l+2}} = \frac{2}{f} \cdot \frac{r_{k,l} \cdot r_{k,l+2}}{\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} r_{i,j} \cdot r_{i,j+1}} \cdot p \cdot \frac{f}{f_{k,l}+f_{k,l+2}}$$

$$= \frac{r_{k,l} \cdot r_{k,l+2}}{\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} r_{i,j} \cdot r_{i,j+1}} \cdot \frac{1}{r_{k,l}+r_{k,l+2}} \cdot \frac{2p}{f} = \frac{r_{k,l} \cdot r_{k,l+2}}{\left(\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} r_{i,j} \cdot r_{i,j+1}\right)\left(r_{k,l}+r_{k,l+2}\right)} \cdot O.$$

For the sake of notational convenience, we let

$$M_{k,l}^{(2)} = \frac{r_{k,l} \cdot r_{k,l+2}}{\left(\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} r_{i,j} \cdot r_{i,j+1}\right)\left(r_{k,l}+r_{k,l+2}\right)}, \text{ so that } O_{k,l}^{(2)} = M_{k,l}^{(2)} \cdot O.$$

7.6   As in section 5, where we computed accident rates for pairs of adjacent paths, accident rates for pairs of paths separated by two separation standards can likewise be computed. Only one of the constants mentioned in 5.1 changes with the increase in separation, viz. the lateral overlap probability. We recognize that change by letting $C^{(2)}$ denote the product of the (new) lateral overlap probability, the vertical overlap probability, the longitudinal overlap probability for proximate aeroplanes, and the sum of the reciprocals of passing times. We let $a_{k,l}^{(2)}$ denote the accident rate for paths $P_{k,l}$ and $P_{k,l+2}$ and compute it by the Reich model formula: $a_{k,l}^{(2)} = C^{(2)} O_{k,l}^{(2)} = C^{(2)} M_{k,l}^{(2)} O$. The accident rate $a_{k,l}^{(2)}$ is expressed in accidents per flight hour.

7.7   We can also express the rate of accidents between aeroplanes assigned to paths $P_{k,l}$ and $P_{k,l+2}$ as an hourly rate. The paths experience a traffic load of

$$\left(Tf_{k,l} + Tf_{k,l+2}\right)\frac{\text{flight hours}}{24 \text{ hours}}, \text{ or } \frac{T}{24}\left(f_{k,l}+f_{k,l+2}\right)\frac{\text{flight hours}}{\text{hour}},$$

so it follows that their hourly accident rate is

$$a_{k,l}^{(2)} \frac{\text{accidents}}{\text{flight hour}} \cdot \frac{T}{24}\left(f_{k,l}+f_{k,l+2}\right)\frac{\text{flight hours}}{\text{hour}} = \frac{T}{24} a_{k,l}^{(2)}\left(f_{k,l}+f_{k,l+2}\right)\frac{\text{accidents}}{\text{hour}}.$$

Hourly accident rates can be added to each other, and when we do so we find that the route system's total hourly accident rate for accidents due to the loss of two separation standards of planned separation is

$$\frac{T}{24}\sum_{k=1}^{m}\sum_{l=1}^{n-2} a_{k,l}^{(2)}\left(f_{k,l}+f_{k,l+2}\right)\frac{\text{accidents}}{\text{hour}}.$$

7.8   Since the entire route system experiences $\dfrac{Tf}{24}\ \dfrac{\text{flight hours}}{\text{hour}}$, we can re-express its total accident rate (from the loss of two separation standards of planned lateral separation) in traditional terms as

$$\dfrac{\dfrac{T}{24}\displaystyle\sum_{k=1}^{m}\sum_{l=1}^{n-2} a_{k,l}^{(2)}\left(f_{k,l}+f_{k,l+2}\right)\ \dfrac{\text{accidents}}{\text{hour}}}{\dfrac{Tf}{24}\ \dfrac{\text{flight hours}}{\text{hour}}} = \sum_{k=1}^{m}\sum_{l=1}^{n-2} a_{k,l}^{(2)}\left(r_{k,l}+r_{k,l+2}\right)\ \dfrac{\text{accidents}}{\text{flight hour}}.$$

From 7.5 and 7.6 we know that each $a_{k,l}^{(2)} = C^{(2)} M_{k,l}^{(2)} O$, and each $M_{k,l}^{(2)} = \dfrac{r_{k,l}\cdot r_{k,l+2}}{\left(\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} r_{i,j}\cdot r_{i,j+1}\right)\left(r_{k,l}+r_{k,l+2}\right)}$,

so that:

$$\text{each } a_{k,l}^{(2)} = C^{(2)}\cdot \dfrac{r_{k,l}\cdot r_{k,l+2}}{\left(\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} r_{i,j}\cdot r_{i,j+1}\right)\left(r_{k,l}+r_{k,l+2}\right)}\cdot O,$$

$$\text{and each } a_{k,l}^{(2)}\left(r_{k,l}+r_{k,l+2}\right) = C^{(2)}\cdot \dfrac{r_{k,l}\cdot r_{k,l+2}}{\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} r_{i,j}\cdot r_{i,j+1}}\cdot O.$$

Substituting these terms into the sum above, we re-express the total accident rate, in accidents per flight hour, as

$$\sum_{k=1}^{m}\sum_{l=1}^{n-2}\left[C^{(2)}\cdot \dfrac{r_{k,l}\cdot r_{k,l+2}}{\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} r_{i,j}\cdot r_{i,j+1}}\cdot O\right] = C^{(2)}\cdot\left[\dfrac{\displaystyle\sum_{k=1}^{m}\sum_{l=1}^{n-2} r_{k,l}\cdot r_{k,l+2}}{\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} r_{i,j}\cdot r_{i,j+1}}\right]\cdot O = C^{(2)}\cdot R^{(2)}\cdot O = C^{(2)}\cdot O^{(2)},$$

which is exactly the result expected from applying the Reich model.

7.9   Paragraphs 7.2 through 7.8 derive analogues of most of the significant formulae of sections 3 and 5 for the case in which co-altitude flight paths are separated by twice the route system's minimum lateral separation. The same reasoning can be used to derive analogous formulae for the case in which co-altitude flight paths are separated by some other multiple of the route system's minimum lateral separation. Without repeating the derivations, we simply list the definitions of symbols and the principal results for the case in which the flight paths are separated by *s* multiples of the minimum lateral separation for any integer *s* between 1 and *n*−1:

$p_{i,j}^{(s)} \equiv$ the number of proximate pairs consisting of an aeroplane assigned to $P_{i,j}$ and an aeroplane assigned to $P_{i,j+s}$

$$p^{(s)} = \sum_{i=1}^{m} \sum_{j=1}^{n-s} p_{i,j}^{(s)} \equiv \text{the total number of the lateral pairs obtained from routes separated by a distance}$$

$O(s) \equiv$ the occupancy that would be computed by counting only proximate pairs (of aeroplanes) from co-altitude paths separated by s separation standards

$$O(s) \equiv \frac{2p^{(s)}}{f}$$

$$p_{k,l}^{(s)} = \frac{r_{k,l} \cdot r_{k,l+s}}{\displaystyle\sum_{i=1}^{m} \sum_{j=1}^{n-1} r_{i,j} \cdot r_{i,j+1}} \cdot p$$

$$p^{(s)} = \sum_{k=1}^{m} \sum_{l=1}^{n-s} p_{k,l}^{(s)} = \frac{\displaystyle\sum_{k=1}^{m} \sum_{l=1}^{n-s} r_{k,l} \cdot r_{k,l+s}}{\displaystyle\sum_{i=1}^{m} \sum_{j=1}^{n-1} r_{i,j} \cdot r_{i,j+1}} \cdot p$$

$$R^{(s)} = \frac{\displaystyle\sum_{k=1}^{m} \sum_{l=1}^{n-s} r_{k,l} \cdot r_{k,l+s}}{\displaystyle\sum_{i=1}^{m} \sum_{j=1}^{n-1} r_{i,j} \cdot r_{i,j+1}}$$

$$O^{(s)} = R^{(s)} O$$

$$O_{k,l}^{(s)} = \frac{r_{k,l} \cdot r_{k,l+s}}{\left(\displaystyle\sum_{i=1}^{m} \sum_{j=1}^{n-1} r_{i,j} \cdot r_{i,j+1}\right)\left(r_{k,l} + r_{k,l+s}\right)} \cdot O$$

$$M_{k,l}^{(s)} \equiv \frac{r_{k,l} \cdot r_{k,l+s}}{\left(\displaystyle\sum_{i=1}^{m} \sum_{j=1}^{n-1} r_{i,j} \cdot r_{i,j+1}\right)\left(r_{k,l} + r_{k,l+s}\right)}$$

$$O_{k,l}^{(s)} = M_{k,l}^{(s)} \cdot O$$

$C^{(s)} \equiv$ the product of the lateral overlap probability for routes separated by s multiples of the lateral separation minimum, the vertical overlap probability, the longitudinal overlap probability for proximate aeroplanes, and the sum of reciprocals of passing times

$a_{k,l}^{(s)} \equiv$ the accident rate for paths $P_{k,l}$ and $P_{k,l+s}$, expressed in accidents per flight hour;

$$a_{k,l}^{(s)} = C^{(s)} O_{k,l}^{(s)} = C^{(s)} M_{k,l}^{(s)} O.$$

8. CONCLUSION

8.1   The formulae derived in sections 3, 5, and 6 give analysts the tools needed to compute the accident rates of individual components of oceanic route systems. The distribution of traffic on some of the most heavily used route systems is known to be markedly non-uniform, and results obtained from applying these formulae will allow decision-makers to consider not only an entire route system's accident rate, but also the rates of the most heavily used flight paths, routes and flight levels.

8.2   Furthermore, the formulae derived in section 7 give analysts the ability to account for risk due to the loss of planned lateral separations equal to all integer multiples of the separation minimum and thereby more accurately compute a route system's total lateral risk.

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Attachment A to Appendix 14

CONSISTENCY WITH ICAO GUIDANCE

A.1   The *Air Traffic Services Planning Manual* (Doc 9426) includes a discussion of the "steady state flow model" of occupancy (Part II, Section 2, Chapter 4, Appendix C). The present Attachment A to Appendix 14 demonstrates that the steady state flow model infers the same occupancy multiplier derived in section 3 of Appendix 14. The notation used in Doc 9426 differs from that used in Appendix 14, and the reader should not assume that symbols used in Attachment A to Appendix 14 have the same meanings they have in Appendix 14 proper, unless they are specifically identified as having those meanings. In particular, subscripts in Doc 9426 — and in the first part of Attachment A— reverse the order of route and flight level used in the body of Appendix 14. Moreover, Doc 9426 uses symbols "T" and "*f*" whose meanings are entirely different from those of the "*T*" and "*f*" used in the body of Appendix 14.

A.2   Doc 9426 posits a system of *t* parallel routes operating on *f* flight levels. The traffic flow on the path at route *i* and flight level *j* is $m_{i,j}$ aircraft per hour. Each route has length *L* NM, and the average speed of the aircraft on each route is *V* kt. The system is observed for *T* hours. Aircraft on adjacent paths and the same flight level are considered proximate when they are longitudinally within S_x NM of each other. During the *T* hours in which the system is observed, T_y denotes the total time during which aircraft pairs are proximate, *H* denotes the total number of flight hours, and $E_y = \dfrac{2T_y}{H}$ denotes the system's occupancy. Table A-14-5 shows how the symbols in Doc 9426 correspond to those in the body of Appendix 14.

Table A-14-5.   Use of symbols

Symbol in Doc 9426	Symbol in Appendix 14	Meaning
t	*n*	Number of routes
f	*m*	Number of flight levels
$m_{i,j}$	$\dfrac{f_{j,i}}{24}$	Traffic flow, in aircraft per hour, on route *i* at flight level *j*
$\dfrac{L}{V}$	*T*	Average time, in hours, needed to fly an oceanic route
T	24	Time, in hours, during which the system is monitored
$\dfrac{S_x}{V}$	$\dfrac{15}{60}$	Flight time, in hours, corresponding to a proximity interval
E_y	*O*	Occupancy

A.3.   On average, it takes each aircraft $\dfrac{L}{V}$ hours to pass through the system. Therefore, the path at route

i and flight level j experiences $\dfrac{L}{V} m_{i,j}$ flight hours per hour and a total of $\dfrac{LT}{V} m_{i,j}$ flight hours during the

T hours of system monitoring. Adding the contributions of all the flight paths, we find that the entire system

experiences $H = \dfrac{LT}{V} \sum\limits_{i=1}^{t} \sum\limits_{j=1}^{f} m_{i,j}$ flight hours during the T hours of monitoring.

A.4   If $2 \leq i \leq t$, then the path at route i and flight level j has an adjacent path, at route $i-1$ and flight level

j, whose flow is $m_{i-1,j}$ aircraft per hour and which holds, on average, $\dfrac{L}{V} m_{i-1,j}$ aircraft. The average interval

between aircraft on the adjacent path is $\dfrac{L \text{ NM}}{\dfrac{L}{V} m_{i-1,j} \text{ aircraft}} = \dfrac{V}{m_{i-1,j}} \dfrac{\text{NM}}{\text{aircraft}}$. Within a longitudinal distance

S_x of an aircraft flying on the path at route i and flight level j, we expect to find   that

$\dfrac{2S_x \text{ NM}}{\dfrac{V}{m_{i-1,j}} \dfrac{\text{NM}}{\text{aircraft}}} = \dfrac{2S_x}{V} m_{i-1,j}$ aircraft on the adjacent path. During the $\dfrac{L}{V}$ hours in which the aircraft on route

i and flight level j flies through the system, it experiences (on average) $\dfrac{2S_x}{V} m_{i-1,j} \dfrac{L}{V}$ hours of proximity time.

Since $m_{ij}T$ aircraft use the path at route i and flight level j during the monitoring period, that path experiences $\dfrac{2S_x}{V} m_{i-1,j} \dfrac{L}{V} m_{i,j} T$

hours of proximity time with its lower-numbered adjacent path (during that period). Adding the proximity times

of all the paths, we find that the entire system generates $T_y = \dfrac{2S_x LT}{V^2} \sum\limits_{i=2}^{t} \sum\limits_{j=1}^{f} m_{i-1,j} \, m_{i,j}$ hours of proximity time

during the monitoring period.

A.5   Doc 9426 then computes the route system's occupancy, E_y, by the equation

$$E_y = \frac{2T_y}{H} = \frac{2 \cdot \dfrac{2S_x LT}{V^2} \sum\limits_{i=2}^{t} \sum\limits_{j=1}^{f} m_{i-1,j} \, m_{i,j}}{\dfrac{LT}{V} \sum\limits_{i=1}^{t} \sum\limits_{j=1}^{f} m_{i,j}} = \frac{4S_x}{V} \cdot \frac{\sum\limits_{i=2}^{t} \sum\limits_{j=1}^{f} m_{i-1,j} \, m_{i,j}}{\sum\limits_{i=1}^{t} \sum\limits_{j=1}^{f} m_{i,j}}.$$

A.6   Consider a single pair of adjacent paths at the same flight level, viz. the paths at routes l and $l+1$, on

flight level k. As was shown in A.3, during the monitoring period those paths respectively generate $\dfrac{LT}{V} m_{l,k}$

flight hours and $\dfrac{LT}{V} m_{l+1,k}$ flight hours for a total of $\dfrac{LT}{V} (m_{l,k} + m_{l+1,k})$ flight hours. As was shown in A.4,

during the monitoring period they experience $\dfrac{2S_x}{V} m_{l,k} \dfrac{L}{V} m_{l+1,k} T$ proximity hours $= \dfrac{2S_x LT}{V^2} m_{l,k} m_{l+1,k}$

proximity hours. Dividing twice the proximity time by the total flight hours yields the pair's occupancy:

$$\frac{2 \cdot \dfrac{2 S_x LT}{V^2} m_{l,k} m_{l+1,k}}{\dfrac{LT}{V}(m_{l,k} + m_{l+1,k})} = \frac{4 S_x}{V} \frac{m_{l,k} m_{l+1,k}}{m_{l,k} + m_{l+1,k}}.$$

A.7   Using the results of A.5 and A.6, we now see that the ratio of the path pair's occupancy to the entire system's occupancy can be written as

$$\frac{\dfrac{4 S_x}{V} \dfrac{m_{l,k} m_{l+1,k}}{m_{l,k} + m_{l+1,k}}}{\dfrac{\dfrac{4 S_x}{V} \sum\limits_{i=2}^{t} \sum\limits_{j=1}^{f} m_{i-1,j} m_{i,j}}{\sum\limits_{i=1}^{t} \sum\limits_{j=1}^{f} m_{i,j}}} = \frac{m_{l,k} m_{l+1,k}}{\left(\sum\limits_{i=2}^{t} \sum\limits_{j=1}^{f} m_{i-1,j} m_{i,j} \right) \cdot \dfrac{m_{l,k} + m_{l+1,k}}{\sum\limits_{i=1}^{t} \sum\limits_{j=1}^{f} m_{i,j}}}$$

$$= \frac{m_{l,k} T \cdot m_{l+1,k} T}{\left(\sum\limits_{i=2}^{t} \sum\limits_{j=1}^{f} m_{i-1,j} T \cdot m_{i,j} T \right) \cdot \dfrac{m_{l,k} T + m_{l+1,k} T}{\sum\limits_{i=1}^{t} \sum\limits_{j=1}^{f} m_{i,j} T}}.$$

A.8   Each $m_{i,j} T$ in Doc 9426 is equivalent to $f_{j,i}$ in the notation used in the body of this appendix. The number of routes is called "t" in Doc 9426 and "n" in the body of this appendix; and the number of flight levels is called "f" in Doc 9426 and "m" in this appendix. Rewriting the ratio of occupancies using the notation from this appendix, we find the ratio to be

$$\frac{f_{k,l} f_{k,l+1}}{\left(\sum\limits_{i=2}^{n} \sum\limits_{j=1}^{m} f_{j,i-1} f_{j,i} \right) \cdot \dfrac{f_{k,l} + f_{k,l+1}}{\sum\limits_{i=1}^{n} \sum\limits_{j=1}^{m} f_{j,i}}}.$$

A.9   We can then rewrite the sum of products in the denominator as

$$\sum_{i=2}^{n} \sum_{j=1}^{m} f_{j,i-1} f_{j,i} = \sum_{j=1}^{m} \sum_{i=2}^{n} f_{j,i-1} f_{j,i} = \sum_{j=1}^{m} \sum_{i-1=1}^{n-1} f_{j,i-1} f_{j,i} = \sum_{x=1}^{m} \sum_{y=1}^{n-1} f_{x,y} f_{x,y+1} = \sum_{i=1}^{m} \sum_{j=1}^{n-1} f_{i,j} f_{i,j+1},$$

and we also rewrite the total number of flights in the system during the monitoring period as

$$\sum_{i=1}^{n}\sum_{j=1}^{m} f_{j,i} \ = \ \sum_{j=1}^{m}\sum_{i=1}^{n} f_{j,i} \ = \ \sum_{x=1}^{m}\sum_{y=1}^{n} f_{x,y} \ = \ \sum_{i=1}^{m}\sum_{j=1}^{n} f_{i,j}.$$

Substituting these expressions into the ratio of occupancies, we rewrite the ratio as

$$\frac{f_{k,l}\, f_{k,l+1}}{\left(\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} f_{i,j}\, f_{i,j+1}\right) \cdot \dfrac{f_{k,l}+f_{k,l+1}}{\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n} f_{i,j}}}.$$

We then recall that in the notation in the body of this appendix, $f \equiv \displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n} f_{i,j}$, so that we can also write the

ratio as

$$\frac{f_{k,l}\, f_{k,l+1}}{\left(\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} f_{i,j}\, f_{i,j+1}\right) \cdot \dfrac{f_{k,l}+f_{k,l+1}}{f}}.$$

Dividing the numerator and denominator by f^2 and remembering that $r_{i,j} \equiv \dfrac{f_{i,j}}{f}$, we finally rewrite the ratio

of occupancies as

$$\frac{\dfrac{f_{k,l}}{f}\cdot\dfrac{f_{k,l+1}}{f}}{\left(\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} \dfrac{f_{i,j}}{f}\cdot\dfrac{f_{i,j+1}}{f}\right) \cdot \left(\dfrac{f_{k,l}}{f}+\dfrac{f_{k,l+1}}{f}\right)} \ = \ \frac{r_{k,l}\cdot r_{k,l+1}}{\left(\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n-1} r_{i,j}\cdot r_{i,j+1}\right)\cdot\left(r_{k,l}+r_{k,l+1}\right)},$$

which is exactly the definition of $M_{k,l}$ in 3.7 (QED).

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Attachment B to Appendix 14

UPPER BOUNDS FOR OCCUPANCY

B.1   This attachment uses the notation from the *Air Traffic Services Planning Manual*, as described in A.1 and A.2.

B.2   In occupancy computations the distance S_x NM is often taken to be the distance covered by an aeroplane moving at V kt during a time period equal to the minimum longitudinal separation. That is, $\dfrac{S_x}{V}$ hours is viewed as the minimum longitudinal separation. The maximum flow on each path is then the reciprocal of the minimum separation, i.e. $\dfrac{V}{S_x}$ (flights) per hour.

B.3   Paragraph A.5 expresses E_y, the route system's occupancy, as

$$\frac{4S_x}{V} \cdot \frac{\displaystyle\sum_{i=2}^{t}\sum_{j=1}^{f} m_{i-1,j}\, m_{i,j}}{\displaystyle\sum_{i=1}^{t}\sum_{j=1}^{f} m_{i,j}},$$

which can be rewritten as

$$4 \cdot \frac{\displaystyle\sum_{i=2}^{t}\sum_{j=1}^{f} m_{i-1,j}\, m_{i,j}}{\dfrac{V}{S_x}\displaystyle\sum_{i=1}^{t}\sum_{j=1}^{f} m_{i,j}} ,\ \text{or as}\ \ 4 \cdot \frac{\displaystyle\sum_{i=2}^{t}\sum_{j=1}^{f} m_{i-1,j}\, m_{i,j}}{\displaystyle\sum_{i=1}^{t}\sum_{j=1}^{f} \dfrac{V}{S_x}\, m_{i,j}} ,\ \text{or as}\ \ 4 \cdot \frac{\displaystyle\sum_{i=2}^{t}\sum_{j=1}^{f} m_{i-1,j}\, m_{i,j}}{\displaystyle\sum_{j=1}^{f} \dfrac{V}{S_x}\, m_{1,j} + \displaystyle\sum_{i=2}^{t}\sum_{j=1}^{f} \dfrac{V}{S_x}\, m_{i,j}} .$$

B.4   Since all of the flows are non-negative and the maximum flow rate $\dfrac{V}{S_x}$ is positive, the three sums that appear in this last expression for E_y contain only non-negative terms. Therefore, the sums are necessarily non-negative. Since $\dfrac{V}{S_x} \geq m_{i-1,j}$ for all $i = 2, ..., t$ and all $j = 1, ..., f$, the second sum in the denominator,

$\displaystyle\sum_{i=2}^{t}\sum_{j=1}^{f} \dfrac{V}{S_x}\, m_{i,j}$, is greater than or equal to the numerator, $\displaystyle\sum_{i=2}^{t}\sum_{j=1}^{f} m_{i-1,j}\, m_{i,j}$. *A fortiori*, the entire denominator is greater than or equal to the numerator, and the quotient is less than or equal to 1. Thus $E_y \leq 4$.

B.5   Figure A-14-1 depicts one flight level in a system of seven heavily loaded parallel routes. The horizontal lines on the figure represent the flight paths, and the small rectangular boxes represent aeroplanes flying along them. The aeroplanes are all moving at the same speed, 480 kt (8 NM per minute), and the minimum longitudinal separation is 15 minutes, which thus corresponds to a distance of 120 NM. The
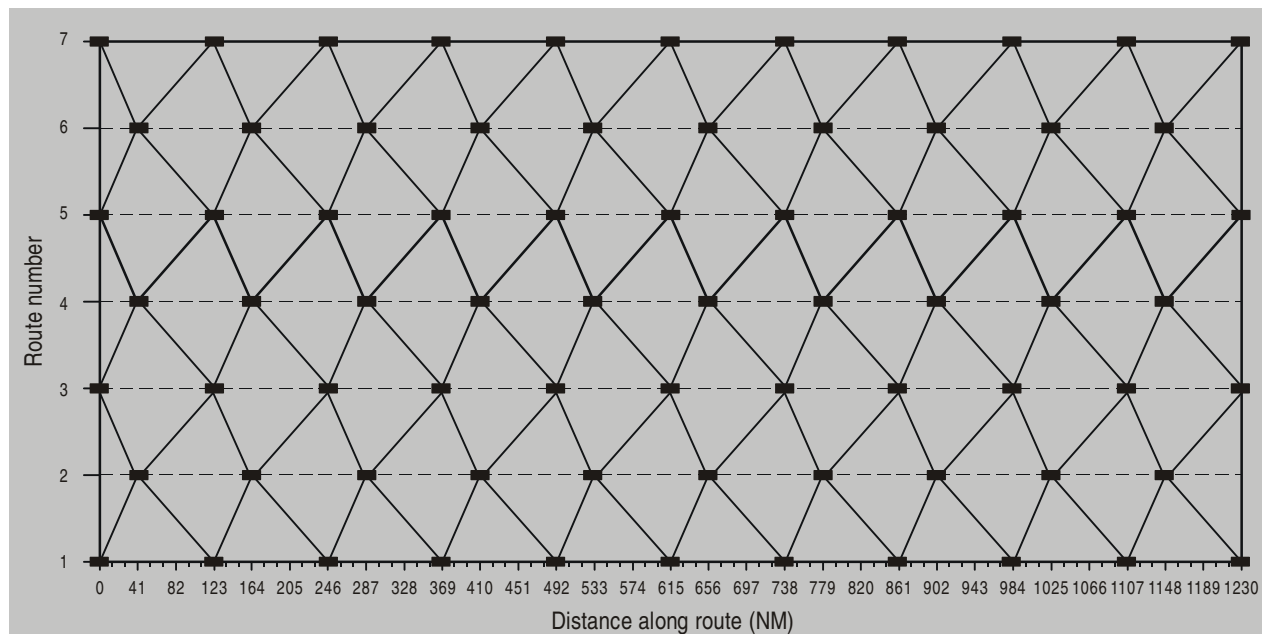
**Figure A-14-1.   Proximity pairings for seven routes,
with proximity time = longitudinal separation minimum**

aeroplanes on each path are spaced at just slightly more than that distance, i.e. 123 NM apart. As indicated in 1.1, aeroplanes on adjacent paths (at the same flight level) are said to be proximate whenever their positions (in the longitudinal sense) are separated by no more than 15 minutes of flying time. Thus, in the figure two aeroplanes are proximate as long as their respective horizontal position coordinates are within 120 NM of each other. Each of the intervals marked on route 1 represents 41 NM, so two aeroplanes are proximate as long as their horizontal positions differ by one or two intervals. The diagonal line segments on the figure connect proximate aeroplanes, and it is clear that (except for the aeroplanes at the left and right margins of the figure) every aeroplane on the interior routes is proximate to four others, while every aeroplane on the outermost routes is proximate to two others. Each diagonal line segment represents a unique proximate pair. Since every aeroplane on an interior route is a member of four distinct pairs, but there are two aeroplanes per pair, we expect that the interior routes will contribute twice as many pairs as flights. It is only the effect of the outermost routes that keeps the entire system's ratio of pairs to flights from reaching 2 and keeps the occupancy (which is twice that ratio) from reaching 4. If the system consisted of infinitely many parallel routes and all of them were interior routes, the occupancy computed for a fully loaded system would then reach the upper bound of 4 (derived in B.4).

B.6   We make this observation more precise by noting that in a fully loaded route system the flow on each path is $\dfrac{V}{S_x}$ flights per hour, and thus over T hours each path experiences $\dfrac{V}{S_x}T$ flights and $\dfrac{V}{S_x}T \cdot \dfrac{L}{V} = \dfrac{LT}{S_x}$ flight hours, each route experiences $f\dfrac{LT}{S_x}$ flight hours, and the entire route system experiences $tf\dfrac{LT}{S_x}$ flight hours. There are 2 outer routes, each of which contributes the same number of proximity hours as flight hours,

and there are $t-2$ interior routes, each of which contributes twice as many proximity hours as flight hours. Therefore, the system exhibits a total of $2f\dfrac{LT}{S_x} + (t-2)\cdot 2f\dfrac{LT}{S_x}$ proximity hours and an occupancy of

$$2\cdot\frac{2f\dfrac{LT}{S_x} + (t-2)\cdot 2f\dfrac{LT}{S_x}}{tf\dfrac{LT}{S_x}} = 2\cdot\frac{2 + (t-2)\cdot 2}{t} = 2\cdot\frac{2t-2}{t} = 4-\frac{4}{t}.$$

As t (the number of routes) increases, the occupancy approaches 4 from below.

B.7   In B.3 we found that E_y, the route system's occupancy, is

$$4\cdot\frac{\displaystyle\sum_{i=2}^{t}\sum_{j=1}^{f} m_{i-1,j}\, m_{i,j}}{\displaystyle\sum_{j=1}^{f}\frac{V}{S_x} m_{1,j} + \sum_{i=2}^{t}\sum_{j=1}^{f}\frac{V}{S_x} m_{i,j}}.$$

If the system consists of exactly two routes, i.e. if $t = 2$, we can rewrite the occupancy as

$$4\cdot\frac{\displaystyle\sum_{j=1}^{f} m_{1,j}\, m_{2,j}}{\displaystyle\sum_{j=1}^{f}\frac{V}{S_x} m_{1,j} + \sum_{j=1}^{f}\frac{V}{S_x} m_{2,j}}.$$

For each flight level j, $\dfrac{V}{S_x}$ is greater than or equal to each of the flows $m_{1,j}$ and $m_{2,j}$, and so $\dfrac{V}{S_x} m_{1,j} \ge m_{1,j} m_{2,j}$ and $\dfrac{V}{S_x} m_{2,j} \ge m_{1,j} m_{2,j}$. Therefore, each of the sums in the denominator is greater than or equal to the numerator, whence the entire quotient is less than or equal to 1/2, and E_y is less than or equal to $4 \cdot (1/2) = 2$.

B.8   Figure A-14-2 is similar to Figure A-14-1, but depicts only two heavily loaded routes. As in Figure A-14-1, each diagonal line segment represents a unique proximate pair of aeroplanes. It is clear from the figure that each aeroplane belongs to two pairs, while each pair consists of two aeroplanes, so except for an extra aeroplane shown at one or the other margin, the number of pairs equals the number of flights. Thus, in a maximally loaded system of two routes, we expect the ratio of pairs to flights to equal 1, and the occupancy, E_y, to equal 2. Note that the formula derived at the end of B.6 remains valid when $t = 2$, since it gives $E_y = 4 - (4/2) = 4 - 2 = 2$.

B.9   The NAT OTS proximity criterion of 15 minutes was probably chosen prior to 1981 when 15 minutes was the minimum longitudinal separation in that route system. However, since 1981 the NAT OTS has used 10 minutes as its longitudinal minimum, and thus the maximum possible flow on any of its paths has increased from four aircraft per hour to six aircraft per hour. In the notation used above, $\dfrac{S_x}{V}$ hours corresponds to the

15-minute separation minimum, and so the 10-minute minimum can be written as $\dfrac{2}{3}\dfrac{S_x}{V}$ hours, and the corresponding maximum flow rate as its reciprocal, $\dfrac{3V}{2S_x}$ (flights) per hour. Following the method in B.3 and B.4, we recall that

$$E_y = 4 \cdot \frac{\displaystyle\sum_{i=2}^{t}\sum_{j=1}^{f} m_{i-1,j}\, m_{i,j}}{\displaystyle\sum_{j=1}^{f}\frac{V}{S_x}m_{1,j} + \sum_{i=2}^{t}\sum_{j=1}^{f}\frac{V}{S_x}m_{i,j}} = 4 \cdot \frac{3/2}{3/2} \cdot \frac{\displaystyle\sum_{i=2}^{t}\sum_{j=1}^{f} m_{i-1,j}\, m_{i,j}}{\displaystyle\sum_{j=1}^{f}\frac{V}{S_x}m_{1,j} + \sum_{i=2}^{t}\sum_{j=1}^{f}\frac{V}{S_x}m_{i,j}}$$

$$= 6 \cdot \frac{\displaystyle\sum_{i=2}^{t}\sum_{j=1}^{f} m_{i-1,j}\, m_{i,j}}{\displaystyle\sum_{j=1}^{f}\frac{3V}{2S_x}m_{1,j} + \sum_{i=2}^{t}\sum_{j=1}^{f}\frac{3V}{2S_x}m_{i,j}}.$$

We then note that $\dfrac{3V}{2S_x} \geq m_{i-1,j}$ for all $i = 2, ..., t$ and all $j = 1, ..., f$, and so the second sum in the denominator,

$\displaystyle\sum_{i=2}^{t}\sum_{j=1}^{f}\frac{3V}{2S_x}m_{i,j}$, is greater than or equal to the numerator, $\displaystyle\sum_{i=2}^{t}\sum_{j=1}^{f} m_{i-1,j}\, m_{i,j}$. *A fortiori*, the entire denominator is greater than or equal to the numerator, whence the quotient is less than or equal to 1, and $E_y \leq 6$.

B.10   Figure A-14-3 illustrates a single flight level in a system of three heavily loaded parallel routes. (More routes would have been depicted if the software used to prepare the figure had been capable of showing them). In this case each interval marked along route 1 represents 27 NM, and consecutive aeroplanes on each path are longitudinally separated by three such intervals, i.e. by 81 NM, which is just slightly greater than the distance (of 80 NM) corresponding to the minimum separation of 10 minutes. Since the proximity criterion remains 15 minutes of flying time (which corresponds to 120 NM), aeroplanes on adjacent routes are proximate if their longitudinal position coordinates are separated by one, two, three or four intervals, i.e. by 27, 54, 81 or 108 NM. As in Figures A-14-1 and A-14-2, the diagonal line segments connect proximate pairs, and it is clear from the figure that every aeroplane on the interior route is proximate to six others, while every aeroplane on the outer routes is proximate to three others. Since each interior route contributes six pairs per aeroplane, but every pair consists of two aeroplanes, the interior routes contribute three times as many pairs as flights. Again, it is only the influence of the outermost routes that keeps the occupancy from reaching its theoretical maximum of $2 \cdot 3 = 6$.

B.11   If the system consists of just two routes (i.e. if $t = 2$), we can rewrite the occupancy derived in B.9 as

$$E_y = 6 \cdot \frac{\displaystyle\sum_{j=1}^{f} m_{1,j}\, m_{2,j}}{\displaystyle\sum_{j=1}^{f}\frac{3V}{2S_x}m_{1,j} + \sum_{j=1}^{f}\frac{3V}{2S_x}m_{2,j}}.$$

Appendix 14. Estimating occupancy and the rate of accidents due to the loss of planned lateral separation for the individual components of a system of parallel routes
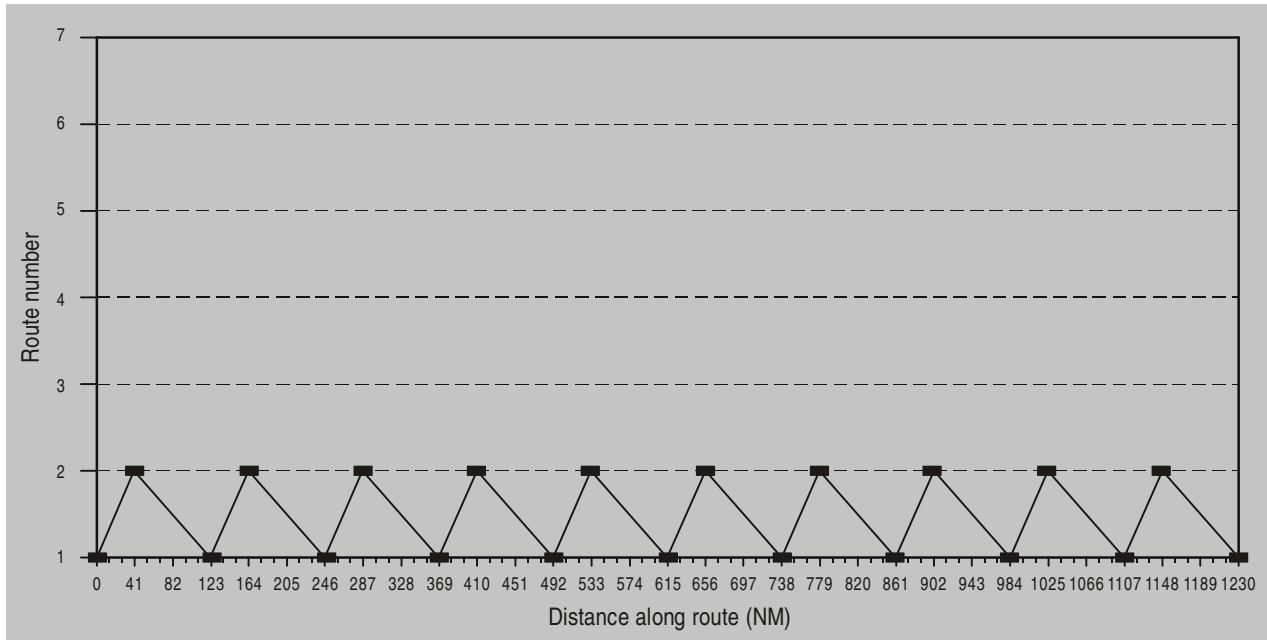
145

**Figure A-14-2.   Proximity pairings for two routes,
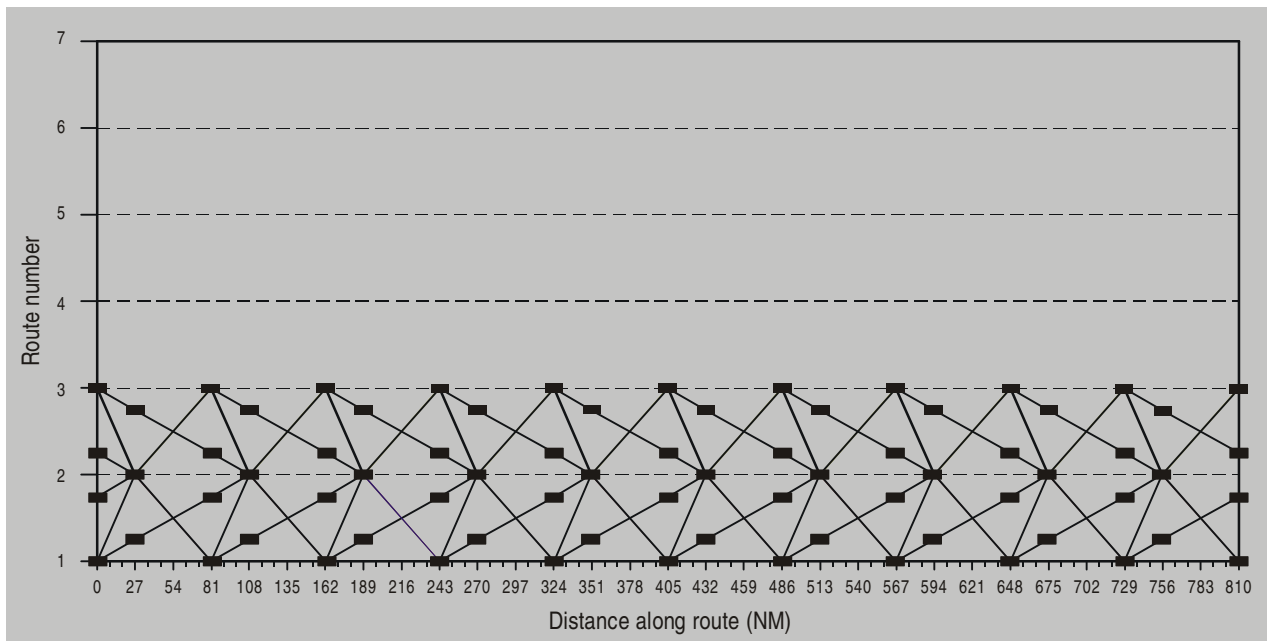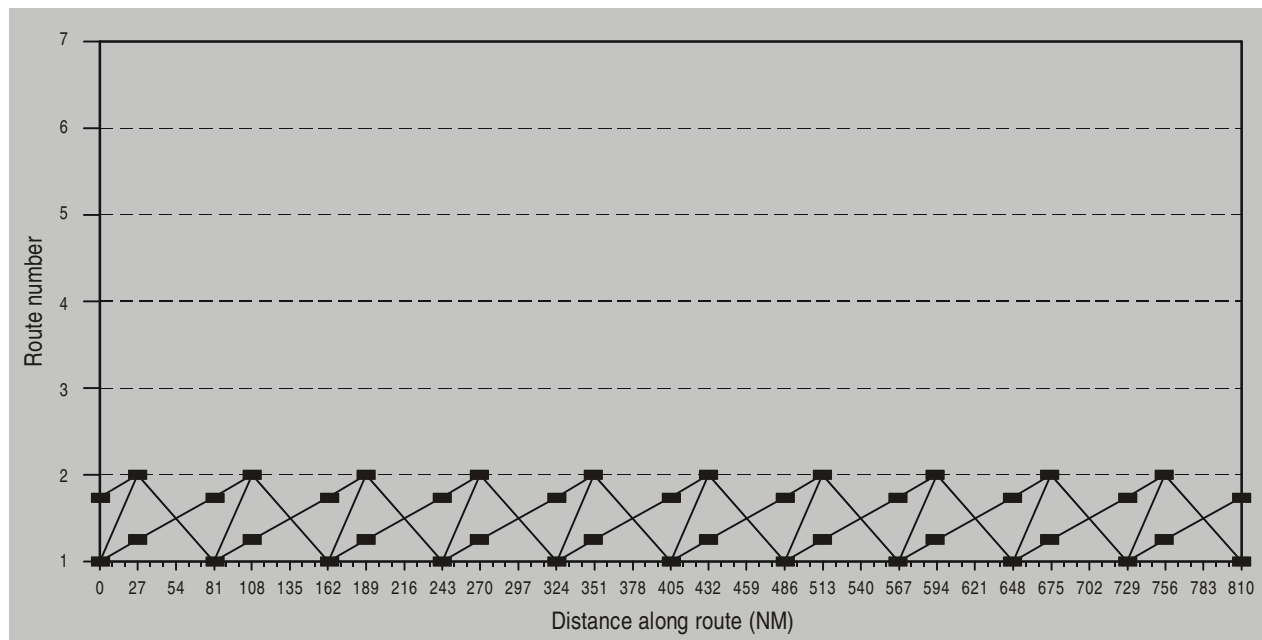with proximity time = longitudinal separation minimum**

**Figure A-14-3.   Proximity pairings for three routes,
with proximity time = 1.5 × longitudinal separation minimum**

**Figure A-14-4.   Proximity pairings for two routes,
with proximity time = 1.5 × longitudinal separation minimum**

For each flight level j, $\dfrac{3V}{2S_x}$ is greater than or equal to each of the flows $m_{1,j}$ and $m_{2,j}$, and so

$\dfrac{3V}{2S_x}m_{1,j} \geq m_{1,j}\,m_{2,j}$ and $\dfrac{3V}{2S_x}m_{2,j} \geq m_{1,j}\,m_{2,j}$. Therefore, each of the sums in the denominator is greater than

or equal to the numerator, whence the entire quotient is less than or equal to 1/2, and E_y is less than or equal to $6 \cdot (1/2) = 3$.

B.12   Figure A-14-4 is similar to Figure A-14-3, but illustrates a system of just two routes. It is clear from the figure that each aeroplane belongs to three pairs, while each pair consists of two aeroplanes, so except for an extra aeroplane shown at one or the other margin, the number of pairs equals 3/2 the number of flights. Thus, in a fully loaded system of two routes, in which the proximity criterion is 3/2 the minimum longitudinal separation, the ratio of proximity time to flight time is expected to equal 3/2, and the occupancy, E_y, to equal 3.

B.13   Paragraphs B.9 through B.12 demonstrate the effect of using a proximity criterion, $\dfrac{S_x}{V}$, that is 3/2 of

the longitudinal separation minimum. More generally, it is clear that another ratio could be substituted for 3/2, and the arguments used in B.9 and B.11 would then yield maximum occupancy values corresponding to that particular ratio. Indeed, let q denote the ratio of proximity criterion to longitudinal separation minimum. Replacing the factor 3/2 by q, in B.9 and B.11, and repeating the arguments of those paragraphs, we find that $4q$ is an upper bound on the occupancy of all multi-route systems, and $2q$ is an upper bound on the occupancy of systems that consist of exactly two routes.

B.14   Imagine (as in B.6) a fully loaded route system in which the proximity time, $\dfrac{S_x}{V}$, divided by the

separation minimum, equals q. Then the separation minimum is $\dfrac{S_x}{qV}$ hours, and the traffic flow on each path

is its reciprocal, $\dfrac{qV}{S_x}$ (flights) per hour. Thus, over T hours, each path experiences $\dfrac{qV}{S_x}T$ flights and

$\dfrac{qV}{S_x}T \cdot \dfrac{L}{V} = \dfrac{qLT}{S_x}$ flight hours, each route experiences $f\dfrac{qLT}{S_x}$ flight hours, and the entire route system

experiences $tf\dfrac{qLT}{S_x}$ flight hours. Since there are q separation intervals per proximity interval, every aeroplane
in the system is (on average) proximate to q aeroplanes on each of the routes adjacent to its own. Thus every
aeroplane on an interior route is proximate to $2q$ others, and every aeroplane on the two outermost routes is
proximate to q others. Each of the $t-2$ interior routes therefore contributes $2q$ proximity hours per flight hour,
while the two outer routes contribute q proximity hours per flight hour. Thus the system exhibits a total of

$$2 \cdot q \cdot f\frac{qLT}{S_x} + (t-2) \cdot 2q \cdot f\frac{qLT}{S_x} \text{ proximity hours}$$

and an occupancy of

$$2 \cdot \frac{2 \cdot q \cdot f\dfrac{qLT}{S_x} + (t-2) \cdot 2q \cdot f\dfrac{qLT}{S_x}}{tf\dfrac{qLT}{S_x}} = 2 \cdot \frac{2q + (t-2) \cdot 2q}{t} = 2 \cdot \frac{2qt-2q}{t} = 4q - \frac{4q}{t}.$$

As t increases, this expression approaches $4q$ from below, and when $t = 2$, the expression simplifies to $2q$. Thus,
in general, the least upper bound for the occupancy of a t-route system is $4q - \dfrac{4q}{t}$.

Appendix 15

NAVIGATION PERFORMANCE REQUIREMENTS FOR THE INTRODUCTION OF 30 NM LATERAL SEPARATION IN OCEANIC AND REMOTE AIRSPACE

1. INTRODUCTION

This appendix presents a method for estimating the risk of collision due to the loss of planned lateral separation in a system of parallel routes having 30 NM between adjacent routes. It considers several different systems of parallel routes and, for each system, derives a navigational performance requirement that must be met in order for the system to operate with no more than the internationally accepted target level of safety (TLS) of 5×10^{-9} accidents per flight hour. In order to eliminate unnecessary verbiage, this appendix does not repeatedly state that such accidents are due to the loss of planned lateral separation. Unless the text specifically refers to the loss of another form of separation, all mention of accidents, accident rates or collision rates refers to those caused by the loss of planned lateral separation.

2. A LIMIT ON TYPICAL NAVIGATIONAL ERRORS

2.1   In establishing standard values of required navigation performance (RNP) for aeroplanes using oceanic route systems, the RGCSP considered the probability that aeroplanes assigned to adjacent parallel routes have laterally overlapping positions. This lateral overlap probability, which is a major determinant of a route system's accident rate, varies with the navigational accuracy of the fleet using the system, and the navigational accuracy can be characterized by a limit on 95 per cent of the typical lateral errors experienced by the fleet's aeroplanes. In examining the functional dependence of lateral overlap probability on this "95 per cent containment limit", the RGCSP observed that over a fairly large range of values, reductions in the containment limit lead to significant reductions in the overlap probability. However, once the containment limit decreases to approximately one-sixth of the separation between the routes, further decreases yield only negligibly small reductions in overlap probability. Thus, if an airspace management authority wishes to establish an RNP for the fleet using a system of parallel routes, it should not ask operators to exhibit better lateral performance than that which produces a 95 per cent containment limit equal to one-sixth of the separation.

2.2   In order to avoid a proliferation of RNP standards, the RGCSP initially established only two such standards for oceanic flight, RNP 20 and RNP 12.6. It later added RNP 10. For en-route flights over continental airspace, the RGCSP adopted RNP 1, RNP 4 and, for a portion of European airspace, RNP 5.

2.3   In recent years some airspace management authorities have expressed an interest in establishing oceanic route systems having 30 NM between adjacent parallel routes. In order for such a system to operate safely, its fleet needs to exhibit lateral performance equivalent to (or better than) RNP 5. However, it is also important to remember that many aeroplanes approved for RNP 5 in continental airspace may not be able to operate at that accuracy in oceanic airspace because their RNP 5 performance may depend on the use of ground-based navigation aids (NAVAIDS). In recent years many new long-range aeroplanes (such as those equipped with Boeing's "FANS-1" package) have been granted RNP 4 approvals, and it is likely that those aeroplanes will be the first to operate on oceanic route systems having 30 NM separation between adjacent parallel routes.

3.   LIMITS ON THE RATE OF
ATYPICAL LATERAL ERRORS

3.1   In the present section several possible configurations of parallel routes are examined. For each configuration a representative set of occupancies (occupancy being a measure of exposure to risk) ranging from 0 to 2 is considered. For each occupancy the maximum tolerable rate of atypical lateral errors corresponding to the TLS of 5×10^{-9} accidents per flight hour is computed. The accident rate is estimated by applying the well-known Reich model for parallel routes, and in so doing the model's traditional notation is used. The meanings of the model's parameters are shown in Table A-15-1, as are values typical of one particular oceanic airspace in the mid- to late 1990s. It is important to remember that fleet characteristics vary from one airspace to another and also change over time. Similar computations for another airspace should use the parameter values expected to prevail there during the time period in which the relevant separation is to be applied.

In the simplest cases, involving two flight paths at the same altitude, the Reich model gives the accident rate as $P_y(S_y) \, P_z(0) \, \dfrac{\lambda_x}{S_x} \, E_y(same) \, K_{same}$ accidents per flight hour when the routes carry traffic in the same direction and as $P_y(S_y) \, P_z(0) \, \dfrac{\lambda_x}{S_x} \, E_y(opp) \, K_{opp}$ accidents per flight hour when the routes carry traffic in opposite directions.

3.2   As was noted above, the lateral overlap probability, $P_y(S)_y$, varies significantly with typical navigational accuracy when the 95 per cent containment limit exceeds one-sixth of the separation between the routes being considered. When the containment limit is less than a sixth of S_y, the lateral overlap probability becomes nearly constant with respect to typical navigational accuracy, but it does vary (almost) linearly with α, the fleet's rate of atypical lateral errors.

3.3   Analysts generally use a double double exponential (DDE) density function to characterize a fleet's lateral errors so that they can thereby describe both typical and atypical errors. The DDE function is often written in the form

$$f(x) = \frac{1-\alpha}{2\lambda_1} e^{-\frac{|x|}{\lambda_1}} + \frac{\alpha}{2\lambda_2} e^{-\frac{|x|}{\lambda_2}}.$$

It is thus a weighted sum of two double exponential densities (also called DE or "first Laplace" densities), the

first of which describes typical errors, and the second of which describes atypical errors. Each of the parameters λ_1 and λ_2 is $\frac{\sqrt{2}}{2}$ times the standard deviation of its respective DE density. The weighting parameter α is the proportion of time during which an "average" aeroplane is committing an atypical error.

3.4   We consider a pair of parallel routes separated by S_y NM and assume that the lateral errors of the aeroplanes using both routes can be characterized by the same DDE density function f. If we randomly select two aeroplanes, one from those assigned to each route, then $P_y(S_y)$, the probability that the two chosen aeroplanes are in lateral overlap, is approximately

$$2\lambda_y\left[\left(\frac{1-\alpha}{2\lambda_1}\right)^2(\lambda_1+S_y)e^{-\frac{S_y}{\lambda_1}} + \left(\frac{\alpha}{2\lambda_2}\right)^2(\lambda_2+S_y)e^{-\frac{S_y}{\lambda_2}} + \frac{\alpha(1-\alpha)}{2}\left(\frac{e^{-\frac{S_y}{\lambda_1}}+e^{-\frac{S_y}{\lambda_2}}}{\lambda_1+\lambda_2} + \frac{e^{-\frac{S_y}{\lambda_2}}-e^{-\frac{S_y}{\lambda_1}}}{\lambda_2-\lambda_1}\right)\right].$$

Of the three terms enclosed in rectangular brackets in this expression for $P_y(S_y)$, the first (describing "core-core interaction") dominates when the 95 per cent containment limit of the fleet's typical errors exceeds $S_y/5$. When the 95 per cent containment limit is less than $S_y/6$, the third term (describing "core-tail interaction") dominates, generally contributing more than 99 per cent of the value of $P_y(S_y)$.

3.5   After choosing a value of same-direction or opposite-direction occupancy, we apply the appropriate formula from the end of 3.1 to derive the value of α at which $P_y(S_y)$ is small enough for the accident rate to equal the TLS. In other words, when the routes carry traffic in the same direction, we find the value of α at which

$$P_y(S_y) = \frac{5 \cdot 10^{-9}}{P_z(0)\left(\frac{\lambda_x}{S_x}\right)E_y(same)\,K_{same}},$$

and when they carry traffic in opposite directions, the value of α is found at which

$$P_y(S_y) = \frac{5 \cdot 10^{-9}}{P_z(0)\left(\frac{\lambda_x}{S_x}\right)E_y(opp)\,K_{opp}}.$$

Using the constants given in Table A-15-1, we find that when the routes carry traffic in the same direction, we seek the value of α for which $P_y(S_y) = \frac{2.33 \times 10^{-8}}{E_y(same)}$, and when they carry traffic in opposite directions, we seek the value of α for which $P_y(S_y) = \frac{2.29 \times 10^{-9}}{E_y(opp)}$.

Table A-15-1.   Meanings of the Reich model parameters

Symbol	Meaning	Value
λ_x	Average aircraft length	0.0348 NM
λ_y	Average aircraft width	0.031 NM
λ_z	Average aircraft height	0.0089 NM
S_x	Distance within which aircraft assigned to different routes are considered to be longitudinally proximate	120 NM
$\overline{V}$	Average aircraft speed	480 kt
$\overline{|\Delta V|}$	Average longitudinal passing speed of same-direction aircraft assigned to different routes	13 kt
$\overline{\dot{y}}$	Average lateral passing speed of aircraft assigned to different routes	75 kt
$\overline{\dot{z}(0)}$	Average vertical passing speed of aircraft assigned to the same flight level	1.5 kt
$P_z(0)$	Probability that two aircraft assigned to the same flight level are in vertical overlap	0.5
$P_y(S_y)$	Probability that two aircraft assigned to routes separated by S_y are in lateral overlap	
$E_y(same)$	Same-direction lateral occupancy	
$E_y(opp)$	Opposite-direction lateral occupancy	
K_{same}	$\dfrac{\overline{|\Delta V|}}{2\lambda_x} + \dfrac{\overline{\dot{y}}}{2\lambda_y} + \dfrac{\overline{\dot{z}(0)}}{2\lambda_z}$	14 80.73
K_{opp}	$\dfrac{\overline{V}}{\lambda_x} + \dfrac{\overline{\dot{y}}}{2\lambda_y} + \dfrac{\overline{\dot{z}(0)}}{2\lambda_z}$	15 087.05

3.6   Figure A-15-1a illustrates two parallel routes separated by 30 NM, carrying traffic in the same direction. Figure A-15-1b shows how the maximum tolerable value of α varies with the same-direction occupancy of the route pair.

3.7   Figure A-15-2a also shows a pair of parallel routes separated by 30 NM, but they are carrying traffic in opposite directions. Figure A-15-2b shows how the maximum tolerable value of α varies with the pair's opposite-direction occupancy. Though the curves in Figures A-15-1b and A-15-2b appear similar, it is important to remember that the numerator 2.29×10^{-9}, used for the opposite-direction example, is an order of magnitude
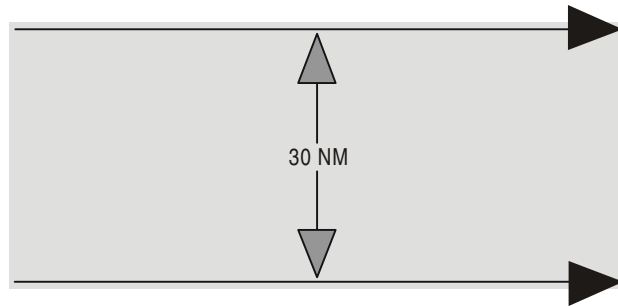
**Figure A-15-1a.   A pair of parallel routes
carrying traffic in the same direction**

smaller than the numerator 2.33×10^{-8} used for the same-direction case, and so the maximum tolerable values
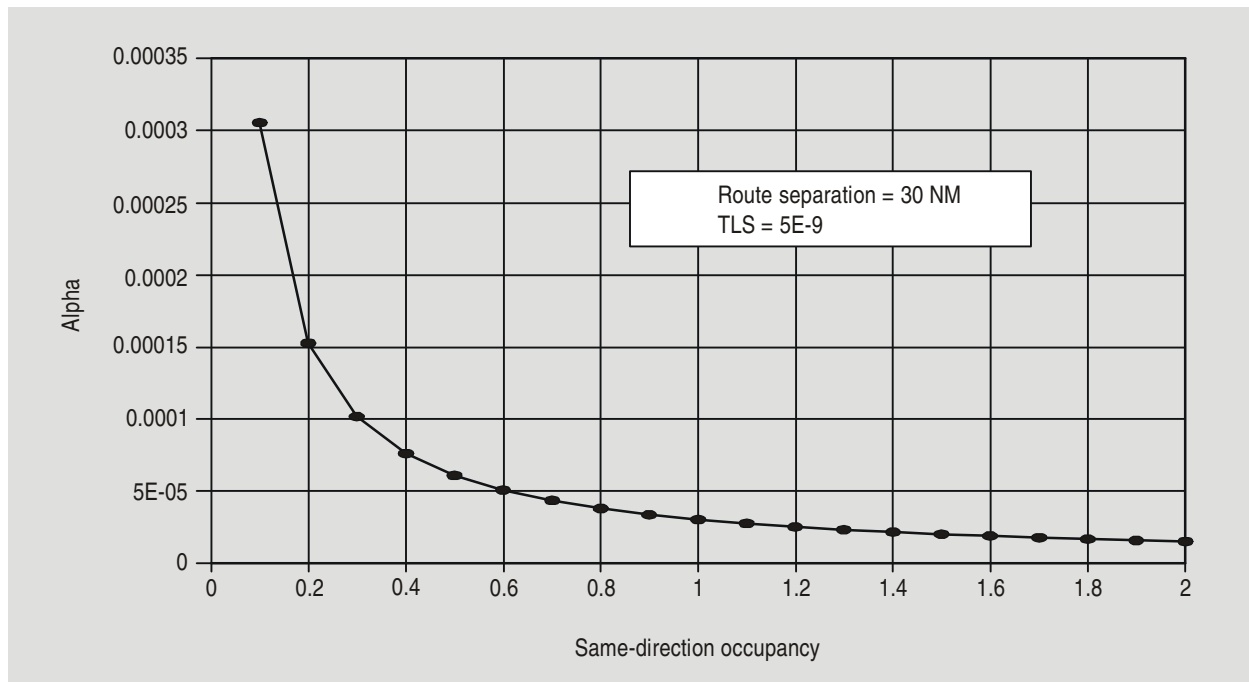

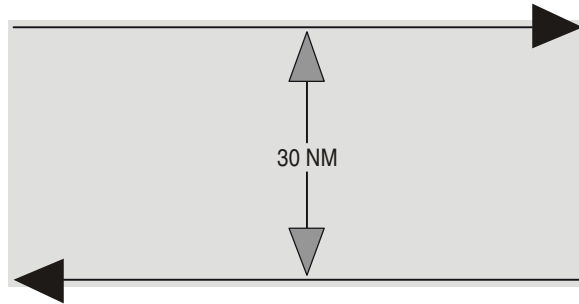**Figure A-15-1b.   Maximum acceptable value of alpha**

**Figure A-15-2a.   A pair of parallel routes
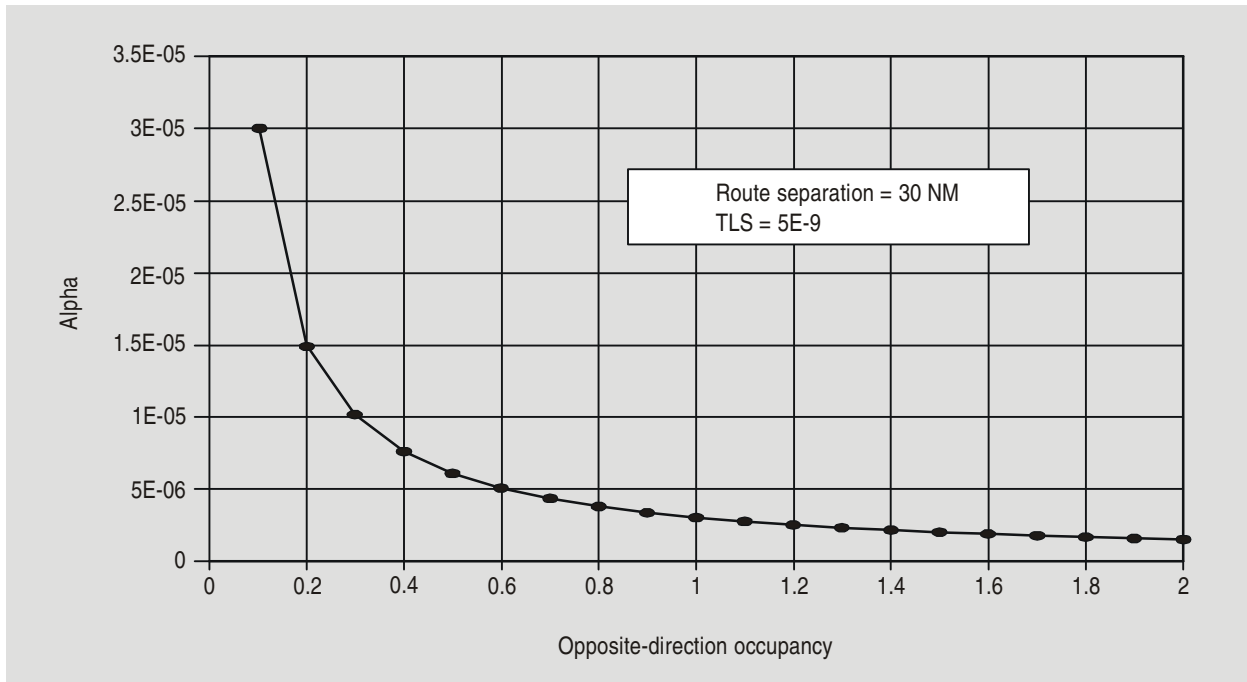carrying traffic in the opposite direction**

Figure A-15-2b.   Maximum acceptable value of alpha

of α shown in Figure A-15-2b are an order of magnitude smaller than the corresponding values shown in Figure A-15-1b. In particular, note that the scales of the vertical axes in the two figures differ by an order of magnitude. (When the fleet using the route system exhibits typical lateral errors whose 95 per cent containment value is less than $S_y/6$, $P_y(S_y)$ varies (almost) directly with α. As was shown in 3.5, $P_y(S_y)$ varies inversely with occupancy. Thus we expect α also to vary inversely with occupancy, and indeed Figures A-15-1b and A-15-2b both look like hyperbolas of the form *constant/x*.)

3.8   The computations of maximum acceptable values of α assume that the fleet's typical performance just satisfies RNP 4. Thus 95 per cent of typical lateral deviations are assumed to be less than 4 NM. It follows that the parameter λ_1 has the value $4/[-\ln(.05)] = 1.3352$. The parameter λ_2 is taken to equal S_y, the distance between the routes — i.e. 30 NM. This is a conservative value in that it maximizes the lateral overlap probability $P_y(S_y)$.

3.9   Figure A-15-3a shows four parallel routes, with traffic moving in the same direction on all of them. For ease of reference, the routes are labelled consecutively as R1, R2, R3 and R4. We assume three flight levels in this example, and we assume a traffic distribution in which the lowest flight level carries 1/4 of the traffic, the middle level carries 5/12 of it, and the upper level carries the remaining 1/3. On each flight level, R1 carries 1/12 of the traffic, R2 carries 1/3 of it, R3 carries 5/12 of it, and R4 carries the remaining 1/6. Thus the example reflects a concentration of traffic on the system's central routes, as is sometimes observed in practice.

3.10   Appendix 14 derives formulae for the occupancies and accident rates of all pairs of flight paths in such a system, as well as for collections of pairs of flight paths. Figure A-15-3b is based on an analysis that first applies the formulae of that appendix to compute accident rates, and then varies α, the rate of atypical errors, to find the value that produces an accident rate equal to the TLS of 5×10^{-9} accidents per flight hour. The figure shows the values of α corresponding to (adjacent flight path) occupancies ranging from 0.1 to 2.0. The analysis supporting Figure A-15-3b differs from most estimates of accident rates (such as those on which Figures A-15-1b and A-15-2b are based) in that it includes not only the rate of accidents due to the loss of planned separation between aeroplanes assigned to adjacent routes, but also the rate of accidents from aeroplanes assigned to non-adjacent routes. In the example used to generate Figure A-15-3b, the inclusion of non-adjacent routes increases the accident rate by almost a seventh and reduces the maximum acceptable α by almost 13 per cent. The example suggests that while accident rates generated from non-adjacent routes constitute a minor contribution to the total rate, that contribution is still far from being negligible or insignificant.

3.11   The analysis mentioned in 1.1 was based on a system of seven parallel routes operating on seven flight levels. It was modelled after the North Atlantic Organized Track System (NAT OTS) as it operated in the mid-1990s, prior to the implementation of a reduced vertical separation minimum (RVSM) of 1 000 feet. An earlier attempt to estimate the maximum acceptable values of α in such a system, discussed at the spring 1995 meeting of RGCSP WG/A, had even attempted to account for the concentration of traffic on the system's central routes and central flight levels, but its analysis of the effect of the concentration was not correct. Applying the results of Appendix 14 is expected to remedy the errors of the earlier analyses.

3.12   Figure A-15-4 graphs the results of an analysis similar to the one that supports Figure A-15-3b. We assume a system of seven parallel routes on seven flight levels, and we take the distribution of traffic on the individual flight paths to be the distribution observed for eastbound NAT OTS traffic on 15 August 1994 (the same distribution used in the spring 1995 analysis). The maximum acceptable values of α shown in Figure A-15-4 are somewhat smaller than the corresponding values shown in Figure A-15-3b, which are themselves somewhat smaller than those shown in Figure A-15-1b. In the analysis supporting Figure A-15-4,
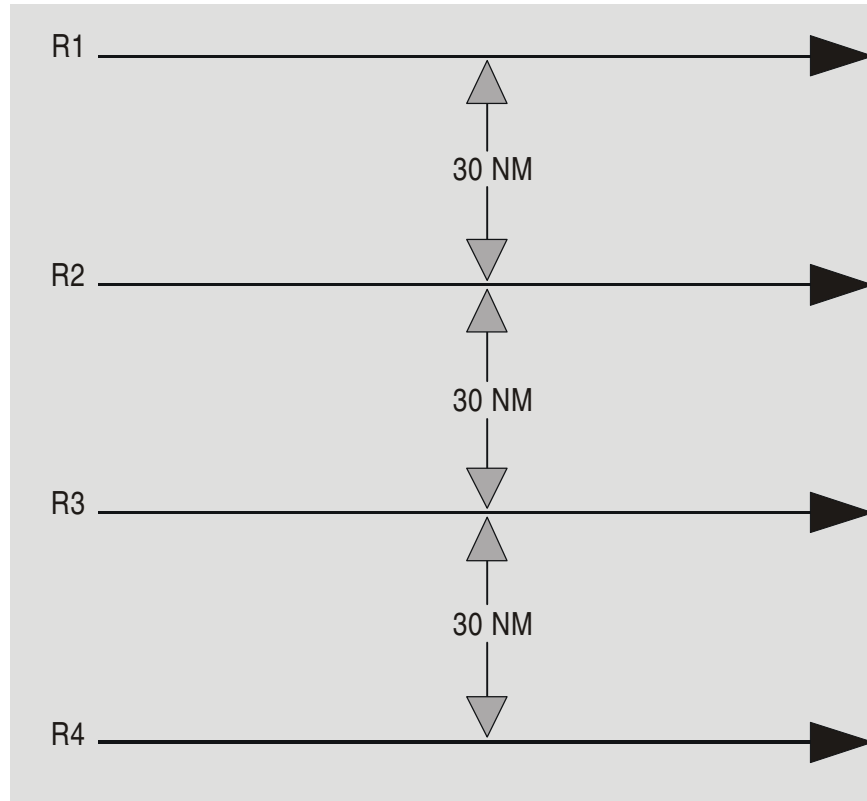
**Figure A-15-3a.   Four parallel routes carrying traffic
in the same direction**

we find that the rate of accidents due to collisions between aeroplanes assigned to (co-altitude) adjacent flight paths constitutes slightly more than 75 per cent of the total accident rate. Aeroplanes assigned to paths separated by twice the minimum lateral separation account for another 20 per cent of the total; those assigned to paths separated by three times the minimum distance contribute more than 4 per cent; and those assigned to paths separated by four times the minimum distance contribute most of the remaining 1 per cent. Thus it can once again be seen that aeroplanes assigned to non-adjacent paths make a significant contribution — nearly 25 per cent in this case — to the route system's total accident rate.

3.13   Recall that the RNP of the airspace determines the parameter λ_1 of the DDE density function and that we have assumed the most conservative value for the parameter λ_2. The analyses supporting Figures A-15-1b, 2b, 3b, and 4 derive the maximum tolerable values of α, the rate of atypical lateral errors, under which the given route systems and occupancies can satisfy the TLS of 5×10^{-9} accidents per flight hour. The three parameters, α, λ_1 and λ_2, completely determine the DDE density function used to describe aircraft performance that just meets the TLS.

3.14   The North Atlantic (NAT) minimum navigation performance specification (MNPS) also states aircraft performance requirements in terms of three parameters. The first of them is a limit on the standard deviation of lateral track-keeping errors. The second, usually denoted η, is a limit on the proportion of total flight time
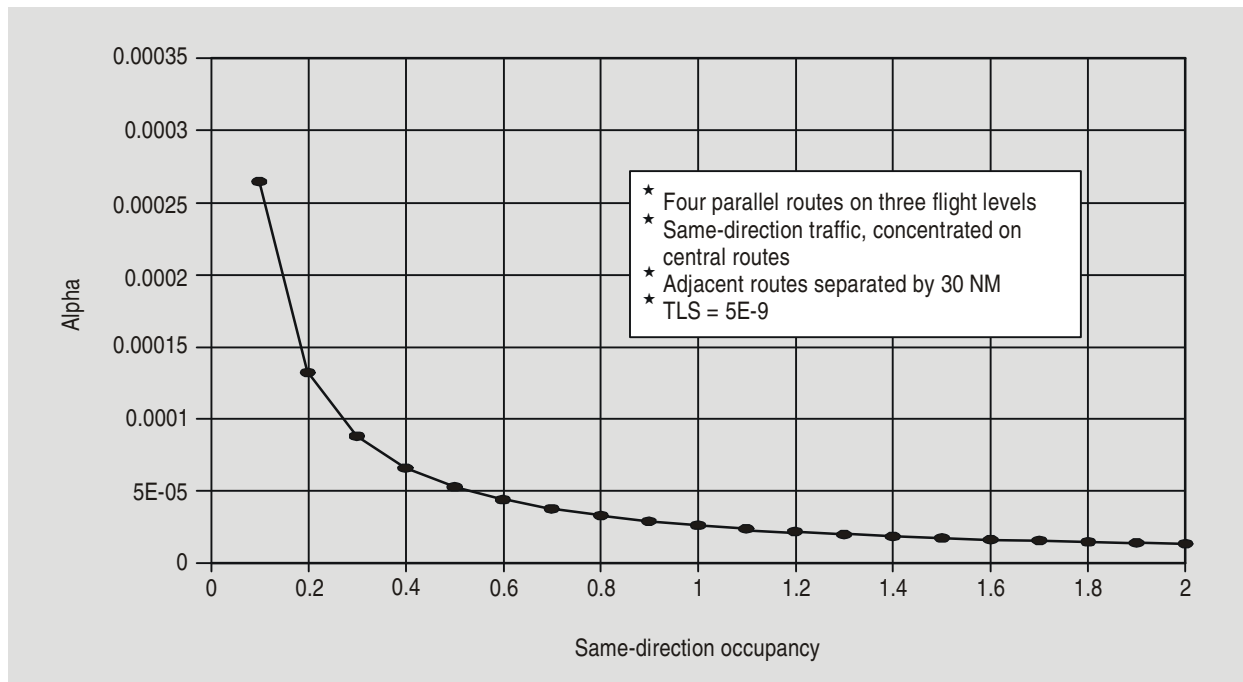
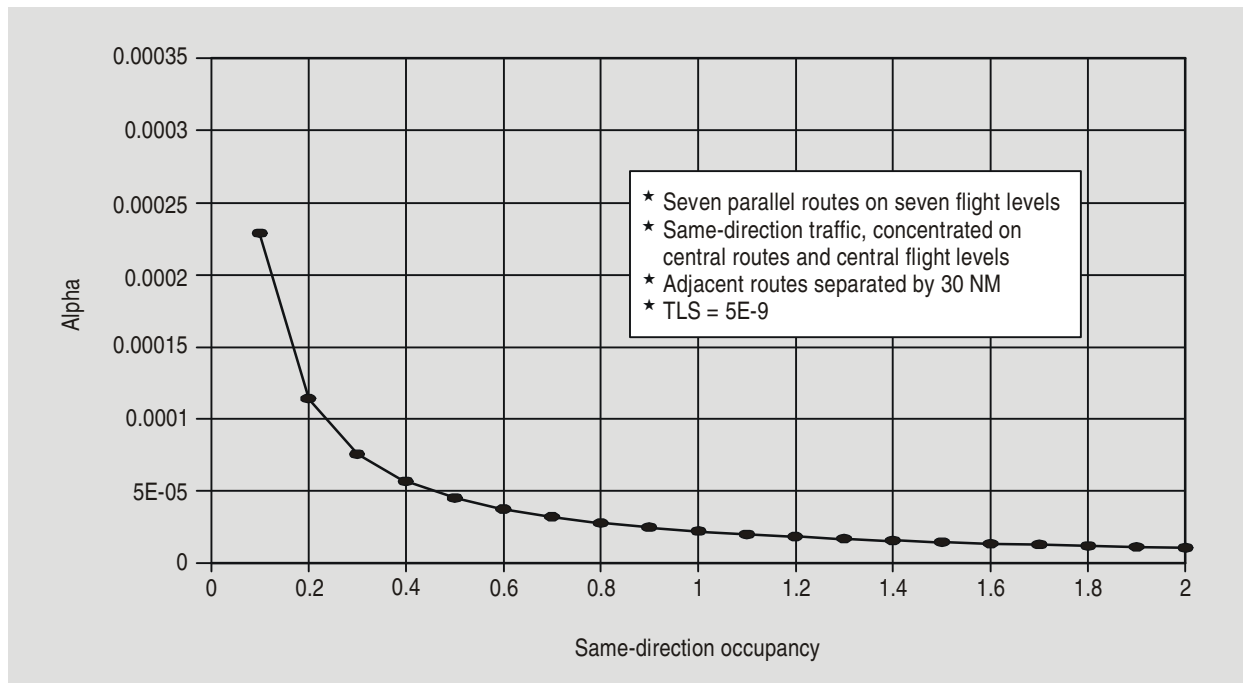**Figure A-15-3b.   Maximum acceptable value of alpha**

Figure A-15-4.   Maximum acceptable value of alpha

spent more than half of a separation standard away from the route centre line, i.e. a limit on the rate of gross lateral errors. The third, called ζ, is a limit on the proportion of flight time spent in the immediate vicinity of an adjacent route's centre line. In the MNPS, the "immediate vicinity" is taken to mean the 20-NM-wide band that covers all points within 10 NM of the centre line. In the present analysis, for which adjacent routes have only 30 NM separation between their centre lines, the "immediate vicinity" is taken to mean the 12-NM-wide band covering all points within 6 NM of the adjacent route's centre line. Fortunately, it is not difficult to translate values of α, λ_1 and λ_2 into corresponding values of the three parameters used in the NAT MNPS. The limit on the standard deviation of lateral track-keeping errors is logically equivalent to an RNP, and values for η and ζ can be obtained by integrating the DDE density function over appropriate intervals or, equivalently, by computing differences of the DDE distribution function evaluated at appropriate points.

3.15   The NAT MNPS requires typical navigation equivalent to RNP 12.6, while (as indicated above) a system having a 30 NM lateral separation minimum is likely to operate with RNP 4. Table A-15-2 shows the values of α that are graphed in Figure A-15-4 (i.e. the maximum tolerable rates of atypical errors for a one-way route system that has seven routes and seven flight levels, has a minimum lateral separation of 30 NM and typically has a traffic distribution resembling that of the eastbound NAT OTS on 15 August 1994). Table A-15-2 also shows the η and ζ values corresponding to each value of α. Even at the lowest occupancy shown in the table, i.e. at 0.1, these η and ζ values are considerably more stringent than the values given in the NAT MNPS, where $\eta = 5.3 \times 10^{-4}$ and $\zeta = 1.3 \times 10^{-4}$.

4.   MEANS OF REDUCING THE RATE OF
GROSS LATERAL ERRORS

Several improvements in avionics, communication systems and air traffic control systems that were to be implemented during the late 1990s were expected to significantly reduce the rate of gross lateral errors and thereby enable reductions in lateral separations. In order to determine whether the proposed systems did indeed have the potential to eliminate oceanic gross errors, researchers from the United States Federal Aviation Administration's Flight Standards Service carefully examined each of the "Table A" and "Table B" errors entered into the database of the North Atlantic Central Monitoring Agency (NAT CMA) between 1986 and 1993. Their examination revealed that the proposed improvements had the potential to eliminate approximately 95 per cent of the listed gross errors. While this result confirmed the potential benefit of pursuing the proposed implementations, it will be necessary to confirm that the various subsystems which are intended to contribute to error reduction actually operate as planned. It is also worth noting that any new system has the potential to cause previously unforeseen errors, and since some of the new systems are highly complex, it may be extremely difficult to determine the causes of such errors.

5.   APPLICATION CONSIDERATIONS

5.1   Each oceanic airspace has characteristics that distinguish it from others, such as the number and length of its routes, and the dimensions, speeds, and navigational characteristics of the aeroplanes that use it. The quantitative results reported in section 3 are derived from typical values of the parameters that are significant in collision risk analyses, but they should not be viewed as applicable to all oceanic airspace. Nonetheless, it would not be difficult to replicate those analyses with the values found to be applicable to any particular route system.

**Table A-15-2. Eta and zeta values computed from
double double exponential (DDE) distribution function**

Lambda1   =   1.3352
Lambda2   =   30

Half lambda2   =
Half separation   =   15

Outer zeta limit =   36

Inner zeta limit =   24

Occupancy	Maximum alpha	Eta	Zeta
0.1	2.29E-04	1.52E-04	3.40E-05
0.2	1.15E-05	8.27E-05	1.70E-05
0.3	7.53E-05	5.95E-05	1.13E-05
0.4	5.72E-05	4.79E-05	8.50E-06
0.5	4.58E-05	4.10E-05	6.80-06
0.6	3.82E-05	3.64E-05	5.67-06
0.7	3.27E-05	3.30E-05	4.86-06
0.8	2.86E-05	3.06E-05	4.25-06
0.9	2.54E-05	2.86E-05	3.78-06
1.0	2.29E-05	2.71E-05	3.40-06
1.1	2.08E-05	2.58E-05	3.10-06
1.2	1.91E-05	2.48E-05	2.84-06
1.3	1.76E-05	2.39E-05	2.62-06
1.4	1.63E-05	2.31E-05	2.43-06
1.5	1.52E-05	2.25E-05	2.27-06
1.6	1.43E-05	2.19E-05	2.13-06
1.7	1.34E-05	2.14E-05	2.01-06
1.8	1.27E-05	2.09E-05	1.89-06
1.9	1.20E-05	2.05E-05	1.80-06
2.9	1.14E-05	2.01E-05	1.71-06

5.2   Appropriate values for some of the parameters used in collision risk analyses can be gathered through surveys of operators; others can be found through examinations of radar-reported aircraft positions; still others may be obtained from careful sorting of the data recorded on flight-progress strips. While some of the processes involved in determining parameter values are labour-intensive and may require weeks or months of effort, most can be accomplished within a reasonable time period. Gross errors, however, occur quite infrequently, and thus an airspace management authority that wishes to determine whether its route system's gross-error rate is acceptably low may need to monitor the system over a period of several years and establish a database such as

that of the NAT CMA. Of course, if the database is not well-maintained — in particular, if it does not include all of the gross errors that occur in the airspace — then the authority may mistakenly conclude that its route system can operate safely with smaller separations than are warranted by the system's true (but unknown) rate of gross errors.

6.   CONCLUSION

Section 3 considers four different examples of route systems having a 30 NM separation between adjacent co-altitude flight paths. For each of those systems, it derives the maximum rate of atypical errors that can be tolerated for each level of occupancy if the system is to meet a TLS of 5×10^{-9} accidents per flight hour.

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Appendix 16

A METHOD OF DERIVING PERFORMANCE STANDARDS FOR AUTOMATIC DEPENDENT SURVEILLANCE (ADS) SYSTEMS

1.   INTRODUCTION

An airspace management authority planning to implement an ADS system may wish to establish performance standards for the system before it embarks upon the preparation of detailed specifications and other procurement documents. This appendix suggests a procedure for obtaining five significant performance standards:

p_w — the minimum acceptable probability that the system prevents a waypoint-insertion lateral error;

p_n — the minimum acceptable probability that the system prevents a preventable non-waypoint-insertion lateral error;

$P(A)$ — the minimum acceptable probability that the sequence of actions referred to in this appendix as an "ADS cycle" is completed;

t — a maximum acceptable time for completion of an ADS cycle; and

m — the minimum acceptable probability that an ADS cycle is completed in time t, given that it is completed.

The procedure is based on two important characteristics of ADS systems: their operation of route conformance functions and their use of event contracts.

2.   OBTAINING VALUES OF p_w AND p_n

2.1   The first step in the procedure assumes that the airspace management authority maintains (or has access to) a monitoring system that records the gross lateral errors committed by the fleet of aeroplanes that use its routes. For example, the Central Monitoring Agency (CMA) of the United Kingdom's National Air Traffic Services maintains statistics on the performance of the North Atlantic fleet; the United States Federal Aviation Administration's Asia-Pacific Approvals Registry and Monitoring Organization (APARMO) performs a similar function for the Pacific fleet; and Spain's Aena operates the South Atlantic Monitoring Agency (SATMA). Although the monitoring system may be employed for various other purposes, the procedure described here is chiefly concerned with its ability to determine a route system's rate of gross lateral errors r. One would usually expect r to be derived empirically and to describe a prevailing rate, but in some contexts it might indicate

an expected future rate, as long as its value could be justified on reasonable analytic grounds. The rate r might be low enough to permit safe operations under current conditions, but be too high for the route system to operate safely after the implementation of a projected change, such as a reduction in the lateral separation between adjacent parallel routes. The implementation of ADS might be needed in order to reduce the gross error rate to an acceptable level.

2.2   Thus the airspace management authority also needs to select a maximum tolerable gross error rate η_R. η_R is the maximum tolerable probability that *after* the implementation of ADS a randomly chosen flight is committing a gross error.

2.3   An examination of gross error data, such as those maintained by the CMA, may suggest a rate of errors attributable to failures of navigation systems. It is reasonable to expect that whenever air traffic controllers are not informed of such a failure, the crew of the affected aeroplane is also unaware of it for, in most cases, if the crew were aware of it, it would notify its servicing ATC unit. If the flight crew has not been informed of a failure, then the navigation system itself, and all aircraft subsystems with which it communicates, must have been unable to detect the failure. The CMA normally requests an investigation of each NAT gross lateral error that comes to its attention, and it retains a record of the cause of the error. If other monitoring agencies do likewise, then a typical airspace management authority should have access to data that will enable it to estimate the fraction, f_u, of all gross lateral errors in its airspace caused by undetectable failures of navigation systems. Since airborne ADS units cannot detect these failures, the ADS system will be unable to prevent the resulting errors, and the fleet's rate of errors after the implementation of ADS will be at least rf_u. If $rf_u > \eta_R$, then the implementation of ADS will not by itself be capable of reducing the system's gross error rate to an acceptable level. The remainder of this appendix therefore applies only to values of f_u less than $\dfrac{\eta_R}{r}$, i.e. values for which $rf_u < \eta_R$.

2.4   It has been observed that, in recent years, in at least one heavily travelled airspace, a large proportion of gross lateral errors have been waypoint-insertion errors. Using data from the relevant monitoring agency, the authority can also estimate f_w, the fraction of all gross errors that are waypoint-insertion errors. The route system's fraction of detectable, and possibly preventable, gross errors is $1 - f_u$. Since all waypoint-insertion errors are candidates for prevention by ADS, it is clear that $f_w < 1 - f_u$, and that $\dfrac{f_w}{1-f_u}$ is the fraction of possibly preventable errors that are waypoint-insertion errors. Thus $1 - \dfrac{f_w}{1-f_u} = \dfrac{1-f_u-f_w}{1-f_u}$ is the fraction of possibly preventable gross errors that are non-waypoint-insertion errors. The underlying rate of (preventable) waypoint-insertion gross errors is rf_w, and the underlying rate of preventable non-waypoint-insertion gross errors is $r(1 - f_u - f_w)$. That is, prior to ADS implementation, rf_w = P(a randomly chosen flight is committing a waypoint-insertion gross error); likewise, $r(1 - f_u - f_w)$ = P(a randomly chosen flight is committing a preventable non-waypoint-insertion gross error). To save space, let the abbreviation "WPIE" denote "waypoint-insertion gross error", and let "NWPIE" denote "non-waypoint-insertion gross error".

2.5   The route conformance function of ADS (much of which is implemented in the ground-based component of the system) is expected to eliminate most WPIEs. Let p_w = P(ADS prevents a WPIE), and let p_n = P(ADS prevents a preventable NWPIE). Then $1 - p_w$ = P(ADS fails to prevent a WPIE) and $1 - p_n$ = P(ADS fails to prevent a preventable NWPIE). After ADS is implemented in a route system, the maximum tolerable rate of gross errors is η_R. However, ADS will not have any effect on the portion of the gross error rate

represented by rf_u, the rate of non-preventable gross errors. If the route system's rate of gross errors after the implementation is to meet the maximum tolerable rate η_R, it must satisfy:

$\eta_R \geq$ P(a randomly chosen flight is committing a preventable gross error)

$= $ P(a randomly chosen flight is committing a WPIE)

$+ $ P(a randomly chosen flight is committing a preventable NWPIE)

$= $ P(ADS fails to prevent a WPIE)rf_w

$+ $ P(ADS fails to prevent a preventable NWPIE)$r(1 - f_u - f_w)$

$= (1 - p_w)rf_w + (1 - p_n)r(1 - f_u - f_w).$

Equivalently,

$$\frac{\eta_R}{r} - f_u \geq (1-p_w)f_w + (1-p_n)(1-f_u-f_w)$$

$$= f_w - f_w\, p_w + 1 - f_u - f_w - (1-f_u-f_w)p_n$$

$$= 1 - f_u - f_w\, p_w - (1-f_u-f_w)p_n$$

and

$$\frac{\eta_R}{r} \geq 1 - f_w\, p_w - (1-f_u-f_w)p_n \,.$$

Thus

$$(1-f_u-f_w)p_n \geq 1 - \frac{\eta_R}{r} - f_w\, p_w$$

or, equivalently,

$$p_n \geq \frac{1 - \dfrac{\eta_R}{r}}{1 - f_u - f_w} - \frac{f_w}{1 - f_u - f_w}p_w. \tag{1}$$

From this inequality it is clear that the set of possible values of the probability p_n is bounded from below by a linear function of the probability p_w. The line that forms the lower bound has p_w-intercept $\dfrac{1 - \dfrac{\eta_R}{r}}{f_w}$ and

p_n-intercept $\dfrac{1 - \dfrac{\eta_R}{r}}{1 - f_u - f_w}$. However, since p_w and p_n are probabilities, they also must lie between 0 and 1.

2.6   Thus, once the authority has chosen values for the parameters η_R, r, f_u and f_w, its choice of the probabilities p_w and p_n is constrained by the linear bound given in inequality (1) and by the requirement that they remain between 0 and 1. The area to the right of the sloping line in Figure A-16-1 illustrates a set of feasible values of p_n and p_w. In this particular example, the parameters have the values $\eta_R = 1.41 \cdot 10^{-5}$, $r = 6.4 \cdot 10^{-5}$, $f_u = 0.05$ and $f_w = 0.90$. Note that where p_w is greater than or equal to 0.8663, p_n can take any value between 0 and 1 because the ADS route conformance function eliminates enough WPIEs that it is not necessary for ADS to eliminate any preventable NWPIEs in order to reduce the overall error rate to η_R. On the other hand, where p_w is less than or equal to 0.8107, the route conformance function is unable to prevent enough waypoint errors to reduce the overall error rate to η_R, even if ADS completely eliminates all other preventable errors. Of course, different inputs would change the graph, but if it remained similar to Figure A-16-1, there is a substantial possibility that the ADS route conformance function would suffice to reduce the overall gross error rate to the required level because its success rate has been predicted to be better than 86.63 per cent.
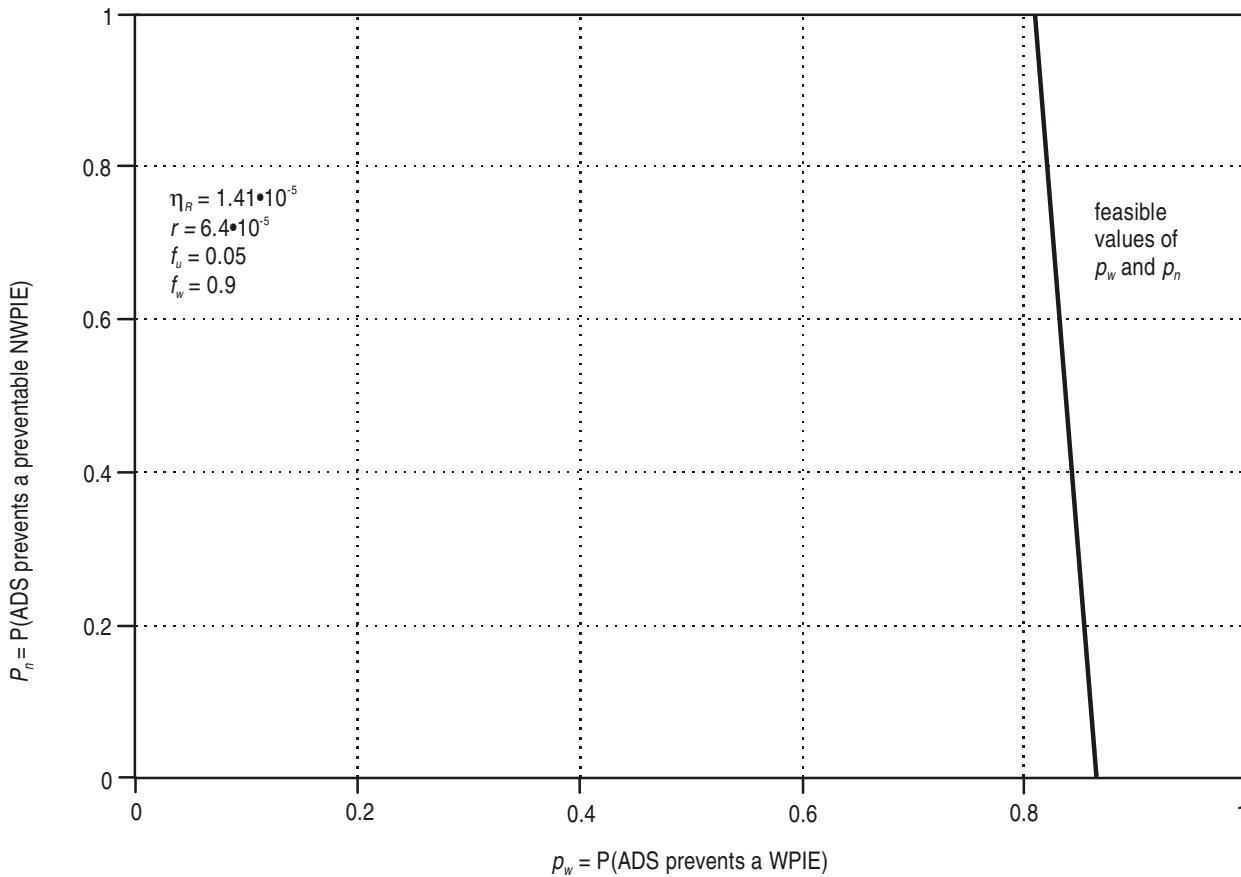

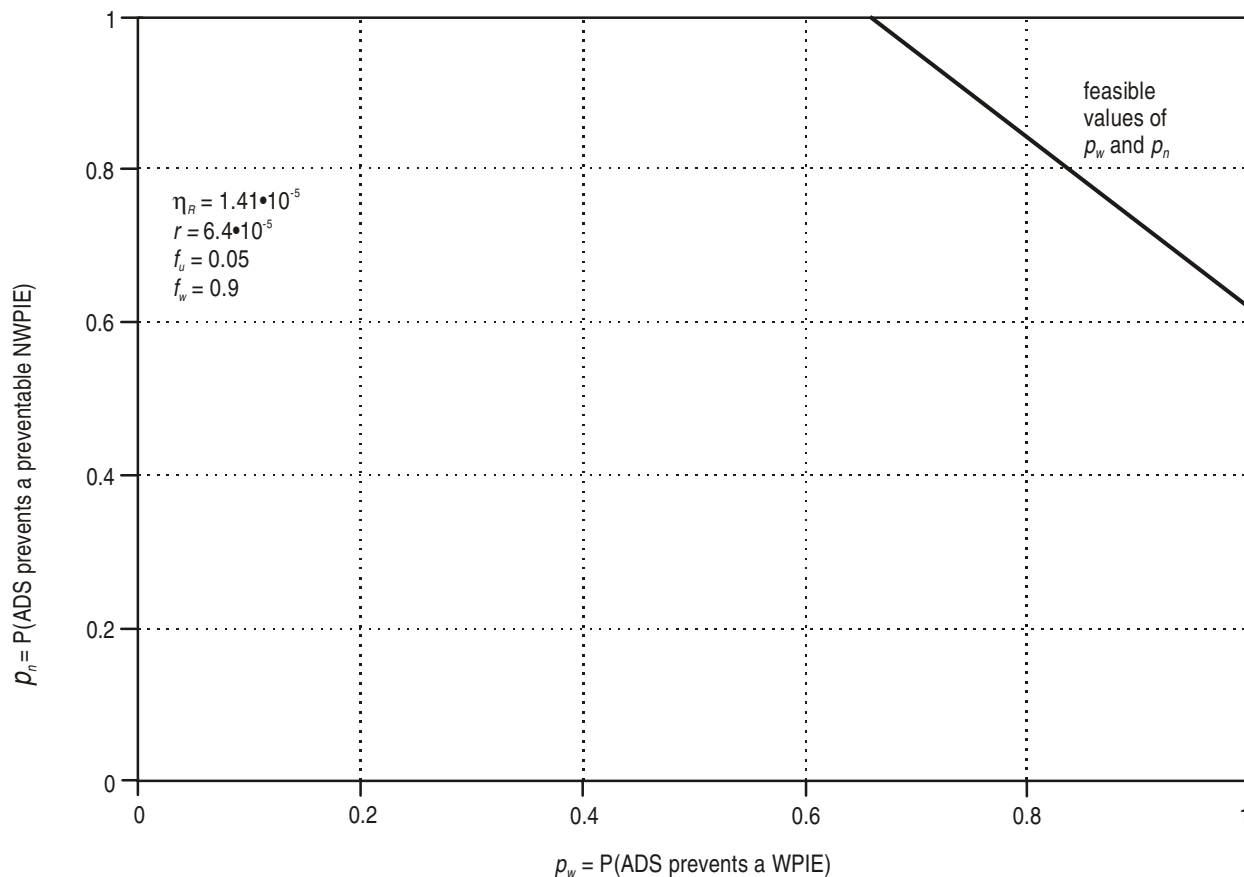**Figure A-16-1.   Relationship between required probabilities**

Figure A-16-2.   Relationship between required probabilities

2.7   The boundary line in Figure A-16-1 is quite steep because f_w, the fraction of gross errors that are waypoint-insertion errors, is a large fraction (90 per cent) of the total. In other words, the overall rate of gross errors is reduced by the required extent if and only if the rate of waypoint-insertion errors is reduced by nearly the same extent. Even if all other gross errors are eliminated, it is still necessary to reduce waypoint-insertion errors by 81.07 per cent, but if ADS eliminates 86.63 per cent of waypoint-insertion errors, it does not need to eliminate any others.

2.8   Figure A-16-2 shows an example in which $f_w = 0.5$, a fraction typical of one heavily used airspace during the early 1990s. In this case, the lower right end of the boundary line shows that even if $p_w = 1$ (i.e. if ADS prevents 100 per cent of waypoint-insertion errors) it still needs to prevent at least 62.14 per cent of other preventable errors in order to reduce the error rate to $\eta_R = 1.41 \cdot 10^{-5}$. Likewise, the top left end of the boundary line shows that if ADS can eliminate 100 per cent of preventable non-waypoint-insertion errors, then it will still need to prevent at least 65.93 per cent of waypoint-insertion errors in order to succeed in reducing the overall rate to η_R.

2.9   Thus the authority can begin its specification of ADS performance parameters by:

a)   estimating r, an underlying rate of gross lateral errors;

b)   selecting a maximum tolerable probability, η_R, that a typical flight will commit a gross lateral error after the implementation of ADS;

c)   estimating f_u, the fraction of gross lateral errors that ADS cannot be expected to prevent (while recognizing that if $rf_u > \eta_R$, then the implementation of ADS will not, by itself, be capable of reducing the system's gross error rate to an acceptable level);

d)   estimating f_w, the fraction of gross lateral errors that are waypoint-insertion errors; and

e)   using inequality (1) to select p_w, the minimum acceptable probability that a waypoint-insertion error will be prevented by ADS, and p_n, the minimum acceptable probability that a (preventable) non-waypoint-insertion error will be prevented by ADS.

2.10   The very significant difference between Figures A-16-1 and A-16-2 (figures that reflect different prevailing conditions in the *same* airspace during different time periods) illustrates an important principle. Different airspace, and even the same airspace operating in different time periods, can exhibit different characteristics, and thus an analysis of safety requirements done for one of them is not necessarily relevant to another. The airspace management authority must be careful to select parameter values that reflect the airspace and the time period relevant to its planning.

3.   MODELLING ADS OPERATION

3.1   A basic scenario

3.1.1   Having chosen or derived values for the parameters listed in 2.9, the airspace management authority can then derive a limit on the maximum acceptable time for the transmission of the sequence of messages used by an ADS system and an associated communications link to prevent the occurrence of a gross lateral error. The remainder of this section explains the model used to derive that limit.

3.1.2   ADS systems send aircraft positions to air traffic controllers when the aircraft are beyond the coverage of surveillance radar. An ADS system consists of both ground-based and airborne components and may also include a space-based communications link. The ground-based component typically arranges a "contract" with the airborne component of each aeroplane that participates in the system, and that contract specifies the kinds of data that are to be reported, as well as the conditions under which reports are to be transmitted.

3.1.3   The messages sent by the airborne component generally incur transmission costs, and in order to keep such costs reasonably low, current ADS systems employ typical reporting rates of approximately one report per 15 minutes. Such low update rates are clearly of very little use in promptly notifying the air traffic control (ATC) system of unauthorized or unintentional deviations from the aeroplane's planned route of flight, since there is only a small probability that such a deviation occurs shortly before the airborne unit sends a scheduled

report. Instead, the ADS system relies on the airborne unit to monitor its aircraft's progress along its cleared route of flight and to automatically report to the ATC system whenever the aeroplane deviates from that route by more than some prescribed distance. In this discussion that distance is called B because it defines a buffer (of width $2B$) about the route's centre line. The angle by which the aeroplane's true path deviates from the route is denoted θ_d.

3.1.4   Suppose that an aeroplane begins to deviate from its cleared route of flight. (The present discussion does not address the issue of whether the deviation is appropriate or not. Some deviations result from human error or equipment malfunction. Others are intentional, and of those, some are obviously necessary to ensure safety of flight.) When the aeroplane has laterally moved distance B from the centre line of its cleared route, the airborne ADS unit should recognize that it is passing beyond the buffer and should generate a report to the ATC system. However, it is not unreasonable to imagine that the airborne ADS unit and the ground-based ATC system might have different definitions of the cleared route. Such an error could result from various causes, but regardless of the cause, the error could lead the airborne unit to understand that it is following the cleared route, even though its path is in fact diverging from that which the ATC system understands the cleared route to be and which it expects the aeroplane to be following. There are, undoubtedly, a variety of other means by which the airborne unit might fail to generate a report to the ATC system, such as failure of its electronic components or execution of faulty software. Whatever they may be, the existence of mechanisms that prevent the generation or transmission of such reports leads to the conclusion that while the probability of correctly generating and transmitting a report may be relatively large, it must still be strictly less than 1. Let p_t denote that probability. In the (highly probable) event that a report is transmitted, let T_1 denote the random variable that is the time between the aeroplane's passage out of the buffer and its transmission of the report.

3.1.5   The report typically passes through an elaborate, complex communications system consisting of many links. Let p_d denote the probability that it reaches the correct ATC system, and in the event that it does so, let the random variable D be the time from its transmission to its reception.

3.1.6   When the ADS report reaches the ATC system, it may be processed by a controller (perhaps aided by a computer), and the controller may respond by transmitting a message telling the pilot to return to the cleared route of flight. Let p_c denote the probability that the ATC system correctly generates and transmits such a message, and in the event that it does so, let the random variable C be the time from the arrival of the ADS report to the transmission of the response.

3.1.7   The response message must also pass through a complex communications system consisting of many links. Let p_u be the probability that it reaches the pilot, and if it does, let the random variable U be the elapsed time between its transmission from the ATC system and its arrival at the pilot's position.

3.1.8   Upon receiving the controller's message, the pilot takes some time to understand it and to decide whether to implement it. Let p_i be the probability that the pilot decides to implement the ATC system's instruction, and let I denote the time between receipt of the controller's message and the pilot's initiation of a constant bank angle turn back towards the cleared route of flight. Let the random variable ϕ be the bank angle.

3.1.9   The aeroplane continues to move away from its assigned route until its course changes by θ_d, at which instant it is moving parallel to the route. (It then continues to turn until it reaches the heading at which the pilot wishes to return to the route.) Thereafter it moves back toward the route. Thus the aeroplane reaches its maximum distance from the route's centre line at the moment when it has turned enough to be flying parallel

to the route. Let Y denote that maximum distance, and let T_2 denote the time between the initiation of the bank and the moment when the aeroplane's heading has turned through an angle θ_d. Let V denote the aeroplane's speed, and let g (≈ 9.8 metres/sec^2) denote the acceleration of gravity. As is shown in the attachment to this appendix, $T_2 = \dfrac{V\theta_d}{g \tan \phi}$.

3.1.10 If the aeroplane is flying in a system of parallel routes, it is viewed as committing a gross lateral error (sometimes also called a gross navigational error or GNE) when its deviation from the centre line of its assigned route exceeds half of the separation between adjacent routes. Let S denote that separation. Also, conservatively assume that the aeroplane continues to deviate from its route and commits a gross error, unless the ADS system (and its associated communications system) works well enough to turn it back to the heading of the route before its excursion takes it $S/2$ away from the centre line. Thus the aeroplane avoids committing a gross error if and only if:

a) the airborne ADS component transmits a report indicating that its aeroplane has passed the boundary of the buffer;

b) the report is successfully received by the appropriate ATC unit;

c) the ATC unit issues a message telling the pilot to return the aeroplane to its cleared route;

d) the ATC unit's message is correctly received by the pilot;

e) the pilot takes the appropriate action to turn the aeroplane; and

f) the aeroplane turns to the heading of its route before its lateral distance from the centre line reaches $S/2$.

Together, events a) through e) constitute an "ADS cycle". Let A denote the mathematical intersection of those events, i.e. the event that the ADS cycle is completed, in that actions a) through e) all occur. Then $P(A) = p_t \cdot p_d \cdot p_c \cdot p_u \cdot p_i$. Given that the cycle is completed, the probability that action f) occurs is the (conditional) probability that $Y < S/2$.

3.1.11 The aeroplane's speed, V, is a random variable in that it takes different values for different aeroplanes, but for any particular cruising aeroplane it is essentially constant. During the periods whose durations are T_1, D, C, U, and I, the aeroplane is travelling straight, and its lateral speed is $V \sin \theta_d$. Thus it laterally travels $V \sin \theta_d (T_1 + D + C + U + I)$ during those five periods.

3.1.12 As is shown in the attachment to this appendix, the lateral distance that the aeroplane travels while turning, during the time period of duration T_2, is $\dfrac{V^2}{g \tan \phi}(1 - \cos \theta_d)$. Thus Y, the aeroplane's greatest lateral distance from the centre line of its cleared route, is given by

$$Y = B + V \sin \theta_d(T_1 + D + C + U + I) + \frac{V^2}{g \tan \phi}(1 - \cos \theta_d). \tag{2}$$

The probability that the aeroplane returns to its cleared route of flight without committing a gross lateral error is then $P[A \text{ and } (Y < S/2)] = P(A) \cdot P(Y < S/2 \mid A)$

$$= p_t \cdot p_d \cdot p_c \cdot p_u \cdot p_i \cdot P\left(B + V \sin \theta_d (T_1 + D + C + U + I) + \frac{V^2}{g \tan \phi} (1 - \cos \theta_d) < S/2 \mid A \right). \qquad (3)$$

3.2   Distributions of random variables

3.2.1   Little, if any, empirical data are available to characterize the random variables T_1, D, C, U, I, V, θ_d and ϕ. It is, however, clear that they are all strictly positive, and their distribution functions must reflect that property.

3.2.2   The times D and U vary with the performance characteristics of the equipment used to accomplish the ADS data link functions. They can also be expected to vary because of delays due to random contention for scarce transmission resources. One expects D and U to have extremely small probabilities of being very close to 0, but their probability density functions can be expected to increase with increasing time, up to some local maxima, and then gradually decrease. Functions such as gamma densities may be reasonable candidates to quantitatively describe these random variables.

3.2.3 T_1, the aeroplane's delay in sending a message to the ATC system, is likely to be far smaller than D and U, but it too may be well described by an appropriate gamma density.

3.2.4   The time intervals C and I depend on the performance of both equipment and humans, but one expects their density functions to have the same properties mentioned in 3.2.2, and so the gamma densities are again likely candidates to describe them.

3.2.5   The distribution of speeds in a given airspace depends on the types of aircraft that use it. However, the speeds used for travel in one particular direction, on a single route or a set of parallel routes, rarely differ from each other by more than 100 kt, i.e. approximately 50 metres/second. Thus it may be possible, for many airspaces, to model aircraft speed V as a random variable uniformly distributed over a relatively small interval.

3.2.6   The bank angle ϕ of a deviating aeroplane may vary with the flight management system of its aeroplane, with the deviation angle θ_d, and with the aeroplane's longitudinal distance from its next reporting point when it begins to turn back toward its cleared route. In the absence of empirical data, a uniform distribution over a small range of angles (perhaps 10 or 15 degrees) may suffice to describe the bank angle.

3.2.7   In the scenario presented above, the aeroplane laterally deviates from its assigned route at a speed of $V \sin \theta_d$ during the time periods whose lengths are T_1, D, C, U and I. If the sum of those times is as little as two or three minutes, but the deviation angle θ_d is relatively large, the lateral speed will be great enough to cause a gross lateral error — even before the aeroplane begins to turn back toward its assigned route. It is clear that (the sine of) θ_d has a very significant effect on the probability that the deviating aeroplane commits a gross lateral error. Large deviation angles lead to very large lateral speeds, and in such cases the aeroplane avoids committing a gross error only when the ADS system works so quickly that it allows the aeroplane to turn back toward its cleared route within a very short time after leaving the buffer. For example, if the aeroplane is moving at 480 kt, and $\theta_d = 30°$, then the lateral speed is 480 kt $\cdot$ sin(30°) = 240 kt or 4 NM per minute. If the distance between the routes is 30 NM, and the buffer's half-width is 5 NM, then the aeroplane commits a gross

error if it goes $(30/2) - 5$ NM $= 10$ NM past the boundary of the buffer, and so (without even considering lateral movement during the aeroplane's turnback toward its route, it's clear that) the aeroplane commits a gross error if $T_1 + D + C + U + I$ exceeds $(10$ NM$)/(4$ NM/min$) = 2\frac{1}{2}$ minutes.

3.3 Deriving a performance parameter
from the basic scenario

3.3.1   When airspace management authorities have accumulated several years of experience in the operation of ADS systems, it may become easy for them to develop requirements for the distributions of T_1, D, C, U and I. However, at the time of the preparation of this appendix, the data needed to develop such requirements were not readily available. Nonetheless, though the distributions of the individual times T_1, D, C, U and I may be difficult to determine, the example in the last paragraph suggests that their sum is far more important than any of the individual terms. Therefore, it makes sense to return to equation (3), which expresses the probability that ADS prevents a deviation from developing into a gross error as

$$p_t \cdot p_d \cdot p_c \cdot p_u \cdot p_i \cdot P\left(B + V \sin \theta_d (T_1 + D + C + U + I) + \frac{V^2}{g \tan \phi}(1 - \cos \theta_d) < S/2 \mid A \right).$$

By selecting a minimum acceptable probability p_n that ADS prevents a preventable NWPIE, the airspace management authority will effectively require that

$$p_t \cdot p_d \cdot p_c \cdot p_u \cdot p_i \cdot P\left(B + V \sin \theta_d (T_1 + D + C + U + I) + \frac{V^2}{g \tan \phi}(1 - \cos \theta_d) < S/2 \mid A \right) \geq p_n, \quad (4)$$

or, equivalently, that $P(Y < S/2 \mid A) \geq \dfrac{p_n}{P(A)}$. Figure A-16-3 shows values of $\dfrac{p_n}{P(A)}$ for six different probabilities $P(A)$ that the ADS cycle is successfully completed, and for a large range of possible values of p_n. That is, Figure A-16-3 shows minimum acceptable values of

$$P(Y < S/2 \mid A) = P\left(B + V \sin \theta_d (T_1 + D + C + U + I) + \frac{V^2}{g \tan \phi}(1 - \cos \theta_d) < S/2 \mid A \right).$$

Through consultations with equipment manufacturers the airspace management authority should be able to estimate a realistic (or even a conservatively large) value for $P(A)$. Dividing the chosen value of p_n by the chosen value of $P(A)$ then yields the minimum acceptable probability m that the ADS cycle is completed quickly enough, given that it is completed. In the event that $p_n /P(A) > 1$, a larger value of $P(A)$ needs to be adopted (since the quotient m must be a probability).

3.3.2   The probability $P(Y < S/2 \mid A) \geq m$ can be re-expressed as

$$P\left(T_1 + D + C + U + I < \frac{S/2 - B - \dfrac{V^2}{g \tan \phi}(1 - \cos \theta_d)}{V \sin \theta_d} \mid A \right) \geq m. \quad (5)$$
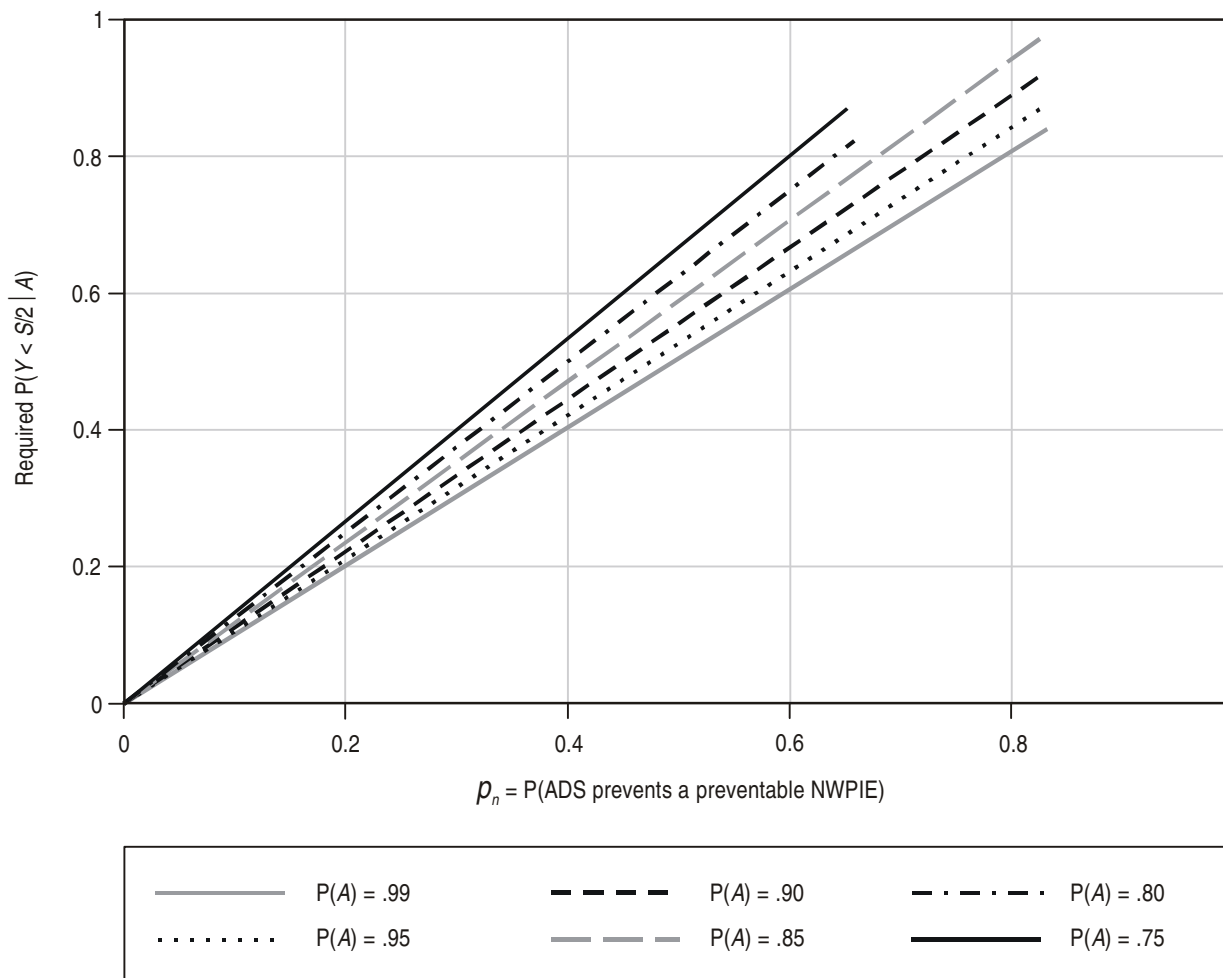
**Figure A-16-3.   Minimum acceptable probability that ADS prevents a
preventable NWPIE, given that the ADS cycle is completed**

By choosing conservative (or "worst-case") values of aircraft speed V, bank angle φ and deviation angle θ_d, the authority can construct a graph such as Figure A-16-4. (The most conservative values of V and θ_d are the largest values that might reasonably be expected; the most conservative value of φ is the smallest value that might be expected.) Figure A-16-4 shows values of $\dfrac{S/2 - B - \dfrac{V^2}{g \tan \varphi}(1 - \cos \theta_d)}{V \sin \theta_d}$ for six different combinations of

V and φ and for a range of deviation angles θ_d extending from 15 degrees to 90 degrees. This particular example takes the distance S between adjacent routes to be 30 NM and takes B, the buffer's half-width, to be 5.66 NM, which is approximately three standard deviations of typical lateral error for an aeroplane that just meets required navigation performance (RNP) level 4. Let t be the value that the airspace management authority derives from

Appendix 16. A method of deriving performance standards for automatic dependent
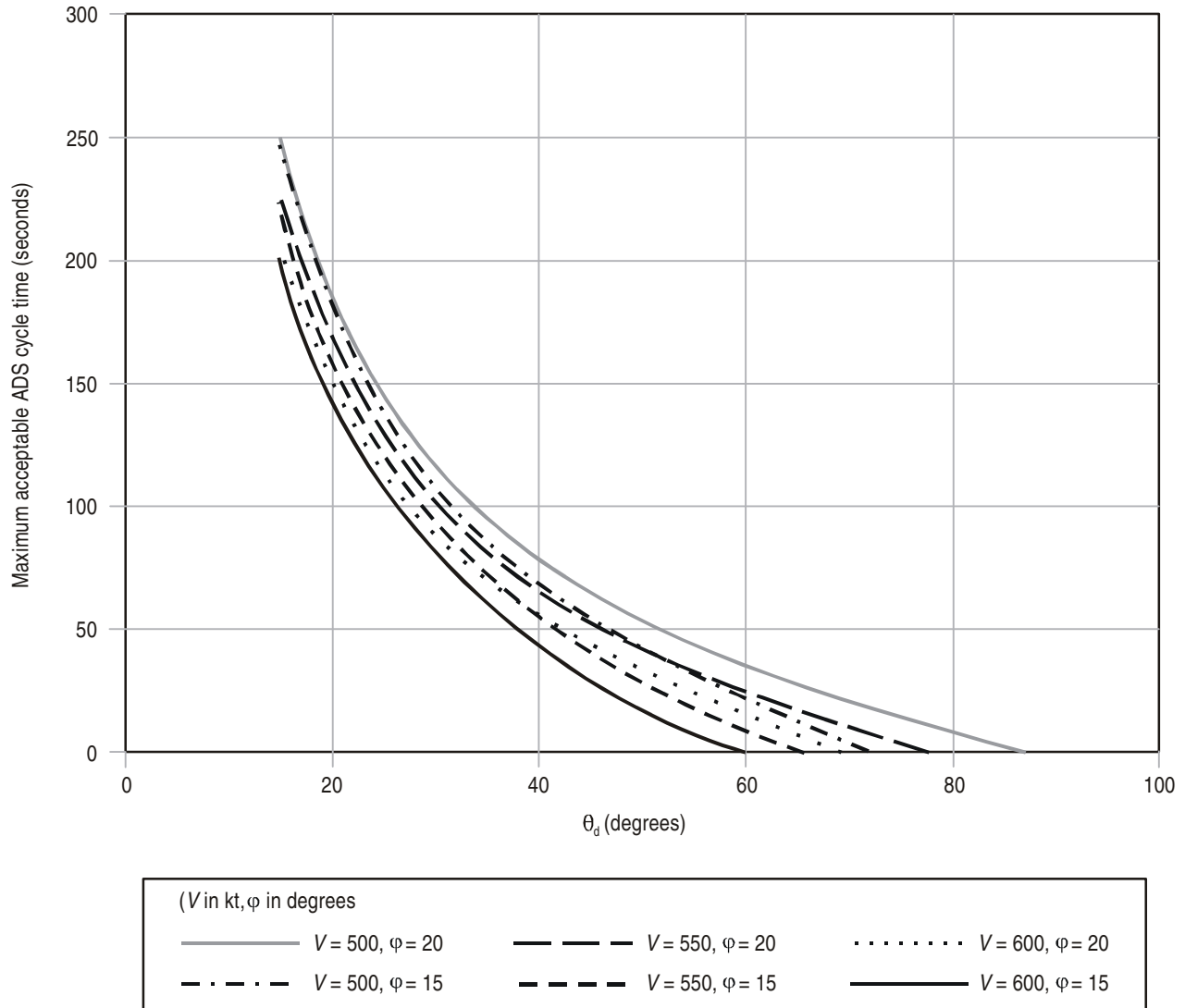*surveillance (ADS) systems*
171

Figure A-16-4.   Maximum acceptable ADS cycle time

a figure such as Figure A-16-4. Then the authority has, in essence, established a requirement that $\mathbf{P}\!\left(T_1 + D + C + U + I < t \mid A\right) \geq \mathbf{m}$ or, in other words, that

$$P(\text{ADS cycle time} < t \mid \text{the ADS cycle is completed}) \geq m . \tag{6}$$

3.3.3   The analysis of the preceding paragraphs does not include the effect of wind. It has been shown, however (reference 1) that the effect of wind on maximum ADS cycle time can be well approximated to within one second in almost all cases of interest, by using the maximum ground speed in place of airspeed in the above analysis. The airspace management authority can take the most conservative case by selecting the highest

airspeed V_A and wind speed V_W that might reasonably be expected and then using $V = V_A + V_W$ in equation (5). For example, the case of an aircraft with maximum airspeed $V_A = 500$ kt and maximum wind speed $V_W = 200$ kt can be satisfactorily approximated by using a value $V = 700$ kt in equation (5).

4. SUMMARY

By following the procedure detailed in sections 2 and 3 of this appendix, an airspace management authority that is planning to implement an ADS system can obtain values for five significant ADS performance parameters:

p_w — the minimum acceptable probability that the system prevents a waypoint-insertion lateral error;

p_n — the minimum acceptable probability that the system prevents a preventable non-waypoint-insertion lateral error;

P(*A*) — the minimum acceptable probability that the sequence of actions referred to in this appendix as an "ADS cycle" is completed;

t — a maximum acceptable time for completion of an ADS cycle; and

m — the minimum acceptable probability that an ADS cycle is completed in time *t*, given that it is completed.

5. REFERENCES

1. Silverman, B.H., *The Effect of Wind on Calculation of Maximum Allowable ADS Cycle Time,* SASP-WG/WHL/1-WP/3, 17 April 2002.

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Attachment to Appendix 16

DESCRIPTION OF A TURNING AEROPLANE

1.   This simple model of a turning aeroplane ignores the effect of winds and also ignores the brief period during which the aeroplane rolls into a bank. Let M (kg) denote the mass of an aeroplane, and let g (m/sec^2) denote the acceleration of gravity. While the aeroplane is flying straight and level, its wings, moving through the air, generate lift which equals its weight, Mg. That is, the wings generate a force of magnitude Mg, directed upward. If the aeroplane goes into a bank at a constant angle φ, it needs to increase lift in order to continue to generate a vertical force component Mg and thereby maintain its altitude. As is shown in Figure A-16-5, the force normal to the wings is then Mg sec φ, and the horizontal force on the aeroplane is Mg tan φ.Therefore, the magnitude of the aircraft's horizontal acceleration is g tan φ.

2.   Figure A-16-6 illustrates a rectangular coordinate system established so that when the aeroplane begins its turn, at time 0, it is at the origin and is headed along the X-axis (the horizontal axis) in the positive direction. Let t denote the time elapsed since the beginning of the turn. At any $t \geq 0$, let $(x(t), y(t))$ denote the aeroplane's position, let $\vec{v}(t) = \left(v_x(t), v_y(t)\right)$ denote its velocity, and let $\vec{a}(t) = \left(a_x(t), a_y(t)\right)$ denote its acceleration. Assume that the aeroplane maintains a constant speed V during its turn so that $V = \sqrt{v_x(t)^2 + v_y(t)^2}$ for all $t \geq 0$.

3.   At the beginning of the turn the aeroplane's velocity is $\vec{v}(0) = \left(v_x(0), v_y(0)\right) = (V,0)$. The horizontal component of the force generated by the wings is always directed at a right angle to the aeroplane's velocity vector. That is, the aeroplane's acceleration vector remains perpendicular to its velocity vector throughout the constant bank angle turn. Thus the initial acceleration $\vec{a}(0)$ must be $(0, g \tan \varphi)$ or $(0, -g \tan \varphi)$, depending on whether the turn is to the left or to the right. At an arbitrary time t, when the velocity vector lies along a line whose slope is $v_y(t)/v_x(t)$, the acceleration vector must lie along a line whose slope is the negative reciprocal of $v_y(t)/v_x(t)$, i.e. $-v_x(t)/v_y(t)$. The two unit vectors that lie along such a line are $\dfrac{\left(-v_y(t), v_x(t)\right)}{V}$ and $\dfrac{\left(v_y(t), -v_x(t)\right)}{V}$, and since the magnitude of the acceleration remains constant at $g \tan \varphi$ throughout the turn, $\vec{a}(t)$ must be either $g \tan \varphi \dfrac{\left(-v_y(t), v_x(t)\right)}{V}$ or $g \tan \varphi \dfrac{\left(v_y(t), -v_x(t)\right)}{V}$, depending on the direction of the turn. For the sake of argument, assume that the turn is to the left, as shown in Figure A-16-6.

Then:

$$\vec{a}(t) = g \tan \varphi \frac{\left(-v_y(t), v_x(t)\right)}{V}.$$

Since $\vec{a}(t) = \dfrac{d\vec{v}}{dt}(t)$, it follows that

$$\frac{dv_x}{dt}(t) = a_x(t) = \frac{-g \tan \varphi}{V} v_y(t), \text{ and } \frac{dv_y}{dt}(t) = a_y(t) = \frac{g \tan \varphi}{V} v_x(t).$$
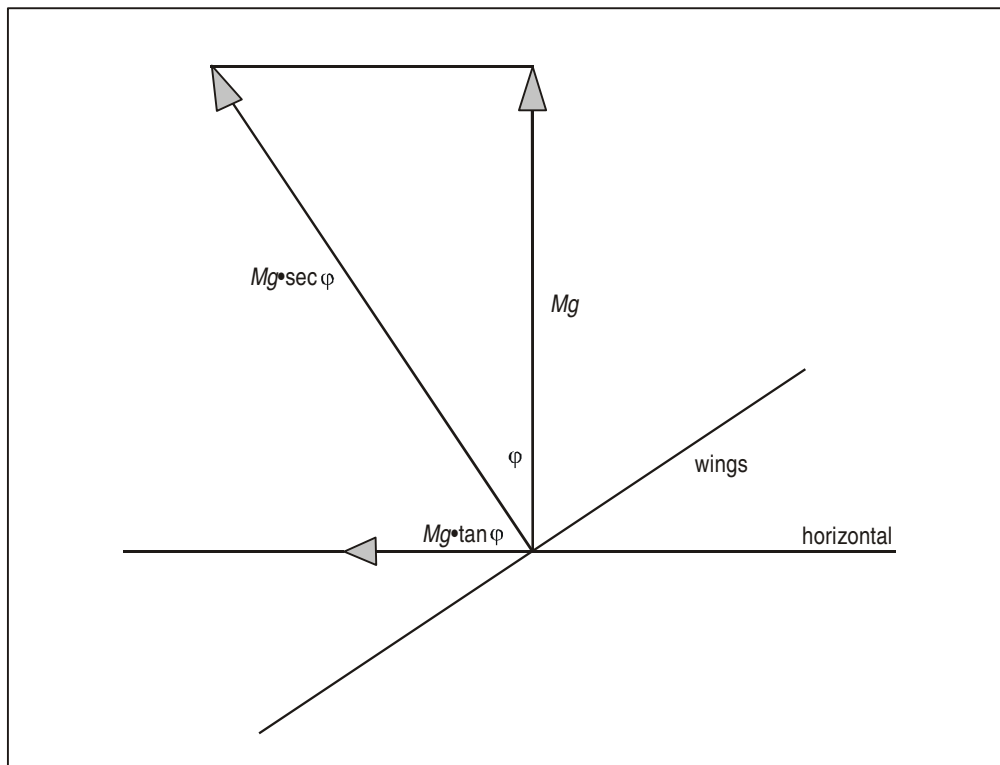
Figure A-16-5.   Forces acting on a banked aeroplane

These differential equations have solutions

$$v_x(t) = V \cos\left(\frac{g \tan \phi}{V} t \right)$$

and

$$v_y(t) = V \sin\left(\frac{g \tan \phi}{V} t \right),$$

which also fit the initial conditions $v_x(0) = V$ and $v_y(0) = 0$.

4.   The aeroplane begins its turn at the origin, $(0,0)$. At time t, its position, $(x(t), y(t))$, satisfies $x(t) = \int_0^t v_x(u)\,du$ and $y(t) = \int_0^t v_y(u)\,du$. Substituting the expressions for v_x and v_y obtained in the last paragraph then yields two simple integrations, the results of which are

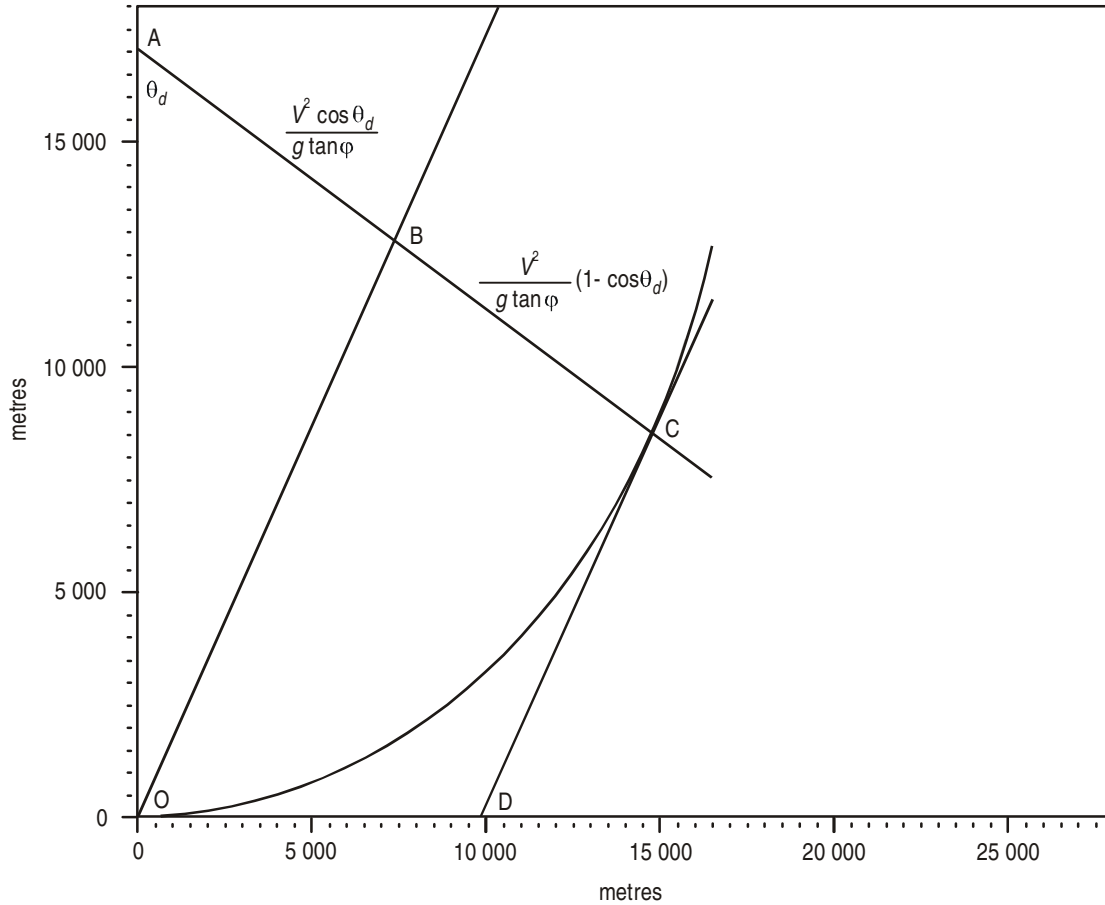$$x(t) = \frac{V^2}{g \tan \phi} \sin\left(\frac{g \tan \phi}{V} t \right)$$

and

**Figure A-16-6.   Lateral movement of an aeroplane
while turning back toward its assigned route**

$$y(t) = \frac{V^2}{g \tan \phi} \left(1 - \cos\left(\frac{g \tan \phi}{V} t \right) \right).$$

That is, at time t the aeroplane's position is

$$(x(t), y(t)) = \frac{V^2}{g \tan \phi} \left(\sin\left(\frac{g \tan \phi}{V} t \right), 1 - \cos\left(\frac{g \tan \phi}{V} t \right) \right).$$

5.   At time t the aeroplane's distance from the point $\left(0, \dfrac{V^2}{g \tan \phi} \right)$ is then

$$\sqrt{ \left(\frac{V^2}{g \tan \phi} \sin\left(\frac{g \tan \phi}{V} t \right) \right)^2 + \left(\frac{V^2}{g \tan \phi}\left(1 - \cos\left(\frac{g \tan \phi}{V} t \right) \right) - \frac{V^2}{g \tan \phi} \right)^2 },$$

which is easily simplified to $\dfrac{V^2}{g\tan\phi}$. Since this distance is *not* a function of t, but is a constant, the aircraft's

path during its turn must lie along the circle of radius $\dfrac{V^2}{g\tan\phi}$ about the point $\left(0, \dfrac{V^2}{g\tan\phi}\right)$. In Figure A-16-6

the aeroplane's path is shown as arc OC, and the circle's centre, $\left(0, \dfrac{V^2}{g\tan\phi}\right)$, is labelled point A.

6.   When the aircraft begins its turn, it is headed along the X-axis, so that the angle between its velocity vector and the X-axis is 0. As the turn progresses, the velocity vector rotates away from the X-axis. Since the aeroplane's path lies along a circle, the central angle subtended by the path at time t, $\theta(t)$, is also the angle by which the velocity vector has rotated away from the X-axis. The tangent of that angle is $\dfrac{v_y(t)}{v_x(t)}$, and the

expressions derived above for $v_x(t)$ and $v_y(t)$ immediately yield the result $\tan\theta = \tan\left(\dfrac{g\tan\phi}{V}t\right)$, from which

it follows (since the tangent function is one-to-one on $\left[0, \dfrac{\pi}{2}\right)\sigma$) that $\theta = \dfrac{g\tan\phi}{V}t$, or $t = \dfrac{V\theta}{g\tan\phi}$.

7.   Let θ_d denote the angle at which the aeroplane deviates from its cleared route before beginning its turn. It needs to turn through the same angle before it can begin to head back toward the route. It reaches its maximum distance from the route at the instant when it has turned through that angle and is (momentarily) heading parallel to the route. In Figure A-16-6, that occurs when the aeroplane is at point C. The tangent to the circle at C intersects the X-axis at a point D, so line segment $\overline{CD}$ is parallel to the cleared route. Line segment $\overline{OB}$ is drawn parallel to segment $\overline{CD}$ (and is therefore also parallel to the cleared route), and it passes through O = (0,0), the point at which the aeroplane began its turn.

8.   Since $\overline{CD}$ is tangent to the circle at C, the radius $\overline{AC}$ is perpendicular to $\overline{CD}$, $\overline{OB}$ and the cleared route of flight, and so the length of segment $\overline{BC}$ is the distance that the aeroplane moved away from the cleared route between the time it began its turn and the time it started moving back toward the route. Triangle ABO is a right

triangle whose hypotenuse $\overline{AO}$ has length $\dfrac{V^2}{g\tan\phi}$, and angle OAB has measure θ_d. Thus the length of segment

$\overline{AB}$ must be $\dfrac{V^2}{g\tan\phi}\cos\theta_d$. Since the radius $\overline{AC}$ has length $\dfrac{V^2}{g\tan\phi}$, the length of segment $\overline{BC}$ must be

$\dfrac{V^2}{g\tan\phi}(1-\cos\theta_d)$.

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ICAO TECHNICAL PUBLICATIONS

The following summary gives the status, and also describes in general terms the contents of the various series of technical publications issued by the International Civil Aviation Organization. It does not include specialized publications that do not fall specifically within one of the series, such as the Aeronautical Chart Catalogue or the Meteorological Tables for International Air Navigation.

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Regional Supplementary Procedures (SUPPS) have a status similar to that of PANS in that they are approved by the Council, but only for application in the respective regions. They are prepared in consolidated form, since certain of the procedures apply to overlapping regions or are common to two or more regions.

The following publications are prepared by authority of the Secretary General in accordance with the principles and policies approved by the Council.

Technical Manuals provide guidance and information in amplification of the International Standards, Recommended Practices and PANS, the implementation of which they are designed to facilitate.

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