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Doc 9501

Environmental Technical Manual

Volume III — Procedures for the CO_2 Emissions Certification of Aeroplanes
Second Edition, 2020

Approved by and published under the authority of the Secretary General

INTERNATIONAL CIVIL AVIATION ORGANIZATION

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AMENDMENTS

Amendments are announced in the supplements to the *Products and Services Catalogue;* the Catalogue and its supplements are available on the ICAO website at www.icao.int. The space below is provided to keep a record of such amendments.

RECORD OF AMENDMENTS AND CORRIGENDA

AMENDMENTS		
No.	Date	Entered by

CORRIGENDA		
No.	Date	Entered by

FOREWORD

The *Environmental Technical Manual* (Doc 9501), Volume III — *Procedures for the CO_2 Emissions Certification of Aeroplanes*, Second Edition, includes material that has been approved by the ICAO Committee on Aviation Environmental Protection (CAEP) during its eleventh meeting (CAEP/11) in February 2019. This manual is to be periodically revised under the supervision of the CAEP Steering Group and is intended to make the most recent information available to certificating authorities, aeroplane certification applicants and other interested parties in a timely manner, aiming at achieving the highest degree of harmonization possible. The technical procedures and equivalent procedures described in the manual are consistent with currently accepted techniques and modern instrumentation. This edition and subsequent revisions that may be approved by the CAEP Steering Group will be posted on the ICAO website (http://www.icao.int/) under "publications" until the latest approved revision is submitted to CAEP for formal endorsement and subsequent publication by ICAO.

Comments on this manual, particularly with respect to its application and usefulness, would be appreciated from all States. These comments will be taken into account in the preparation of subsequent editions. Comments concerning this manual should be addressed to:

The Secretary General
International Civil Aviation Organization
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Montréal, Québec, Canada H3A 2R2

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ACRONYMS AND ABBREVIATIONS

A	Area (m^2)
CAEP	Committee on Aviation Environmental Protection
C_D	Drag coefficient
CFD	Computational fluid dynamics
CG	Centre of gravity
CI	Confidence interval
C_L	Lift coefficient
CO_2	Carbon dioxide
CO_2 MV	CO_2 emissions evaluation metric value (kg/km)
ETM	Environmental technical manual
g	Gravitational acceleration (m/s^2)
h	Altitude (m)
LHV	Lower heating value (MJ/kg)
M	Mach number
MTOM	Maximum take-off mass (kg)
MV	Metric value
P	Pressure (Pa)
Re	Radius of the Earth (m)
RE	Reynolds number
RGF	Reference geometric factor
SAR	Specific air range (km/kg)
SEM	Semi-empirical methods
SFC	Specific fuel consumption (kg/h/N)
STC	Supplemental Type Certificate
T	Temperature (K)
TAS	True airspeed (km/h)
TC	Type Certificate
TOM	Take-off mass (kg)
V	Speed (m/s)
Wf	Total aeroplane fuel flow (kg/h)
W	Weight (N)
WV	Weight variant
δ	Ratio of atmospheric pressure at a given altitude to the atmospheric pressure at sea level
φ	Latitude (degrees)
ρ	Density (kg/m^3)
σ	Ground track angle (degrees) / Standard deviation in confidence interval section 3.3
ζ	Degrees of freedom in confidence interval section 3.3
μ	Mean in confidence interval section 3.3

Chapter 1

INTRODUCTION

1.1 PURPOSE

The aim of this manual is to promote uniformity in the implementation of the technical procedures of Annex 16 — *Environmental Protection*, Volume III — *Aeroplane CO_2 Emissions* by providing: 1) guidance to certificating authorities, applicants and other interested parties regarding the intended meaning and stringency of the Standards in the current edition of the Annex; 2) guidance on specific methods that are deemed acceptable in demonstrating compliance with those Standards; and 3) equivalent procedures resulting in effectively the same CO_2 emissions evaluation metric that may be used in lieu of the procedures specified in the appendices of Annex 16, Volume III.

1.2 DOCUMENT STRUCTURE

1.2.1 Chapter 1 provides general information regarding the use of this manual. Chapter 2 provides general guidelines on the interpretation of Annex 16, Volume III. Chapter 3 brings technical guidelines for the certification of aeroplanes against Annex 16, Volume III, including equivalent procedures.

1.2.2 Guidance is provided in the form of explanatory information, acceptable methods for showing compliance, and equivalent procedures.

1.3 EQUIVALENT PROCEDURES

1.3.1 The procedures described in the Annex, as supplemented by the means of compliance information provided in this manual, shall be used unless an equivalent procedure is approved by the certificating authority. Equivalent procedures should not be considered as limited only to those described herein, as this manual will be expanded as new equivalent procedures are developed. Also, their presentation does not imply limitation of their application or commitment by certificating authorities to their further use.

1.3.2 The use of equivalent procedures may be requested by applicants for many reasons, including:

a) to make use of previously acquired or existing data for the aeroplane; and

b) to minimize the costs of demonstrating compliance with the requirements of Annex 16, Volume III by keeping aeroplane test time, equipment and personnel costs to a minimum.

1.4 EXPLANATORY INFORMATION

Explanatory information has the following purpose:

a) to explain the intent of the Annex 16, Volume III Standards;

b) to state current policies of certificating authorities regarding compliance with the Annex; and

c) to provide information on critical issues concerning approval of applicants' compliance methodology proposals.

1.5 CONVERSION OF UNITS

Conversions of some non-critical numerical values between United States Customary (English) and SI units are shown in the context of acceptable approximations.

1.6 REFERENCES

1.6.1 Unless otherwise specified, references throughout this document to "the Annex" relate to Annex 16 — *Environmental Protection*, Volume III — *Aeroplane* CO_2 *Emissions*, Second Edition**.**

1.6.2 References to sections of this manual are defined only by the section number to which they refer. References to documents other than the Annex are numbered sequentially (e.g., Reference 1, Reference 2, etc.). A list of these documents is provided in Appendix 1 of this manual and a bibliography can be found in Appendix 2.

Chapter 2

GENERAL GUIDELINES

2.1 APPLICABILITY OF ANNEX 16, VOLUME III

2.1.1 The *Convention on International Civil Aviation*, Article 3 specifically states that it is not applicable to state aircraft and provides some examples (see below), but this can also include specific flights carrying official government representatives:

"*a)* This Convention shall be applicable only to civil aircraft, and shall not be applicable to state aircraft.

b) Aircraft used in military, customs and police services shall be deemed to be state aircraft."

2.1.2 In addition, Annex 16, Volume III, Part II, Chapter 2, 2.1 excepts amphibious aeroplanes, aeroplanes initially designed or modified for specialized operational requirements and used as such, aeroplanes designed with zero reference geometric factor (RGF), and those aeroplanes specifically designed or modified and used for fire-fighting purposes. These are typically special categories of aeroplanes which are limited in numbers and have specific technical characteristics resulting in very different CO_2 metric values compared to all other aeroplane types in the proposed applicability scope.

2.1.3 Examples of specialized operational requirements include:

a) aeroplanes that are initially certified as civil aeroplanes during the production process but immediately converted to military aeroplanes;

b) a required capacity to carry cargo that is not possible by using less specialized aeroplanes (e.g. ramped, with back cargo door);

c) a required capacity for very short or vertical take-offs and landings;

d) a required capacity to conduct scientific, research or humanitarian missions exclusive of commercial service; or

e) similar factors.

2.1.4 Type designs to be certified

2.1.4.1 Annex 16, Volume III, Part I, Chapter 1 defines maximum take-off mass (MTOM) as *"*the highest of all take-off masses for the type design*"*. Part II, Chapter 2, 2.3 defines the three reference masses at which the 1/SAR value shall be established, and these masses are calculated based on the MTOM.

2.1.4.2 Applicants may develop multiple take-off mass (TOM) variants of a specific type design for operational purposes. As stated above, only the highest MTOM of a specific airframe/engine combination is required to be certified against Annex 16, Volume III. As stated in Annex 16, Volume III, Part II, Chapter 2, 2.3.2, certification at MTOM also certifies all TOM variants. These TOM variants would have the same CO_2 emissions evaluation metric value as MTOM.

2.1.4.3 Annex 16, Volume III, Part II, Chapter 2, 2.3.2 also states that *"*applicants may voluntarily apply for the approval of CO_2 metric values for take-off masses less than MTOM." The purpose of this statement is to allow the applicant to apply for approval of a separate CO_2 emissions evaluation metric value for a TOM lower than MTOM. In that case, the reference aeroplane masses and the maximum permitted CO_2 emissions evaluation metric value would be based on the TOM instead of MTOM. The CO_2 emissions evaluation metric value for this TOM could then also be used for any TOM variant of even lower mass. Applicants can apply for approval for separate CO_2 emissions evaluation metric values for as many, or as few, TOM variants as they desire.

2.1.5 Appropriate margin to regulatory level

2.1.5.1 If an applicant chooses to voluntarily certify a lower TOM variant, as discussed in 2.1.4, it should be kept in mind that an underlying principle in applying the CO_2 Standard is that the highest weight variant (MTOM) has the lowest margin to the regulatory limit level. The 1/SAR value used in the CO_2 metric system is calculated as an average of three reference masses (high, medium and low).

2.1.5.2 In establishing the reference conditions for specific air range (SAR) determination, it is expected that the highest SAR value will be sought at the maximum range cruise condition at the optimum altitude (Annex 16, Volume III, Chapter 2, 2.5). It is noted that a greater non-linearity in the 1/SAR versus mass relationship could be introduced by a constraint unrelated to the aerodynamic and propulsive efficiency of the aeroplane (e.g. an altitude pressurization limitation). In this instance, particular care should be taken to ensure that that the principle of the highest weight variant having the lowest margin to the regulatory limit level continues to hold.

2.2 CHANGE TO AN AEROPLANE TYPE DESIGN

2.2.1 Change to a CO_2-certified aeroplane type design

2.2.1.1 Annex 16, Volume III, Part I, Chapter 1 includes the following definition:

"***Derived version of a CO_2-certified aeroplane.*** An aeroplane which incorporates a change in the type design that either increases its maximum take-off mass, or that increases its CO_2 emissions evaluation metric value by more than:

a) 1.35 per cent at a maximum take-off mass of 5 700 kg, decreasing linearly to;

b) 0.75 per cent at a maximum take-off mass of 60 000 kg, decreasing linearly to;

c) 0.70 per cent at a maximum take-off mass of 600 000 kg; and

d) a constant 0.70 per cent at maximum take-off masses greater than 600 000 kg.

Note.— In some States, where the certificating authority finds that the proposed change in design, configuration, power or mass is so extensive that a substantially complete investigation of compliance with the applicable airworthiness regulations is required, the aeroplane requires a new Type Certificate."

2.2.1.2 The Note clarifies that it is the certificating authority, based on airworthiness regulations, that determines whether or not a change requires an application for a new Type Certificate (ref. ANAC RBAC 21.19, EASA Part 21.A.19, FAA: Title 14 of the Code of Federal Regulations, Chapter I, Subchapter C, Part 21.19, IAC AP-21 Subpart B, paragraph 21/19, TCCA CAR 521.153). If the change requires a new Type Certificate from the airworthiness perspective, it shall be certified in accordance with 2.1.1 a), b) or c) of the Annex considering the date of application for this new Type Certificate.

2.2.1.3 Conversely, if the certificating authority does not determine a change requires an application for a new Type Certificate, then it is a derived version for the CO_2 emissions certification perspective if the conditions of 2.2.1.1 of this ETM are met. In the derived version case, the change will be certified using the same amendment of Annex 16, Volume III and the same limit line as the aeroplane type design from which it is derived, or any later amendment at the option of the applicant.

2.2.1.4 According to the definition quoted in 2.2.1.1, a CO_2-certified aeroplane type design that incorporates a change in the type design that increases its MTOM shall be considered a derived version, and the applicant shall demonstrate compliance with Annex 16, Volume III. In addition, a CO_2-certified aeroplane type design that incorporates a change in the type design that increases its certified CO_2 emissions evaluation metric value by more than the above-mentioned thresholds shall be considered a derived version, and the applicant shall demonstrate compliance with Annex 16, Volume III.

2.2.1.5 A change to a CO_2-certified aeroplane type design that neither increases its MTOM, nor increases its CO_2 emissions evaluation metric value by more than the above-mentioned thresholds, is considered a no-CO_2 change. This definition of the no-CO_2 change thresholds is also referred to as the “no-CO_2-change criterion”. For a no-CO_2 change, the CO_2 emissions evaluation metric value of the changed type design shall be considered the same as the parent type design.

2.2.1.6 Visualization of the no-CO_2-change criterion thresholds is provided in Figure 2-1. The trend line equations may be used to evaluate what the no-CO_2 change threshold is for any MTOM.

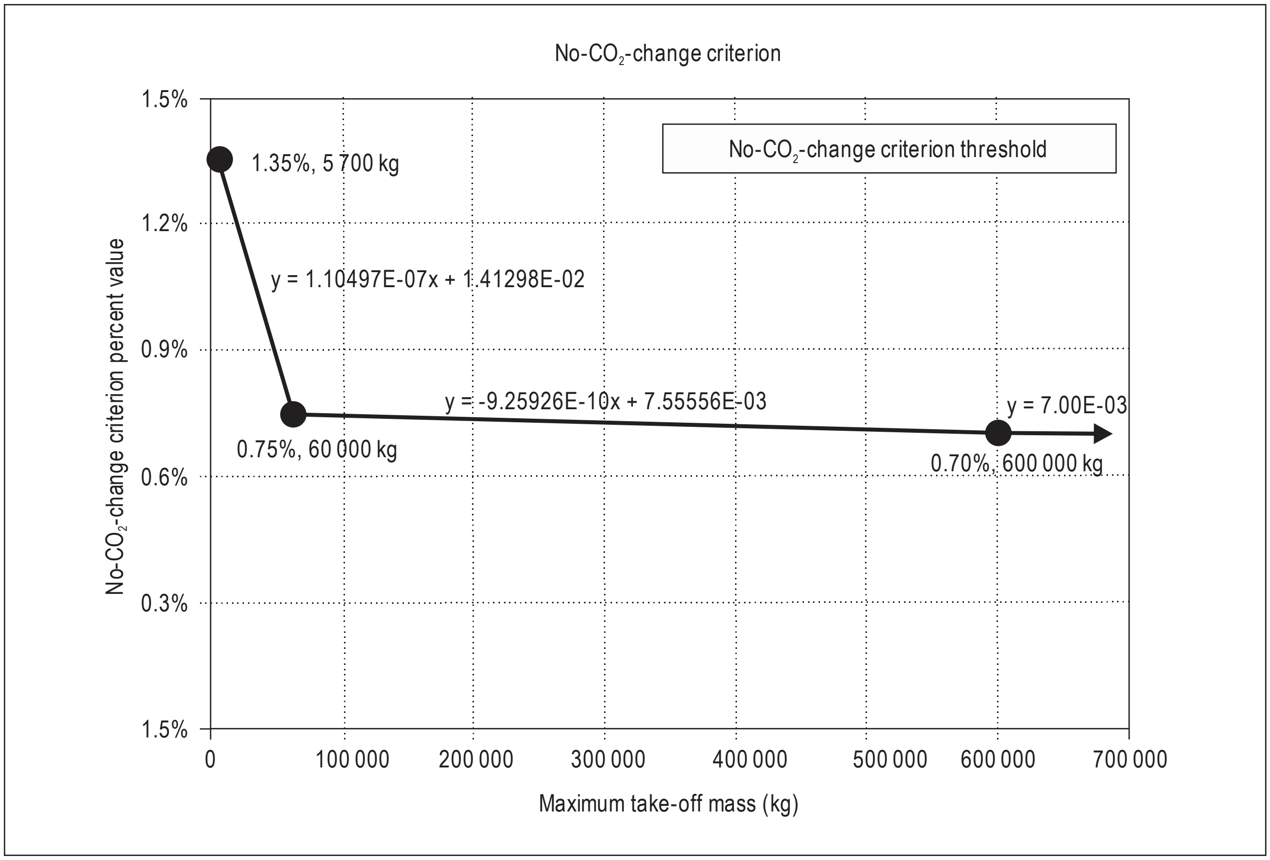


Figure 2-1. Visualization of the no-CO_2-change criterion

2.2.1.7 The evaluation of some changes can be done by simpler equivalent procedures, as detailed in 3.4.2.

2.2.1.8 *Assessment of a change in the CO_2 approved aeroplane type design against the no-CO_2-change criterion*

The CO_2 emissions Standard was developed taking into account the principle of not tracking cumulative changes to a type design, and this principle was considered when the thresholds of the no-CO_2-change criterion were established. This implies that each individual change in the type design should be assessed against the thresholds of the no-CO_2-change criterion, regardless of other changes already certified or to be certified.

2.2.1.9 This principle does not preclude manufacturers from grouping several modifications on the aeroplane into one change to the type design when assessing the impact to the CO_2 metric value against the no-CO_2-change criterion. Any assessment of a grouping of modifications does not apply to each modification individually, or to a different grouping of modifications.

2.2.2 Certification process for a change to the type design

2.2.2.1 It is the responsibility of each certification authority to implement Annex 16, Volume III within their domestic type certification process. The process for approving changes to a type design may depend on a number of elements. In general, the certification authority would review each change to decide whether or not it is an adverse CO_2 change. If the adverse change is above the no-CO_2 change threshold, the authority would then approve the new CO_2 metric value (CO_2 MV) if compliance can be demonstrated. However, there are often specific arrangements (e.g. processes, change classifications), put in place by a certification authority to ensure effective and efficient implementation, that can influence the level of involvement of the certification authority in this process. For example, an applicant (e.g. original equipment manufacturers) with appropriate expertise and processes in place may be granted privileges by the certification authority to approve no-CO_2 changes themselves. The application of this privilege by the design organization is then audited by the certification authority on a regular basis in order to renew this approval of the process. In line with the above, Table 2-1 below provides an overview of the general process and an example of a potential specific process that could be used by the certification authority and the applicant requesting approval of a design change.

Table 2-1. Process to address design changes

Adverse change on the CO_2 MV	*General process*	*Example of specific process*
Case 1: Type design change has an adverse effect on the CO_2 MV and is BELOW X per cent[1] of the no-CO_2 change threshold.	Certification authority reviews application for type design change and approves the no change in the CO_2 MV if compliance against the requirements has been demonstrated.	Applicant has a process that has been approved by the certification authority to review the design change and approve the no change in the CO_2 MV if compliance against the requirements has been demonstrated.
Case 2: Type design change has an adverse effect on CO_2 MV that is ABOVE X per cent and BELOW 100 per cent of the no-CO_2 change threshold.		Applicant has been granted privileges by the certification authority to review the design change and approve the no change in the CO_2 MV if compliance has been demonstrated. Applicant could consult with the certification authority to confirm that the no-CO_2 change threshold has not been exceeded when the metric value (MV) is near to the no-CO_2 change threshold. OR Certification authority reviews application for type design change and approves the no change in the CO_2 MV if compliance against the requirements has been demonstrated.
Case 3: Type design change has an adverse effect on CO_2 MV and is ABOVE the no-CO_2 change threshold.	Certification authority reviews application for design change and approves the new CO_2 MV if compliance against the requirements has been demonstrated.	Certification authority reviews application for design change and approves the new CO_2 MV if compliance against the requirements has been demonstrated.

[1] As guidance, X per cent could equal 50 per cent.

2.2.2.2 For Case 3, the applicant would need to submit a certification report in order for the certification authority to approve the new CO_2 MV. In the case where the no-CO_2 change threshold is not exceeded, two cases with different margins to the no-CO_2 change threshold are considered.

2.2.2.3 If the type design change leads to a decrease in the CO_2 MV, there are two possibilities for the applicant:

a) if the applicant wishes to gain credit for this reduction, then the applicant may voluntarily apply for this change to be approved by the certification authority following the process as described in Case 3; or

b) if the applicant does not wish to gain credit for this reduction, then the applicant would follow the process as described in Case 1.

2.2.2.4 Bilateral agreements between Contracting States, following a review of each other's certification systems, may also influence the process linked to the validation of type design changes by other Contracting States.

2.2.2.5 In all of the above cases, it is the responsibility of the certification authority to establish the rules for the classification of CO_2 design changes and their level of involvement.

2.2.3 Change to a non-CO_2-certified aeroplane type design

2.2.3.1 This section is intended to provide guidelines in relation to derived versions of non-CO_2-certified aeroplanes. Annex 16, Volume III, Part I, Chapter 1 includes the following definition:

"***Derived version of a non-CO_2-certified aeroplane.*** *An individual aeroplane that conforms to an existing Type Certificate, but which is not certified to Annex 16, Volume III, and to which a change in the type design is made prior to the issuance of the aeroplane's first certificate of airworthiness that increases its CO_2 emissions evaluation metric value by more than 1.5 per cent or is considered to be a significant CO_2 change.*"

2.2.3.2 This definition of a derived version of a non-CO_2 certified aeroplane is the criteria to trigger the certification of an aeroplane type design, for which the baseline type design was not certified against the CO_2 Standard. This is captured in the Annex in Part II, Chapter 2, 2.1.1 d) and e):

"*d) derived versions of non-CO_2-certified subsonic jet aeroplanes, including their subsequent CO_2-certified derived versions, of greater than 5 700 kg maximum certificated take-off mass for which the application for certification of the change in type design was submitted on or after 1 January 2023;*

e) derived versions of non-CO_2 certified propeller-driven aeroplanes, including their subsequent CO_2-certified derived versions, of greater than 8 618 kg maximum certificated take-off mass, for which the application for certification of the change in type design was submitted on or after 1 January 2023;"

2.2.3.3 Following the certification to the CO_2 Standard of this changed type design, any subsequent application for certification of a change (after this type design was certified for the CO_2 Standard) will be assessed against the derived version of a CO_2-certified aeroplane criteria in order to decide if a new MV determination is mandatory or not. A visual representation of this concept is shown in Figure 2-2 below.

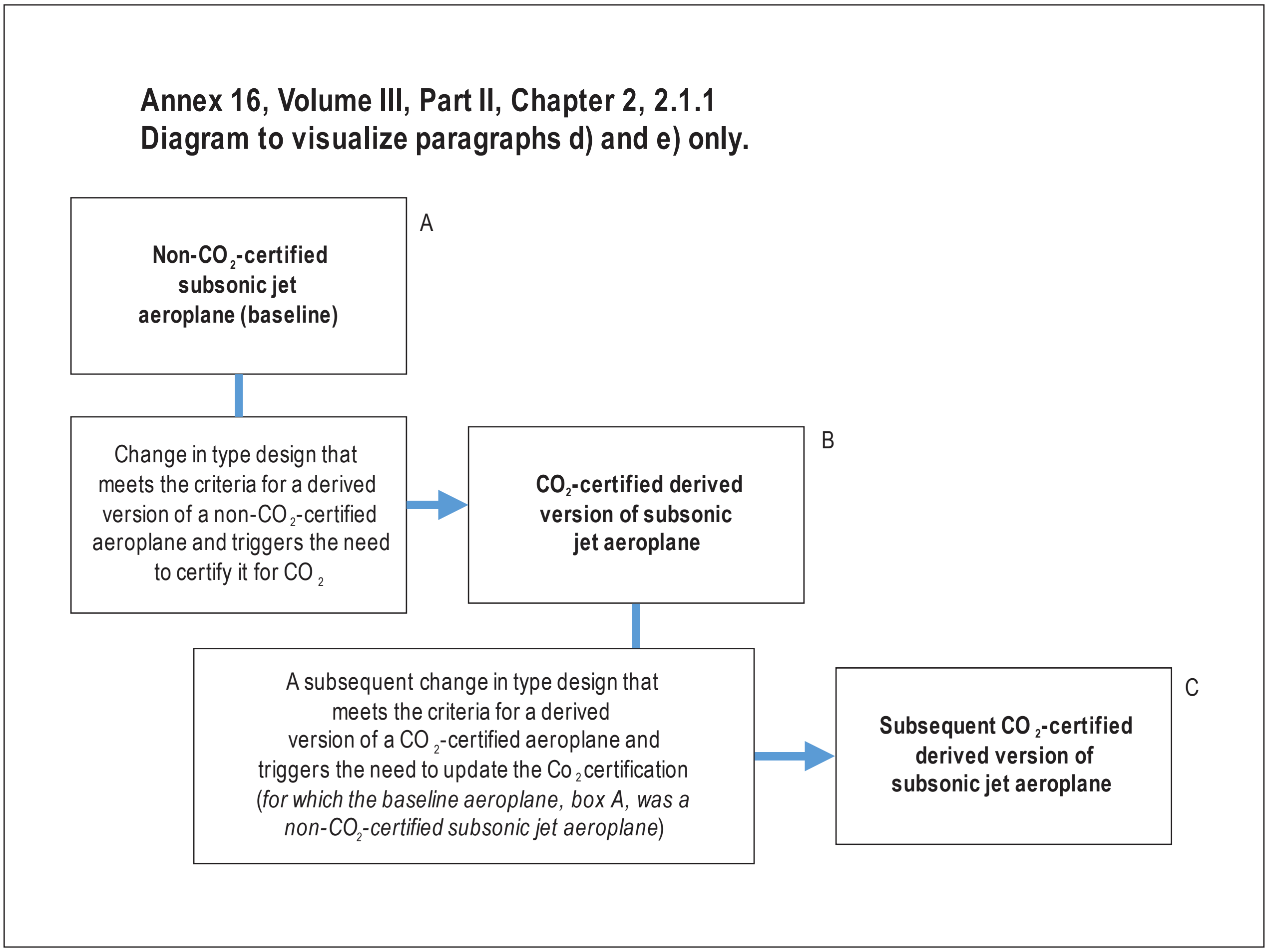


Figure 2-2. Visual representation of "derived versions of non-CO_2-certified subsonic jet aeroplanes, including their subsequent CO_2-certified derived versions"

2.2.3.4 Guidance on the derived versions of CO_2-certified aeroplane type designs is in section 2.2.1 of this ETM.

2.3 CO_2 EMISSIONS EVALUATION METRIC COMPLIANCE DEMONSTRATION PLANS

Prior to undertaking a CO_2 certification demonstration, the applicant should submit to the certificating authority a CO_2 compliance demonstration plan. This plan contains a complete description of the methodology and procedures by which an applicant proposes to demonstrate compliance with the CO_2 certification Standards specified in Annex 16, Volume III. Approval of the plan and the proposed use of any equivalent procedures or technical procedures not included in the Annex remains with the certificating authority. CO_2 compliance demonstration plans should include the following information:

a) *Introduction.* A description of the aeroplane CO_2 certification basis.

b) *Aeroplane description.* Type, model number and the specific design to be certificated.

Note.— The certificating authority should require that the applicant demonstrate and document the conformity of the test aeroplane, particularly with regard to those parts which might affect its CO_2 emissions evaluation metric.

c) *Aeroplane CO_2 certification methodology.* Means of compliance, equivalent procedures from this manual, and technical procedures from Annex 16, Volume III.

d) *Plans for tests.* The plans for test should include:

1) *Test description.* Test methods to comply with the test environment and flight path conditions of the Annex, as appropriate.

Note.— Plans for tests shall either be integrated into the basic CO_2 compliance demonstration plan or submitted separately and referenced in the basic plan.

e) *Deliverables*. List the documents that should show compliance with Annex 16, Volume III (test and analysis reports, including RGF determination).

2.4 ENGINE INTERMIX

2.4.1 Applicants will typically demonstrate compliance with the Standards in Annex 16, Volume III, Chapter 2 for an aeroplane type design where all engines are of the same design. However, an applicant may wish to demonstrate compliance of an aeroplane type design where not all the engines are of the same design. Such a design is commonly referred to as an "engine intermix" arrangement.

2.4.2 In such a case, the applicant may, subject to the approval of the certificating authority, demonstrate compliance in one of three ways:

a) in accordance with the test procedures defined in Annex 16, Volume III, Chapter 2, 2.6, and for which the test aeroplane shall be representative of the intermix arrangement for which certification is requested; or

b) in cases where the CO_2 metric value has been established for aeroplanes on which each of the intermix engine models has been exclusively installed, compliance can be demonstrated on the basis of either:

1) the average of the CO_2 emissions evaluation metrics for aeroplanes on which each of the intermix engine models has been exclusively installed; or

2) the highest CO_2 emissions evaluation metric for aeroplanes on which each of the intermix engine models has been exclusively installed.

Note.— Annex 16, Volume III, Chapter 2, 2.1.2 states that in the case of time-limited engine changes, Contracting States may not require a demonstration of compliance with the Standards of Annex 16, Volume III.

2.5 EXEMPTIONS

2.5.1 Introduction

2.5.1.1 Annex 16, Volume III, Part II, Chapter 2, 2.1.3 raises the possibility for certificating authorities to exempt aeroplane units from the applicability requirements in Annex 16, Volume III, Part II, Chapter 2, 2.1.1 a) to g).

2.5.1.2 In addition, Part II, Chapter 1, 1.11 indicates that Contracting States shall recognize valid exemptions agreed by another Contracting State provided that the process for granting exemption is acceptable. It is recommended to follow the acceptable process and criteria as described in this ETM. For example, certificating authorities may decide to exempt low volume production aeroplanes in exceptional circumstances, taking into account the justifications listed in 2.5.2.1 c).

2.5.1.3 In order to promote a harmonized global approach to the granting, implementing and monitoring of these exemptions, this section provides guidelines on the process and criteria for issuing exemptions from the CO_2 Standard agreed at CAEP/10 (Part II, Chapter 2, 2.4).

2.5.2 Exemption process

2.5.2.1 *Application*

In order for the competent authority to review an application, the applicant should submit to the competent authority a formal application letter for the manufacture of the exempted aeroplanes with a copy to all other relevant organizations and involved competent authorities. The letter should include the following information in order for the competent authority to be in a position to review the application:

a) *Administration*

   1) name, address and contact details of the applicant.

b) *Scope of application for exemptions*

   1) aeroplane type (e.g. new or in-production type, model designation, Type Certificate (TC) number and TC date);

   2) number of aeroplane exemptions requested;

   3) anticipated duration (end date) of continued production of exempted aeroplanes;

   4) designation of to whom the aeroplanes will be originally delivered.

c) *Justification for the exemptions*. In applying for an exemption, an applicant should, to the extent possible, address the following factors (with quantification) in order to support the merits of the exemption request:

   1) technical issues from an environmental and airworthiness perspective which may have delayed compliance;

   2) economic impacts on the manufacturer, operator(s) and the aviation industry at large;

3) environmental effects. This should consider the amount of additional CO_2 that will be emitted as a result of the exemption, including items such as the amount by which the aeroplane model exceeds the Standard, taking into account any other aeroplane models in the aeroplane family covered by the same Type Certificate and their relation to the Standard;

4) interdependencies. The impact of changes to reduce CO_2 on other environmental factors, including community noise and NOx emissions;

5) the impact of unforeseen circumstances and hardship due to business circumstances beyond the manufacturer's control (e.g. employee strike, supplier disruption or calamitous event);

6) projected future production volumes and plans for producing a compliant version of the aeroplane model for which exemptions are sought;

7) for new type design aeroplanes only, provide a demonstration that the maximum use of fuel efficient technology relative to CAEP/10 new types regulatory limit was reasonably applied to the design of the aeroplane;

8) equity issues in granting exemptions among economically competing parties (e.g. provide the rationale for granting an exemption when another manufacturer has a compliant aeroplane and does not need an exemption, taking into account the implications for operator fleet composition, commonality and related issues in the absence of the aeroplane for which exemptions are sought); and

9) any other relevant factors.

2.5.2.2 *Evaluation criteria*

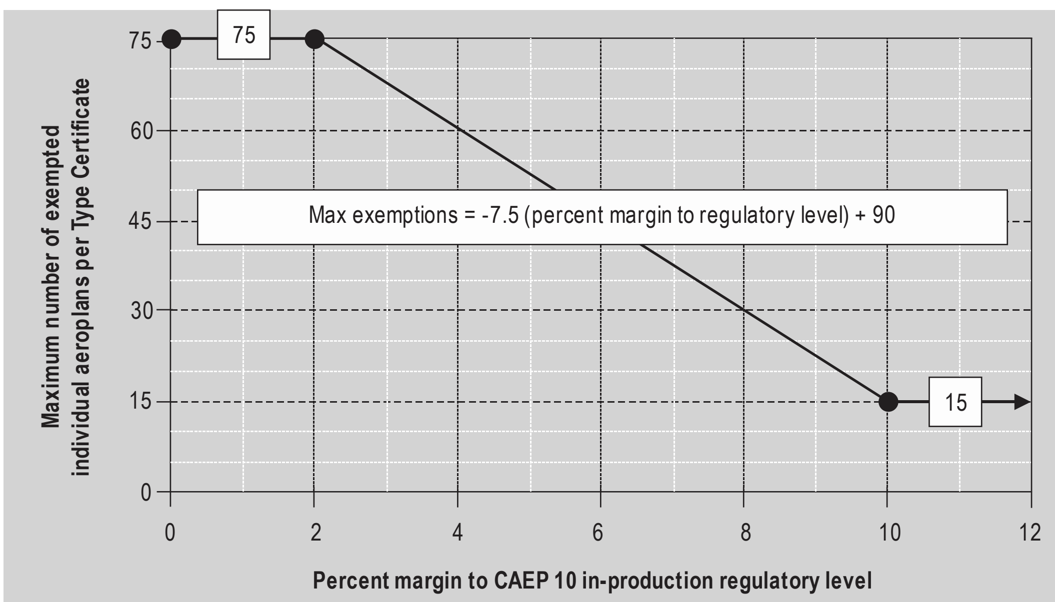
2.5.2.2.1 The evaluation of an exemption application should be based on the justification provided. The total number of exempted aeroplanes should be agreed at the time the application is approved and based on the considerations explained in 2.5.2.1 c).

2.5.2.2.2 The proposed maximum number of potential exemptions should be inversely proportional to the % margin of the CO_2 metric value from the regulatory level (Part II, Chapter 2, 2.4). Those aeroplane types with a smaller % margin to the regulatory level should be permitted a larger number of exemptions compared to the aeroplane types with a larger % margin.

2.5.2.2.3 Following the recommendation in Part II, Chapter 1, 1.11 to use an acceptable process, the number of aeroplanes exempted per Type Certificate would normally not exceed the proposed maximum number in the tables and figures below.

Table 2-2. Maximum in-production exemptions

% margin to CAEP/10 in-production regulatory level	*Maximum exemptions total*
0 to 2	75
>2 to 10	-7.5 x (per cent margin to CAEP/10 regulatory level)+90
More than 10	15



[A1] [A2]

Figure 2-3. The graphical representation of in-production exemptions for the CAEP/10 aeroplane CO_2 emissions Standard

Table 2-3. Maximum new type design exemptions

% margin to CAEP/10 new type regulatory level	*Maximum exemptions total*
0 to 2	40
>2 to 4	-20 x (per cent margin to CAEP/10 regulatory level)+80
More than 4	0

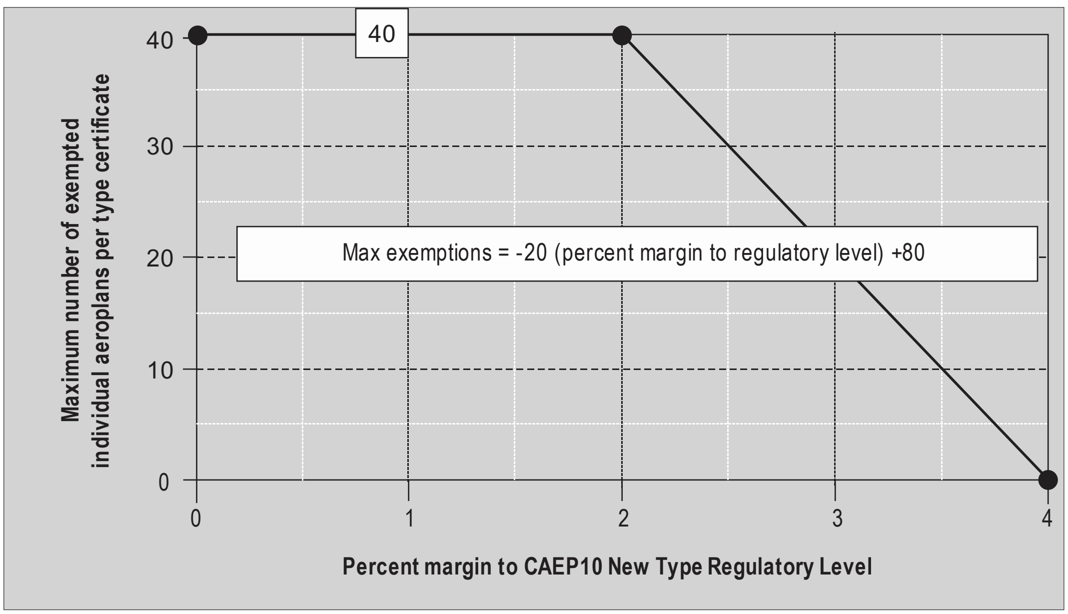


Figure 2-4. The graphical representation of new type design exemptions for the CAEP/10 aeroplane CO_2 emissions Standard

2.5.2.2.4 The maximum number of exemptions should be reviewed during the CAEP/13 cycle (2022 — 2025).

2.5.2.3 *Review*

The competent authority should review, in a timely manner, the application using the information provided in 2.5.2.1 and the evaluation criteria in 2.5.2.2. The analysis and conclusions from the review should be communicated to the applicant through a formal response. If the application is approved, the response should clearly state the scope of the exemptions that have been granted. If the application is rejected, the response should include a detailed justification.

2.5.3 Registration and communication

Oversight of the granted exemptions should include the following elements:

a) The competent authority should publish details of the exempted aeroplanes in an official public register, including aeroplane model and maximum number of permitted exemptions.

b) The applicant should have a quality control process for maintaining oversight and managing the production of aeroplanes which have been granted exemptions.

c) An exemption should be recorded in the aeroplane statement of conformity[2] which states conformity with the Type Certificate (proposed standard text: "Aeroplane exempted from Annex 16, Volume III, Chapter 2, 2.1.1 [x][3]").

d) The applicant should provide to the competent authority, on a regular basis and appropriate to the limitation of the approval, details on the actual exempted aeroplanes which have been produced (e.g. model, aeroplane type and serial number).

e) Exemptions for new aeroplanes should be processed and approved by the competent authority for the production of the exempted aeroplanes in coordination with the competent authorities responsible for the design of the aeroplane and the issuance of the initial Certificate of Airworthiness.

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[2] For example: European Aviation Safety Agency (EASA) Form 52, United States Federal Aviation Administration (FAA) Form 8130-94 or equivalent forms from other competent authorities.

[3] Relevant applicability subparagraph a) to g) would need to be filed for the exempted aeroplane.

Chapter 3

SAR DETERMINATION PROCEDURES

3.1 SAR MEASUREMENT PROCEDURES

3.1.1 Flight test procedures

3.1.1.1 *Fuel properties*

3.1.1.1.1 One of the important factors when determining the CO_2 emissions of an aeroplane, according to Annex 16, Volume III, is the fuel used in the flight tests.

3.1.1.1.2 Annex 16, Volume III, Part II, Chapter 2, 2.6.3 states: *"Note.— The fuel used for each flight test should meet the specification defined in either ASTM D1655-15*[1]*, DEF STAN 91-91 Issue 7, Amendment 3*[2]*, or equivalent.*" Equivalent fuel specifications accepted for the purposes of CO_2 emissions certification are the following:

a) Brazil: CNP-08, QAV-1;

b) China: GB6537 Number 3 Jet Fuel;

c) France: DCSEA 134;

d) Russia: GOST 10227-86 or 52050-2006, RT;

e) USA: ASTM International D1655;

f) UK: DEF STAN 91-91; and

g) similar specifications from other Member States, subject to the approval of the certificating authority.

3.1.1.1.3 The Annex, Part II, Chapter 2, 2.5.1 specifies the reference conditions to which the test conditions shall be corrected. The reference fuel lower heating value is specified as 43.217 MJ/kg (18 580 BTU/lb). Appendix 1, 3.2.1 c), Recommendation 1) states "the fuel lower heating value should be determined in accordance with methods which are at least as stringent as those defined in ASTM specification D4809-13[3]." This method is estimated to have an accuracy level of the order of 0.23 per cent.

3.1.1.1.4 The Annex, Appendix 1, 3.2.1 d) states that "A sample of fuel shall be taken for each flight test to determine its specific gravity and viscosity when volumetric fuel flow meters are used." The fuel's specific gravity and viscosity need not be determined if volumetric fuel flow meters are not used.

[1] ASTM D1655-15 entitled *Standard Specification for Aviation Turbine Fuels*.
[2] Defence Standard 91-91, Issue 7, Amendment 3, entitled *Turbine Fuel, Kerosene Type, Jet A-1*.
[3] ASTM D4809-13 entitled *Standard Test Method for Heat of Combustion of Liquid Hydrocarbon Fuels by Bomb Calorimeter (Precision Method)*.

3.1.1.1.5 Examples of acceptable methods to determine the fuel's specific gravity and viscosity are ASTM International D4052[4] and ASTM International D445[5]. Other methods may be used, subject to the approval of the certificating authority.

3.2 SAR DATA ANALYSIS

3.2.1 Data selection

Selection of data used to show compliance to the Standard encompasses both the selection of flight test data gathered during each test condition used to obtain an individual SAR point, as well as the distribution of the resulting corrected SAR points in relation to the three reference masses and the reference conditions.

3.2.1.1 *Selection of flight test data*

3.2.1.1.1 There are multiple methods employed by aeroplane manufacturers in selecting flight test data for analysis, reflecting a variety of tools and practices. Whichever method is chosen, the flight test data encompassed within the selected range of time is expected to meet the stability criteria detailed in Annex 16, Volume III, Appendix 1, 3.2.3.1, or alternative stability criteria approved by the certificating authority, as per 3.2.3.2. Test data that do not meet these stability criteria should normally be discarded. However, if such test data appear to be valid when compared with data that meet the stability criteria, and the overall stability of the conditions is reasonably bounded, these data can be retained, subject to the approval of the certificating authority.

3.2.1.1.2 One acceptable method is to employ an algorithm that automatically selects the data that meet all the stability criteria and discards data that do not. This method could be used to select the longest possible duration SAR point that meets the required stability criteria, or to select multiple SAR points of the minimum requirement duration (one minute), providing these points are separated by a minimum of two minutes or by an exceedance of the stability criteria, as specified in Annex 16, Volume III, Appendix 1, 3.2.2.2. Using a defined algorithm to select data in an automated process allows repeatable and consistent application to other SAR points. This method may also yield a greater number of SAR points to be used in defining the CO_2 metric value and should represent a good statistical distribution. However, because the amount of test data included in each SAR point is maximized, the resulting SAR points could exhibit more scatter than if additional selection criteria are used.

3.2.1.1.3 Another method is to more closely examine the collected flight test data and select the timeframe to be used to define the SAR point by choosing the best or most stable data available and ignoring less stable data that technically still meet the stability criteria. Examples of this are presented in Figures 3-1 and 3-2.

3.2.1.1.4 Figure 3-1 shows that the plotted parameters stay within the tolerances allowed by the stability criteria for the duration of the test condition (the changing altitude after the end of the condition reflects pilot input to leave steady flight and transition to the next test condition). While all parameters are within the required tolerances, fluctuations in ambient temperature and Mach are evident. Figure 3-2 shows the same data, but with a manually selected range of shorter duration where the parameters are more stable.

[4] ASTM D4052-11 entitled *Standard Test Method for Density and Relative Density of Liquids by Digital Density Meter*.

[5] ASTM D445-15 entitled *Standard Test Method for Kinematic Viscosity of Transparent and Opaque Liquids (and Calculation of Dynamic Viscosity)*.

3.2.1.1.5 Selecting data that meet more demanding stability criteria, instead of using all data that meet the stability criteria indicated in Annex 16, Volume III, may allow the applicant to filter out observed instabilities caused by air quality, changing environmental conditions, flight control inputs and aeroplane system dynamics. This could result in a SAR point that is actually more representative of actual aeroplane performance.

3.2.1.1.6 Whichever approach is taken to select data to define SAR points, it is important that the methodology be applied as consistently as possible to minimize potential unseen bias in the resulting distribution of SAR points.

3.2.1.1.7 Another important aspect to consider when selecting data is to ensure that the time interval chosen is representative of the aeroplane's performance and not indicative of a larger trend. For example, the first plot in Figure 3-3 shows a trend line drawn through ground speed data over a 60-second time interval. This ground speed data meets the stability criteria and, taken alone, would indicate the need for an energy correction. However, if the ground speed data trace was continued over a longer time interval, it becomes apparent that it exhibits cyclic behaviour. Cyclic data need not be discarded necessarily, but the applicant should ensure that an appropriate time interval is selected such that the arithmetic average is representative. In the example shown in Figure 3-3, an energy correction would be inappropriate.

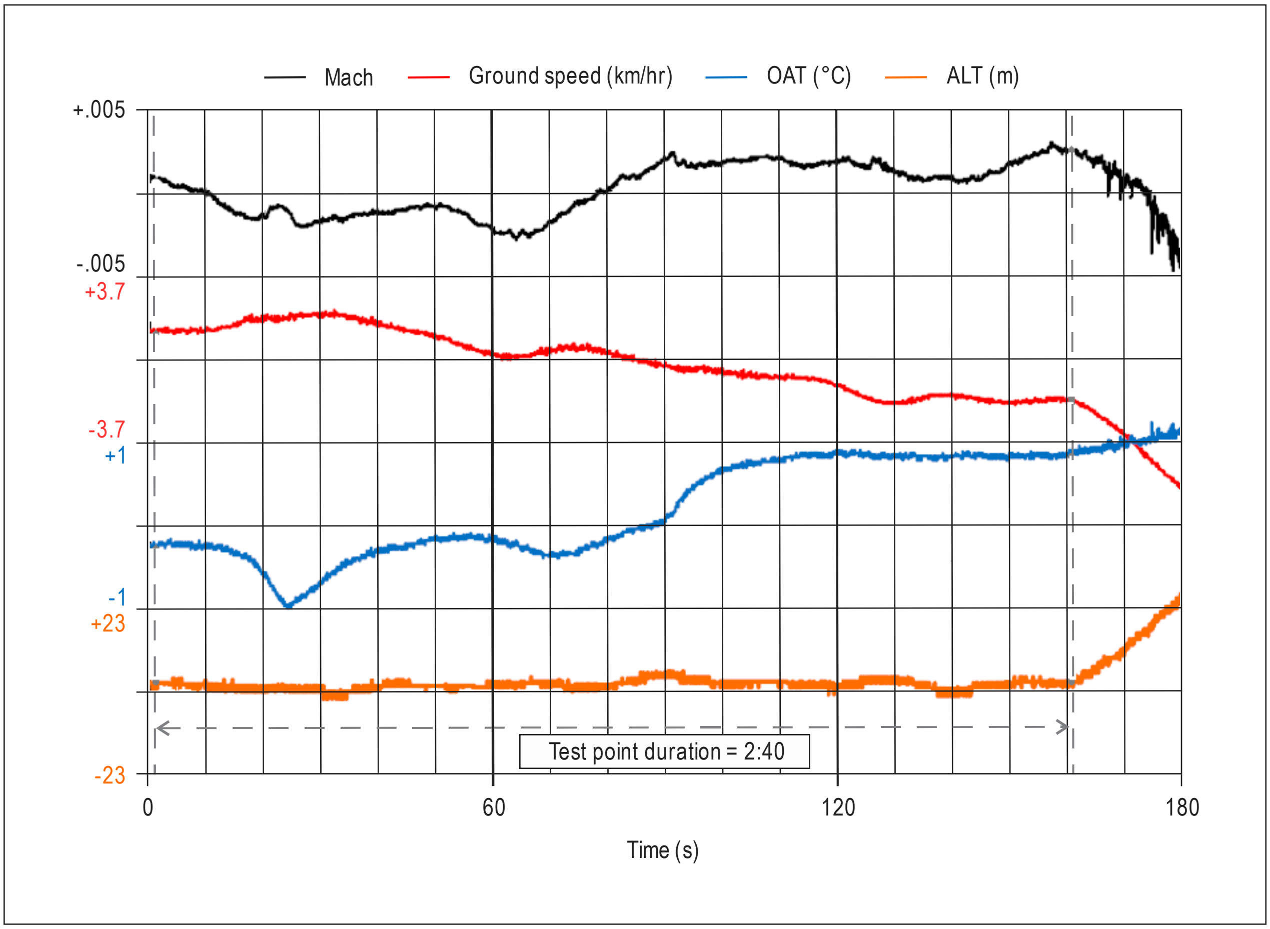

Figure 3-1. Flight test data time interval selection – Example 1

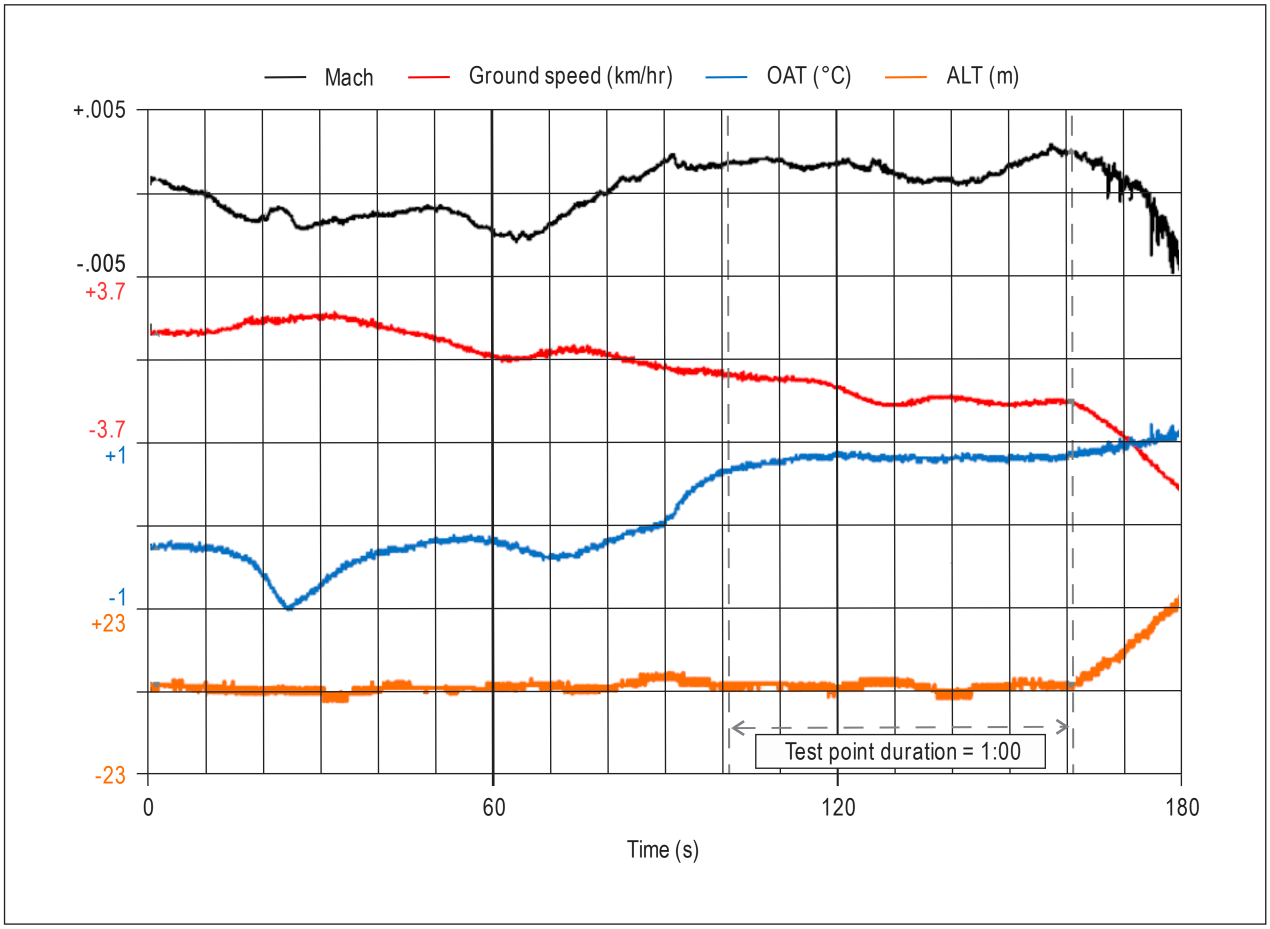

Figure 3-2. Flight test data time interval selection – Example 2

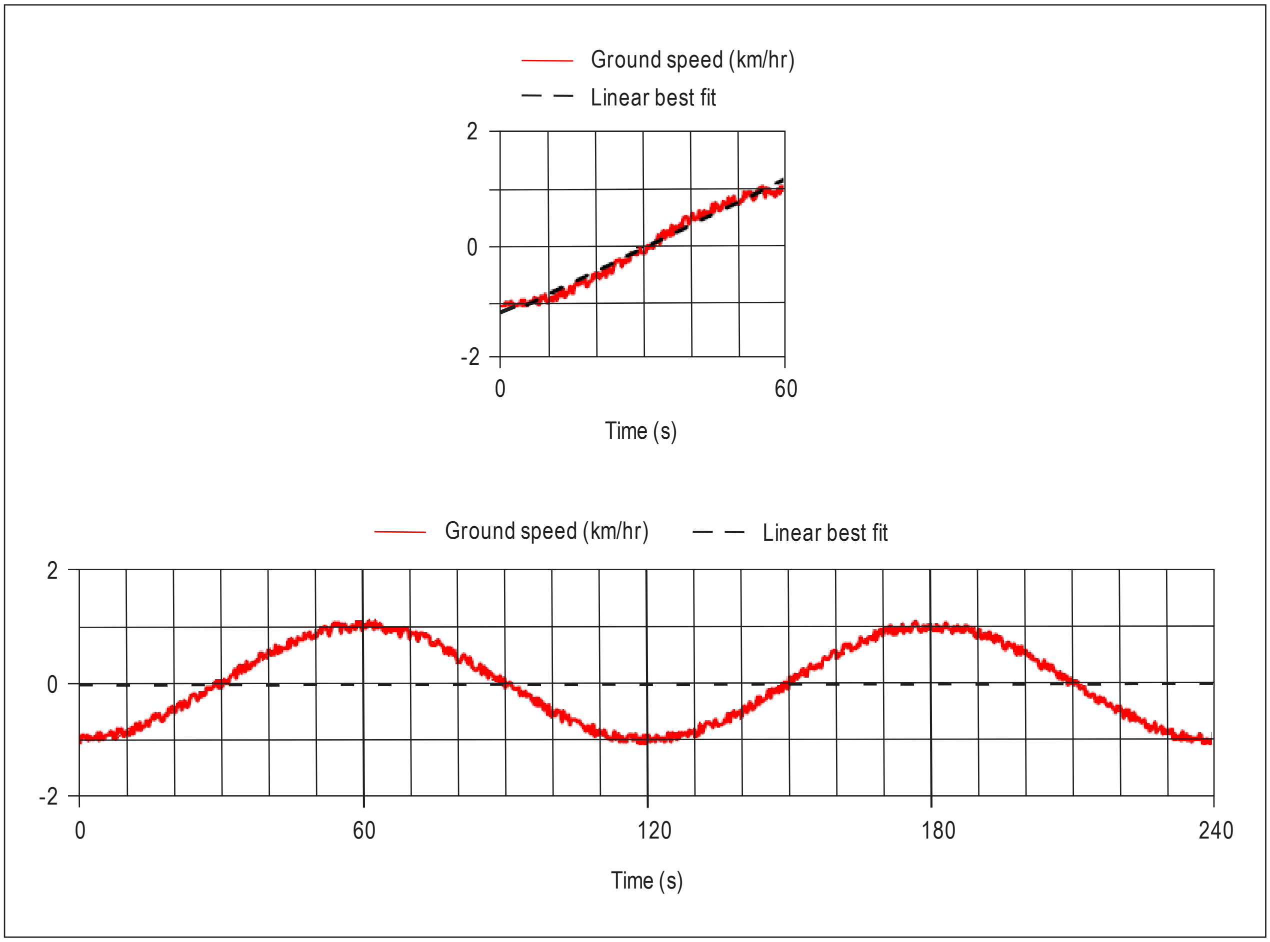

Figure 3-3. Cyclic behaviour example

3.2.1.2 *Distribution of resulting SAR points*

3.2.1.2.1 Once the individual SAR points have been selected and corrected to reference conditions, they should be examined to ensure they present an accurate representation of aeroplane performance.

3.2.1.2.2 For example, if direct flight test is being used to collect 6 SAR points targeting one reference mass, those 6 points, when corrected to reference conditions, should result in a reasonable grouping. If 5 of the points form a reasonable grouping and one point is a clear outlier, the offending point may require closer scrutiny to ensure it is actually representative. In such a situation, collection of additional data may be warranted or, if appropriate, and subject to the approval of the certificating authority, the offending data point could be discarded.

3.2.1.2.3 If the applicant conducts tests across a range of weights to build a regression line of SAR versus weight, the collected SAR points should be reasonably distributed across the weight range. If a large portion of the regression line is unsupported by data, or is anchored by a single SAR point, then the SAR determined for one of the reference masses may be suspect. This is an important aspect to consider during the development of the certification plan and flight test programme. As with the direct test method, if a single SAR point appears to be an outlier compared to the rest of the data points, it should be examined more closely and could potentially be discarded.

3.2.1.2.4 The applicant should investigate the collection of SAR points for potential sources of unintended bias, for example, if all of the data points collected were during periods where groundspeed was increasing. If all of the test points require a large energy correction in one direction, resulting in all SAR points being significantly increased (or decreased), further scrutiny may be required to ensure a bias is not introduced, depending on the test data and correction techniques being used.

3.2.2 Corrections to reference conditions

3.2.2.1 *General.* The guidance provided here represents one set of methods, but not the only acceptable methods, for correcting the SAR test data to the reference conditions specified in Annex 16, Volume III, Part II, Chapter 2, 2.5.

3.2.2.1.1 The corrections identified in paragraphs 3.2.2.3 through 3.2.2.12 cover corrections that should be made to the tested values of aeroplane weight, drag and fuel flow to correct for any differences between the conditions at which the aeroplane was tested and the reference conditions. For steady-state cruise flight, it is a reasonable approximation to assume lift equals weight and thrust equals drag. Since drag is a function of lift, fuel flow is a function of thrust, SAR is a function of fuel flow, and the CO_2 MV is a function of SAR, any deviation from the reference conditions that affect weight, drag, or fuel flow will affect SAR and the CO_2 MV. Each of these corrections will result in either increasing or decreasing the test SAR and the resulting CO_2 MV.

3.2.2.1.2 As stated in Annex 16, Volume III, Appendix 1, 5.2.2, if the applicant considers that a particular correction is unnecessary then acceptable justification shall be provided to the certificating authority. Any correction that would result in an increase in SAR could be considered to be optional. The effect of not making such a correction would be to penalize the CO_2 emissions evaluation metric value.

3.2.2.1.3 Table 3-1 lists all the corrections and the conditions under which a given correction would normally be considered to be optional:

Table 3-1. Corrections to reference conditions

Correction	*Paragraph*	*Type of correction*	*Conditions under which correction would normally be considered to be optional*	*Remarks*
Latitude effect on g	3.2.2.3.4.5.1, 3.2.2.3.4.5.3, 3.2.2.3.5.3	Apparent gravity	Test latitude greater than 45.5 degrees	Gravitational acceleration is greatest at the poles and lowest at the equator. Testing at latitudes greater than 45.5 degrees will penalize SAR and the CO_2 MV as the aeroplane will be heavier.

Correction	*Paragraph*	*Type of correction*	*Conditions under which correction would normally be considered to be optional*	*Remarks*
Altitude effect on g	3.2.2.3.4.5.2, 3.2.2.3.4.5.3, 3.2.2.3.5.4	Apparent gravity	Test altitude lower than reference altitude	Increasing the height above the Earth's surface reduces the gravitational acceleration. Therefore, testing at an altitude lower than the reference altitude will penalize SAR and the CO_2 MV due to the effect of altitude on gravitational acceleration.
Centrifugal effect on g	3.2.2.3.4.6, 3.2.2.3.4.6.1, 3.2.2.3.5.5	Apparent gravity	Headwind (negative wind)	The reduction in velocity (relative to the earth) provided by a headwind increases gravitational acceleration due to the centrifugal effect. This increases the aeroplane weight, which penalizes SAR and the CO_2 MV.
Coriolis effect on g	3.2.2.3.4.7, 3.2.2.3.4.7.1, 3.2.2.3.5.6	Apparent gravity	Test true track angles from 180 to 360 degrees	Flying in a westerly direction, the opposite direction as the Earth's rotation, will increase gravitational acceleration due to the Coriolis effect. This increases the aeroplane weight which penalizes SAR and the CO_2 MV.
Acceleration/deceleration (energy)	3.2.2.4	Drag	(dV_G/dT) greater than 0 (positive acceleration in terms of ground speed)	An accelerated flight condition (excess energy) results in a higher drag force, penalizing SAR and the CO_2 MV.
Reynolds number	3.2.2.5	Drag	Test outside air temperature higher than standard air temperature	Higher temperatures result in higher drag forces due to Reynold number effects. At temperatures higher than the reference temperature, SAR and the CO_2 MV will be penalized.
CG position	3.2.2.6	Drag	Test CG position forward of the reference CG position	A test CG position forward of the reference CG position results in higher drag, penalizing SAR and the CO_2 MV.

Correction	*Paragraph*	*Type of correction*	*Conditions under which correction would normally be considered to be optional*	*Remarks*
Aeroelastics	3.2.2.7	Drag	-	Wing aeroelastics may not be a concern depending on the size/weight/payload of the aeroplane and on wing stiffness.
Fuel lower heating value (LHV)	3.2.2.8	Fuel flow	Test fuel LHV less than 43.217 MJ/kg	Testing with fuel that has an LHV lower than the reference value requires more fuel to be burned to achieve the same engine thrust level, hence penalizing SAR and the CO_2 MV.
Altitude effect on fuel flow	3.2.2.9	Fuel flow	May always be considered optional	Testing at any altitude other than the optimum altitude will result in penalizing SAR and the CO_2 MV due to higher fuel burn at off-optimum altitude.
Temperature effect on fuel flow	3.2.2.10	Fuel flow	Test outside air temperature higher than standard day temperature	Testing at a temperature higher than the reference temperature will result in penalizing SAR and the CO_2 MV due to higher fuel burn at higher temperatures.
Engine deterioration level	3.2.2.11	Fuel flow	-	In general, this correction should not be made. See paragraph 3.2.2.11.
Electrical and mechanical power extraction and bleed flow	3.2.2.12	Fuel flow	Test electrical and mechanical power extraction and bleed flow higher than the reference electrical and mechanical power extraction and bleed flow	Testing with higher than reference power extraction and bleed flow will require more fuel to be burned, hence penalizing SAR and the CO_2 MV.

3.2.2.1.4 Although the reference conditions include airspeed values selected by the applicant under Annex 16, Volume III, 2.5.1 b), there is no correction identified in Annex 16, Volume III, Appendix 1, 5.2 for airspeed. The selection of a reference airspeed is used to determine the target airspeed for acquiring SAR test data for determination of the CO_2 MV. For applicants using either SAR data clustered around each of the three reference masses (see Annex 16, Volume III, Appendix 1, 6.2) or SAR data obtained over a range of masses (see Annex 16, Volume III,

Appendix 1, 6.3), no correction is made for any differences between the target reference airspeed and the test airspeed. Since the reference airspeed is generally expected to be the optimum airspeed (see the Note to Annex 16, Volume III, 2.5.1), any difference between the test airspeed and the reference airspeed will penalize SAR and the CO_2 MV. For applicants using first principle models to show compliance (see 3.4.4), the reference airspeed is used to identify the portion of the performance model to be validated and the airspeed (or Mach number) at which the SAR value for each reference mass is to be computed from the performance model.

3.2.2.1.5 The overall process for determining SAR for the reference conditions for each test data point consists of the following steps:

3.2.2.1.5.1 Determine the aeroplane mass weight from the aeroplane mass for both the test condition and the reference conditions for gravitational acceleration, as described in 3.2.2.2. Determine the drag correction due to the difference in lift between the test weight and the weight at the reference conditions for gravitational acceleration using the aeroplane's drag model and the relationship that lift equals weight for unaccelerated, level flight, as described in 3.2.2.3.4.10.

Note.— This step is unnecessary when using method 2 for the gravitational acceleration correction, as described in paragraph 3.2.2.3.5.

3.2.2.1.5.2 Determine all other drag corrections as described in 3.2.2.4, 3.2.2.5, 3.2.2.6 and 3.2.2.7 and sum them together with the drag correction from 3.2.2.1.5.1.

3.2.2.1.5.3 Determine the aeroplane drag for the test condition. An approximation can be made that for these unaccelerated, level flight test conditions, the aeroplane drag for the test condition is equal to the engine thrust for the test condition. The engine thrust should be determined from either a calibrated engine or from an engine performance model and the average value of the parameters needed to determine thrust from the engine performance model measured during the test condition.

3.2.2.1.5.4 Add the summed drag corrections from 3.2.2.1.5.2 to the aeroplane drag for the test condition from 3.2.2.1.5.3 to obtain the aeroplane drag corrected to the reference conditions.

3.2.2.1.5.5 Determine the change in engine fuel flow for the corrections that were made for drag. The change in engine fuel flow due to the drag corrections can be determined as follows:

$$\Delta \text{Fuel Flow}_{\text{Drag}} = \text{Fuel Flow}_{\text{Ref Drag}} - \text{Fuel Flow}_{\text{Test Drag}}$$

where:

$\text{Fuel Flow}_{\text{Ref Drag}}$ is the fuel flow from an engine performance model at the aeroplane drag corrected for the reference conditions (assuming thrust equals drag) at the average speed, altitude and temperature for the test condition; and

$\text{Fuel Flow}_{\text{Test Drag}}$ is the fuel flow from an engine performance model at the aeroplane drag for the test condition (assuming thrust equals drag) at the average speed, altitude and temperature for the test condition.

3.2.2.1.5.6 Correct the measured test engine fuel flow to reference conditions as follows:

$$\text{Fuel Flow}_{\text{ref}} = \text{Fuel Flow}_{\text{test}} + \Delta \text{Fuel Flow}_{\text{Drag}} + \Delta \text{Fuel Flow}_{\text{LHV}} + \Delta \text{Fuel Flow}_{\text{alt}} + \Delta \text{Fuel Flow}_{\text{temp}} + \Delta \text{Fuel Flow}_{\text{Corr bleed}}$$

where:

$\text{Fuel Flow}_{\text{test}}$ is the average engine fuel flow measured during the test condition, corrected to the reference engine deterioration level (if applicable) in kg/h;

ΔFuel Flow$_{Drag}$ is the fuel flow correction for drag from 3.2.2.1.5.5 in kg/h;

ΔFuel Flow$_{LHV}$ is the fuel flow correction for fuel lower heating value from 3.2.2.8 in kg/h;

ΔFuel Flow$_{alt}$ is the fuel flow correction for altitude from paragraph 3.2.2.9 in kg/h;

ΔFuel Flow$_{temp}$ is the fuel flow correction for temperature from paragraph 3.2.2.10 in kg/h; and

ΔFuel Flow$_{Corr\ bleed}$ is the fuel flow correction for electrical and mechanical power extraction and bleed flow from 3.2.2.12 in kg/h.

3.2.2.1.5.7 The SAR value corrected to reference conditions is given by the following relationship:

$$SAR_{ref} = \left(\frac{TAS}{Fuel\ Flow_{ref}}\right)$$

where:

SAR$_{ref}$ is the SAR for the reference conditions in km/kg;

TAS is the average aeroplane true airspeed for the test condition in km/h; and

Fuel Flow$_{ref}$ is the engine fuel flow for the reference conditions (see 3.2.2.1.5.6) in kg/h.

3.2.2.2 *Test Weight.* Since the weight of the aeroplane during each test condition cannot be directly measured, it is determined using the following process, in accordance with Annex 16, Volume III, Appendix 1, 3.2.1 b) and 3.2.4:

3.2.2.2.1 Begin with weighing the aeroplane on the ground, where the local gravitational acceleration can be readily determined using the process described in 3.2.2.3.4.5. The pre-flight aeroplane mass is determined from the following relationship:

$$Mass_{weighing} = \frac{Weight_{weighing}}{g_{weighing}}$$

where:

Weight$_{weighing}$ is the aeroplane weight from the scale weighing in N; and

$g_{weighing}$ is the gravitational acceleration at rest at the weighing site latitude and elevation in metres/second^2. See 3.2.2.3.4.5 for determining $g_{weighing}$ using the weighing site latitude and elevation for the test latitude, φ, and test geometric altitude, h, respectively.

3.2.2.2.2 Any change in mass after the weighing, and prior to the test flight, should be accounted for to establish the mass of the aeroplane at the start of the test flight. The mass for each test condition can be determined by subtracting the integrated fuel flow (mass of fuel burned) from the mass of the aeroplane at the start of the test flight. The test weight, in Newtons, for each test condition is determined from the following equation:

$$Weight_{test} = (Mass_{test})(g_{test})$$

where:

$Mass_{test}$ is the average mass of the aeroplane during the test condition in kg; and

g_{test} is the gravitational acceleration at the test latitude, test geometric altitude, test true track, and test ground speed in m/s^2. See 3.2.2.3.4.4 for determining g_{test}.

3.2.2.3 *Apparent gravity.* Acceleration, caused by the local effects of gravity and inertia, affects the test weight of the aeroplane. The apparent gravity at the test conditions varies with latitude, altitude, ground speed, and direction of motion, relative to the Earth's axis. The reference gravitational acceleration is the gravitational acceleration for the aeroplane travelling in the direction of True North in still air at the reference altitude, a geodetic latitude of 45.5 degrees, and based on g_0.

3.2.2.3.1 If the test conditions differ from the conditions for the reference gravitational acceleration, the aeroplane weight at the reference gravitational acceleration will be different than the aeroplane weight at the test conditions. This will result in a SAR and CO_2 MV that are not representative of the SAR and CO_2 MV at the reference gravitational acceleration. If the test conditions are at a gravitational acceleration that is less than the reference gravitational acceleration, then a correction will be necessary, as described in 3.2.2.3.2.

3.2.2.3.2 There are two methods provided below for determining the acceleration correction. Method 1 is a general method for determining the effect of gravitational acceleration on the aeroplane test weight, which is then used along with the aeroplane's drag model to determine the effect on aeroplane drag. This drag correction for gravitational acceleration is then combined with the other drag corrections to determine the effect on aeroplane fuel flow and hence SAR.

3.2.2.3.3 Method 2 is a simplified approach that does not require use of an aeroplane drag model. This method is acceptable for CO_2 emissions certification when SAR is determined by direct flight test measurement of SAR test points in accordance with Annex 16, Volume III, Appendix 1, 2.1. Method 2 is based on the same equations for determining g as Method 1 and provides the same results for SAR data at, or near, optimum flight conditions. For SAR data that is not at, or near, reference conditions, Method 2 may result in small errors in the SAR correction for gravitational acceleration correction, but the overall SAR and CO_2 MV will be conservative because these errors will be much smaller than the SAR and CO_2 MV penalty resulting from testing away from optimum flight conditions.

3.2.2.3.4 *Method 1 for the gravitational acceleration correction*

3.2.2.3.4.1 The process for determining the drag correction due to the effect of gravitational acceleration on aeroplane weight is as follows:

a) Determine the aeroplane test weight, as described in paragraph 3.2.2.2.2.

b) Determine the aeroplane weight for the reference gravitational acceleration conditions from the aeroplane test mass and the reference gravitational acceleration:

$$Weight_{ref\ g} = (Mass_{test})(g_{ref})$$

where:

$Weight_{ref\ g}$ is the aeroplane weight at the reference gravitational acceleration conditions in N;

$Mass_{test}$ is the average mass of the aeroplane during the test condition in kg;

g_{ref} is the gravitational acceleration for the aeroplane travelling in the direction of True North in still air at the reference altitude and a geodetic latitude of 45.5 degrees, and based on g_0, in m/s^2; and

g_0 is the standard acceleration due to gravity at sea level and a geodetic latitude of 45.5 degrees, 9.80665 m/s^2.

c) Determine the drag correction for the difference between $Weight_{test}$ and $Weight_{ref\ g}$, as described in 3.2.2.3.4.9.

3.2.2.3.4.2 The value of gravitational acceleration, g, consists of the following components:

a) Distance from the centre of Earth's mass, which is a function of the geometric altitude and the latitude of the aeroplane relative to a mathematical representation of the shape of the Earth.

b) *Centrifugal effect*. An aeroplane in flight will experience a force acting outwards in the radial direction that is proportional to the square of the ground speed of the aeroplane and inversely proportional to the distance of the aeroplane from the centre of mass of the Earth.

c) *Coriolis effect*. An aeroplane in flight will experience a force that is proportional to its ground speed, the rotation rate of the Earth, the direction of travel, and the latitude. For example, an aeroplane flying east, in the same direction that the Earth rotates, will experience a lower gravitational acceleration, while an aeroplane flying west will experience a higher gravitational acceleration. The effect will be greatest at the equator.

3.2.2.3.4.3 The following equations are based on the World Geodetic System 84 Ellipsoidal Gravity definition. Other formulations and simplifications may provide essentially equivalent results.

3.2.2.3.4.4 The gravitational acceleration experienced for each test data point (g_{test}) is determined as follows:

$$g_{test} = g_{\varphi,alt} + g_{cent} + g_{Coriolis}$$

where:

$g_{\varphi,alt}$ is the component of the gravitational acceleration at zero ground speed, at the test altitude and latitude in m/s^2;

g_{cent} is the component of the gravitational acceleration due to centrifugal effect in m/s^2; and

$g_{Coriolis}$ is the component of the gravitational acceleration due to Coriolis effect in m/s^2.

3.2.2.3.4.5 The gravitational acceleration for the test geometric altitude and latitude component at zero ground speed, $g_{\varphi,alt}$, is determined as follows:

3.2.2.3.4.5.1 First, determine the component of gravitational acceleration at the test latitude at sea level and zero ground speed from the following equation:

$$g_\varphi = \left(9.7803267714\ \frac{1 + 0.00193185138639\ sin^2\varphi}{\sqrt{1 - 0.00669437999013\ sin^2\varphi}}\right)$$

where φ is the test latitude in degrees.

3.2.2.3.4.5.2 The component of gravitational acceleration at zero ground speed for the test geometric altitude and latitude is then determined from the following equation:

$$g_{\varphi,alt} = g_{\varphi}\left(\frac{r_e}{r_e + h}\right)^2$$

where:

g_{φ} is the component of gravitational acceleration at the test latitude at sea level and zero ground speed (see 3.2.2.2.3.4.5.1);

h is the test geometric altitude in m; and

r_e is the radius of the Earth at the test latitude, which is determined from the following equation:

$$r_e = \sqrt{\frac{(a^2 cos\ \varphi)^2 + (b^2 sin\ \varphi)^2}{(a\ cos\ \varphi)^2 + (b\ sin\ \varphi)^2}}$$

where:

a is the Earth's radius at the equator = 6 378 137 m;

b is the Earth's radius at the pole = 6 356 752 m; and

φ is the test latitude in degrees.

3.2.2.3.4.5.3 As an alternative to using the equations in 3.2.2.3.4.5.1 and 3.2.2.3.4.5.2, $g_{\varphi,alt}$ can be approximated by linear interpolation from Table 3-2:

Table 3-2. $g_{\varphi,alt}$

$g_{\varphi,alt}$											
	Geometric Height (ft)										
Latitude (either North or South) (degrees)	0	5,000	10,000	15,000	20,000	25,000	30,000	35,000	40,000	45,000	50,000
0	9.78033	9.77565	9.77099	9.76632	9.76166	9.75700	9.75234	9.74769	9.74304	9.73840	9.73376
10	9.78188	9.77721	9.77254	9.76787	9.76321	9.75855	9.75389	9.74924	9.74459	9.73994	9.73530
20	9.78637	9.78169	9.77702	9.77235	9.76768	9.76302	9.75836	9.75370	9.74905	9.74440	9.73975
30	9.79325	9.78857	9.76302	9.77921	9.77454	9.76987	9.76521	9.76054	9.75588	9.75123	9.74658
40	9.80170	9.79701	9.79232	9.78764	9.78296	9.77829	9.77362	9.76895	9.76428	9.75962	9.75496
45.5	9.80665	9.80196	9.79727	9.79258	9.78790	9.78322	9.77855	9.77387	9.76920	9.76454	9.75988
50	9.81070	9.80601	9.80132	9.79663	9.79194	9.78726	9.78258	9.77790	9.77323	9.76856	9.76390
60	9.81918	9.81448	9.80978	9.80508	9.80039	9.79570	9.79101	9.78633	9.78165	9.77698	9.77230
70	9.82610	9.82139	9.81668	9.81198	9.80728	9.80259	9.79790	9.79321	9.78853	9.78385	9.77917
80	9.83062	9.82590	9.82120	9.81649	9.81179	9.80709	9.80240	9.79771	9.79302	9.78833	9.78365
90	9.83219	9.82747	9.82276	9.81806	9.81336	9.80866	9.80396	9.79927	9.79458	9.78989	9.78521

3.2.2.3.4.6 The component of the gravitational acceleration due to centrifugal effect, g_{cent}, is determined from the following equation:

$$g_{cent} = -\frac{{V_g}^2}{r_e + h}$$

where:

V_g is the ground speed in m/s;

r_e is the radius of the Earth in m at the test latitude, which is determined from the same equation given in 3.2.2.3.4.5.2:

$$r_e = \sqrt{\frac{(a^2 \cos\varphi)^2 + (b^2 \sin\varphi)^2}{(a\cos\varphi)^2 + (b\sin\varphi)^2}}\text{ ; and}$$

h is the test geometric altitude in m.

3.2.2.3.4.6.1 As an alternative to using the equation in 3.2.2.3.4.6, g_{cent} can be approximated by linear interpolation from Table 3-3:

Table 3-3. g_{cent}

g_{cent}									
Ground Speed (kts)	0	100	200	300	400	500	600	700	800
g_{cent}	0.00000	-0.00041	-0.00166	-0.00373	-0.00663	-0.01036	-0.01492	-0.02030	-0.02652

3.2.2.3.4.7 The component of the gravitational acceleration due to Coriolis effect, $g_{Coriolis}$, can be found from the following equation:

$$g_{Coriolis} = -2\,\omega_E\,V_g\,cos\,\varphi\,\sin\sigma$$

where:

ω_E is the Earth's rotation rate = 7.29212 x 10^{-5} radians/s;

V is the aeroplane's ground speed in m/s;

φ is the test latitude in degrees; and

σ is the track angle of the aeroplane as measured from True North in degrees.

3.2.2.3.4.7.1 As an alternative to using the equation in paragraphs 3.2.2.3.4.7, $g_{Coriolis}$ can be approximated by linear interpolation from Table 3-4:

Table 3-4. $g_{Coriolis}$

$g_{Coriolis}$ for ground speeds from 200 to 400 knots													
GS (kts)		*200*				*300*				*400*			
Latitude (either N or S) (degrees)		**0**	**30**	**60**	**90**	**0**	**30**	**60**	**90**	**0**	**30**	**60**	**90**
True Track Angle (degrees)	**0**	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	30	-0.00750	-0.00650	-0.00650	0.00000	-0.01125	-0.00975	-0.00563	0.00000	-0.01501	-0.01300	-0.00750	0.00000
	60	-0.01300	-0.01125	-0.01125	0.00000	-0.01949	-0.01688	-0.00975	0.00000	-0.02599	-0.02251	-0.01300	0.00000
	90	-0.01501	-0.01300	-0.01300	0.00000	-0.02251	-0.01949	-0.01125	0.00000	-0.03001	-0.02599	-0.01501	0.00000
	120	-0.01300	-0.01125	-0.01125	0.00000	-0.01949	-0.01688	-0.00975	0.00000	-0.02599	-0.02251	-0.01300	0.00000
	150	-0.00750	-0.00650	-0.00650	0.00000	-0.01125	-0.00975	-0.00563	0.00000	-0.01501	-0.01300	-0.00750	0.00000
	180	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	210	0.00750	0.00650	0.00650	0.00000	0.01125	0.00975	0.00563	0.00000	0.01501	0.01300	0.00750	0.00000
	240	0.01300	0.01125	0.01125	0.00000	0.01949	0.01688	0.00975	0.00000	0.02599	0.02251	0.01300	0.00000
	270	0.01501	0.01300	0.01300	0.00000	0.02251	0.01949	0.01125	0.00000	0.03001	0.02599	0.01501	0.00000
	300	0.01300	0.01125	0.01125	0.00000	0.01949	0.01688	0.00975	0.00000	0.02599	0.02251	0.01300	0.00000
	330	0.00750	0.00650	0.00650	0.00000	0.01125	0.00975	0.00563	0.00000	0.01501	0.01300	0.00750	0.00000

Table 3-4 (continued)

$g_{Coriolis}$ for ground speeds from 500 to 700 knots												
GS (kts)	*500*				*600*				*700*			
Latitude (either N or S) (degrees)	**0**	**30**	**60**	**90**	**0**	**30**	**60**	**90**	**0**	**30**	**60**	**90**
True Track Angle (degrees)												
0	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
30	-0.01876	-0.01624	-0.00938	0.00000	-0.02251	-0.01949	-0.01125	0.00000	-0.02626	-0.02274	-0.01313	0.00000
60	-0.03249	-0.02814	-0.01624	0.00000	-0.03899	-0.03376	-0.01949	0.00000	-0.04548	-0.03939	-0.02274	0.00000
90	-0.03751	-0.03249	-0.01876	0.00000	-0.04502	-0.03899	-0.02251	0.00000	-0.05252	-0.04548	-0.02626	0.00000
120	-0.03249	-0.02814	-0.01624	0.00000	-0.03899	-0.03376	-0.01949	0.00000	-0.04548	-0.03939	-0.02274	0.00000
150	-0.01876	-0.01624	-0.00938	0.00000	-0.02251	-0.01949	-0.01125	0.00000	-0.02626	-0.02274	-0.01313	0.00000
180	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
210	0.01876	0.01624	0.00938	0.00000	0.02251	0.01949	0.01125	0.00000	0.02626	0.02274	0.01313	0.00000
240	0.03249	0.02814	0.01624	0.00000	0.03899	0.03376	0.01949	0.00000	0.04548	0.03939	0.02274	0.00000
270	0.03751	0.03249	0.01876	0.00000	0.04502	0.03899	0.02251	0.00000	0.05252	0.04548	0.02626	0.00000
300	0.03249	0.02814	0.01624	0.00000	0.03899	0.03376	0.01949	0.00000	0.04548	0.03939	0.02274	0.00000
330	0.01876	0.01624	0.00938	0.00000	0.02251	0.01949	0.01125	0.00000	0.02626	0.02274	0.01313	0.00000

3.2.2.3.4.8 The reference gravitational acceleration, g_{ref}, is the gravitational acceleration for the aeroplane travelling in the direction of True North in still air at the reference altitude and a geodetic latitude of 45.5 degrees. Because the reference gravitational acceleration condition is for the aeroplane travelling in the direction of True North, the reference gravitational acceleration does not include any Coriolis effect. Because the reference condition is for the aeroplane travelling in still air, the effect of the centrifugal effect on the reference gravitational acceleration is determined using the aeroplane's true airspeed (which is the same as the zero wind ground speed). The reference gravitational acceleration can be determined for each test point as mentioned in 3.2.2.3.4.8.1 to 3.2.2.3.4.8.3.

3.2.2.3.4.8.1 Determine the component of gravitational acceleration for the reference altitude and latitude at zero ground speed using the process defined in paragraph 3.2.2.3.4.5, using the reference altitude and 45.5 degrees latitude as the altitude and latitude, respectively, instead of the test values.

3.2.2.3.4.8.2 Determine the component of the reference gravitational acceleration due to centrifugal effect using the process defined in 3.2.2.3.4.6, using the aeroplane's true airspeed as the ground speed.

3.2.2.3.4.8.3 The reference gravitational acceleration, g_{ref}, is the sum of the component of gravitational acceleration for the reference altitude and latitude at zero ground speed determined in 3.2.2.3.4.8.1 and the component of the reference gravitational acceleration due to centrifugal effect determined in 3.2.2.3.4.8.2.

3.2.2.3.4.9 The drag correction for gravitational acceleration (due to the difference between the aeroplane test weight, $Weight_{test}$, and the aeroplane weight at the reference gravitational acceleration conditions, $Weight_{ref\ g}$) is determined from the aeroplane drag model. The aeroplane's drag model provides the drag coefficient (C_D) as a function of the lift coefficient (C_L) and Mach number. The lift coefficient for $Weight_{test}$ and $Weight_{ref\ g}$ can be determined from the following relationship:

$$C_L = \left(\frac{Weight\ /\delta}{70927.5\ M^2 A}\right)$$

where:

C_L is the lift coefficient;

Weight is the weight of the aeroplane in N (either $Weight_{test}$ or $Weight_{ref\ g}$, depending on which lift coefficient is being calculated);

δ is the ratio of the ambient air pressure at the test altitude to the ambient air pressure at sea level;

M is the aeroplane's average Mach number during the test condition; and

A is the aeroplane's reference wing area in m^2.

3.2.2.3.4.10 The drag correction for gravitational acceleration can be determined from the drag equation:

$$\Delta Drag_{grav} = \frac{1}{2}\rho V^2 \left(C_{D_{ref\ g}} - C_{D_{test}}\right) A$$

where:

$\Delta Drag_{grav}$ is the drag correction in N for gravitational acceleration;

ρ is the density of air at the test altitude and test temperature in kg/m^3;

V is the aeroplane's average true airspeed during the test condition in m/s;

A is the aeroplane's reference wing area in m^2;

$C_{Dref\ g}$ is the drag coefficient from the aeroplane's drag model at the C_L for $Weight_{ref\ g}$ from 3.2.2.3.4.9; and

$C_{D\ test}$ is the drag coefficient from the aeroplane's drag model at the test C_L for $Weight_{test}$ from 3.2.2.3.4.9.

3.2.2.3.5 *Method 2 for the gravitational acceleration correction*

3.2.2.3.5.1 The process for determining gravitational acceleration correction to SAR is as follows:

3.2.2.3.5.2 Determine the aeroplane test mass, as described in 3.2.2.2.1.

3.2.2.3.5.3 Determine g_{test}/g_{ref} for the test latitude by linear interpolation from Table 3-5:

Table 3-5. g_{test}/g_{ref} for latitude (latitude correction)

g_{test}/g_{ref} for latitude (latitude correction)										
Test Latitude (either north or south) (degrees)	0	10	20	30	40	50	60	70	80	90
$(g_{test}/g_{ref})_{lat}$	0.9973	0.9975	0.9979	0.9986	0.9995	1.0004	1.0013	1.0020	1.0024	1.0026

3.2.2.3.5.4 Determine g_{test}/g_{ref} to correct for test altitude by linear interpolation from Table 3-6:

Table 3-6. g_{test}/g_{ref} for altitude (altitude correction)

g_{test}/g_{ref} for altitude (altitude correction)											
Reference Altitude - Test Geometric Altitude (ft)	-5000	-4000	-3000	-2000	-1000	0	1000	2000	3000	4000	5000
$(g_{test}/g_{ref})_{alt}$	0.9995	0.9996	0.9997	0.9998	0.9999	1.0000	1.0001	1.0002	1.0003	1.0004	1.0005

3.2.2.3.5.5 Determine g_{test}/g_{ref} for headwind/tailwind (centrifugal correction) by linear interpolation from Table 3-7:

Table 3-7. g_{test}/g_{ref} for headwind/tailwind (centrifugal correction)

g_{test}/g_{ref} for headwind/tailwind (centrifugal correction)						
TAS (kts)		200	300	400	500	600
Tailwind (kts)	300	0.9991	0.9989	0.9986	0.9983	0.9981
	200	0.9995	0.9993	0.9991	0.9990	0.9988
	100	0.9998	0.9997	0.9996	0.9995	0.9994
	0	1.0000	1.0000	1.0000	1.0000	1.0000
Headwind (kts)	-100	1.0001	1.0002	1.0003	1.0004	1.0005
	-200	1.0002	1.0003	1.0005	1.0007	1.0009
	-300	1.0001	1.0004	1.0006	1.0009	1.0011

3.2.2.3.5.6 Determine g_{test}/g_{ref} for track angle (Coriolis correction) by linear interpolation from Table 3-8:

Table 3-8. g_{test}/g_{ref} for track angle (Coriolis correction)

g_{test}/g_{ref} for track angle (Coriolis correction) for ground speeds from 200 to 400 knots

GS (kts)	200				300				400			
Latitude (either north or south) (degrees)	**0**	**30**	**60**	**90**	**0**	**30**	**60**	**90**	**0**	**30**	**60**	**90**
True Track Angle (degrees)												
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
30	0.9992	0.9993	0.9996	1.0000	0.9988	0.9990	0.9994	1.0000	0.9985	0.9987	0.9992	1.0000
60	0.9987	0.9988	0.9993	1.0000	0.9980	0.9983	0.9990	1.0000	0.9973	0.9977	0.9987	1.0000
90	0.9985	0.9987	0.9992	1.0000	0.9977	0.9980	0.9988	1.0000	0.9969	0.9973	0.9985	1.0000
120	0.9987	0.9988	0.9993	1.0000	0.9980	0.9983	0.9990	1.0000	0.9973	0.9977	0.9987	1.0000
150	0.9992	0.9993	0.9996	1.0000	0.9988	0.9990	0.9994	1.0000	0.9985	0.9987	0.9992	1.0000
180	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
210	1.0008	1.0007	1.0004	1.0000	1.0012	1.0010	1.0006	1.0000	1.0015	1.0013	1.0008	1.0000
240	1.0013	1.0012	1.0007	1.0000	1.0020	1.0017	1.0010	1.0000	1.0027	1.0023	1.0013	1.0000
270	1.0015	1.0013	1.0008	1.0000	1.0023	1.0020	1.0012	1.0000	1.0031	1.0027	1.0015	1.0000
300	1.0013	1.0012	1.0007	1.0000	1.0020	1.0017	1.0010	1.0000	1.0027	1.0023	1.0013	1.0000
330	1.0008	1.0007	1.0004	1.0000	1.0012	1.0010	1.0006	1.0000	1.0015	1.0013	1.0008	1.0000

Table 3-8 (continued)

g_{test}/g_{ref} for track angle (Coriolis correction) for ground speeds from 500 to 700 knots													
GS (kts)		**500**				**600**				**700**			
Latitude (either North or South) (degrees)		**0**	**30**	**60**	**90**	**0**	**30**	**60**	**90**	**0**	**30**	**60**	**90**
True Track Angle (degrees)	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	30	0.9981	0.9983	0.9990	1.0000	0.9977	0.9980	0.9988	1.0000	0.9973	0.9977	0.9987	1.0000
	60	0.9967	0.9971	0.9983	1.0000	0.9960	0.9965	0.9980	1.0000	0.9953	0.9960	0.9977	1.0000
	90	0.9961	0.9967	0.9981	1.0000	0.9954	0.9960	0.9977	1.0000	0.9946	0.9953	0.9973	1.0000
	120	0.9967	0.9971	0.9983	1.0000	0.9960	0.9965	0.9980	1.0000	0.9953	0.9960	0.9977	1.0000
	150	0.9981	0.9983	0.9990	1.0000	0.9977	0.9980	0.9988	1.0000	0.9973	0.9977	0.9987	1.0000
	180	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	210	1.0019	1.0017	1.0010	1.0000	1.0023	1.0020	1.0012	1.0000	1.0027	1.0023	1.0013	1.0000
	240	1.0033	1.0029	1.0017	1.0000	1.0040	1.0035	1.0020	1.0000	1.0047	1.0040	1.0023	1.0000
	270	1.0039	1.0033	1.0019	1.0000	1.0046	1.0040	1.0023	1.0000	1.0054	1.0047	1.0027	1.0000
	300	1.0033	1.0029	1.0017	1.0000	1.0040	1.0035	1.0020	1.0000	1.0047	1.0040	1.0023	1.0000
	330	1.0019	1.0017	1.0010	1.0000	1.0023	1.0020	1.0012	1.0000	1.0027	1.0023	1.0013	1.0000

3.2.2.3.5.7 Determine $Mass_{grav}$, as defined in 3.2.2.3.5.7.1. Couple this mass with the tested SAR values, corrected for reference conditions from 3.2.2.1.5.7, in a SAR versus mass regression model to determine the SAR values, at each of the three reference masses, of the CO_2 emissions evaluation metric and to determine the 90 per cent confidence intervals.

3.2.2.3.5.7.1 $Mass_{grav} = Mass_{test} \times (g_{test}/g_{ref})_{lat} \times (g_{test}/g_{ref})_{alt} \times (g_{test}/g_{ref})_{cent} \times (g_{test}/g_{ref})_{Coriolis}$

where:

$Mass_{grav}$ is the mass at which the reference SAR value from paragraph 3.2.2.1.5.7 should be associated in a regression model to provide a SAR versus mass corrected for gravitational acceleration;

$Mass_{test}$ is the test mass, as defined in 3.2.2.2.2, in kg;

$(g_{test}/g_{ref})_{lat}$ is g_{test}/g_{ref} for the test latitude, as determined in 3.2.2.3.5.3;

$(g_{test}/g_{ref})_{alt}$ is g_{test}/g_{ref} for altitude, as determined in 3.2.2.3.5.4;

$(g_{test}/g_{ref})_{cent}$ is g_{test}/g_{ref} for headwind/tailwind (centrifugal correction) from 3.2.2.3.5.5; and

$(g_{test}/g_{ref})_{Coriolis}$ is g_{test}/g_{ref} for track angle (Coriolis correction) from 3.2.2.3.5.6.

3.2.2.4 *Acceleration/deceleration (energy).* Drag determination is based on an assumption of steady, unaccelerated flight. Acceleration or deceleration relative to the ground occurring during a test condition affects the assessed drag level. The reference condition is steady, unaccelerated flight.

3.2.2.4.1 The correction for the change in drag force resulting from acceleration during the test condition can be determined from the following equation:

$$\Delta D_{accel} = -Mass_{test}\left(\frac{dV_g}{dT}\right)$$

where:

ΔD_{accel} is the drag correction in N due to acceleration occurring during the test condition;

$Mass_{test}$ is the average mass of the aeroplane during the test condition, as defined in 3.2.2.2 in kg; and

(dV_g/dT) is the change in ground speed over time during the test condition in m/s^2.

3.2.2.5 *Reynolds number.* The Reynolds number affects aeroplane drag. For a given test condition, the Reynolds number is a function of the density and viscosity of air at the test altitude and temperature. The reference Reynolds number is derived from the density and viscosity of air from the ICAO standard atmosphere at the reference altitude.

3.2.2.5.1 The value of the drag coefficient correction for being off the reference RE condition during the test can be expressed as:

$$\Delta C_{D\ RE} = -B\ log\left[\frac{\frac{1}{M}\left(\frac{RE}{m}\right)_{test}}{\frac{1}{M}\left(\frac{RE}{m}\right)_{Ref}}\right]$$

where:

$\Delta C_{D\ RE}$ is the change in drag coefficient due to being off the reference RE;

B is a value representing the variation of drag with RE for the specific aeroplane (see 3.2.2.5.2);

M is Mach number; and

RE/m is Reynolds number per m.

3.2.2.5.2 One method to obtain B is to use a drag model to obtain the incremental drag variation in response to changing Mach and altitude from a reference cruise condition. The value for B is the value of a single representative slope of a plot of the drag variation, ΔDrag versus $Log_{10}\left[\frac{1}{M}\left(\frac{RE}{m}\right)x10^{-6}\right]$.

3.2.2.5.3 The term $\left[\frac{\frac{1}{M}\left(\frac{RE}{m}\right)_{test}}{\frac{1}{M}\left(\frac{RE}{m}\right)_{Ref}}\right]$ is the term $\frac{1}{M}\left(\frac{RE}{m}\right)$ determined at the temperature and altitude for the test condition, divided by the same term determined at the standard day temperature and the reference altitude for the test mass/δ using the following equation:

$$\frac{1}{M}\frac{RE}{m} = 4.7899\ x\ 10^5\ P_S\left(\frac{T_S + 110.4}{T_S^2}\right)$$

where:

RE/m is Reynolds number per m;

P_S is static pressure in Pa; and

T_S is static temperature in K.

3.2.2.5.4 The effect on aeroplane drag can then be determined from $\Delta C_{D\ RE}$ and the aeroplane drag equation as follows:

$$\Delta D_{RE} = \frac{1}{2}\rho\ V^2 \Delta C_{D\ RE}\ A$$

where:

ΔD_{RE} is the aeroplane drag correction in Newtons due to the test RE being different than the reference RE;

ρ is the density of air at the test altitude and test temperature in kg/m^3;

V is the aeroplane's average true airspeed during the test condition in m/s;

A is the aeroplane's reference wing area in m^2; and

$\Delta C_{D\ RE}$ is the change in drag coefficient due to being off the reference RE, as per 3.2.2.5.1.

3.2.2.6 *CG position.* The position of the aeroplane centre of gravity (CG) affects the drag due to longitudinal trim.

3.2.2.6.1 The drag correction for being off the reference CG position during the test is the difference between the drag at the reference CG position and the drag at the test CG position. This drag correction can be determined by adjusting the longitudinal trim drag, determined from the aeroplane's drag model at the reference CG, to adjust for the test CG position. For example, testing at a CG position aft of the reference CG used in the drag model would yield a positive drag correction (that is, a penalty to the test SAR).

3.2.2.6.2 A method to determine the change in longitudinal trim drag for CG position from the reference is to first determine the amount of longitudinal trim drag ($C_{D\ Trim\ Ref\ CG}$) that exists at the reference CG position as a function of Mach and C_L. This can be done using wind tunnel testing and analytical methods. The change of the longitudinal trim drag with CG, which is specific to an aeroplane type design, can be estimated with wind tunnel and analytical methods, and verified with flight test data. This relationship can then be used to determine the longitudinal trim drag ($C_{D\ Trim\ Test\ CG}$) at the test CG position.

3.2.2.6.3 Once the trim drag coefficients are determined from the aeroplane drag model, the aeroplane drag correction for the CG position can be determined from the drag equation:

$$\Delta D_{Trim\ CG} = \frac{1}{2}\rho\, V^2\left(C_{D\ Trim\ Ref\ CG} - C_{D\ Trim\ Test\ CG}\right)A$$

where:

$\Delta D_{Trim\ CG}$ is the aeroplane drag correction in Newtons due to the test CG being different than the reference CG;

ρ is the density of air at the test altitude and test temperature in kg/m^3;

V is the aeroplane's average true airspeed during the test condition in m/s;

A is the aeroplane's reference wing area in m^2; and

$C_{D\ Trim\ Ref\ CG}$ and $C_{D\ TrimTest\ CG}$ are the trim drag coefficients from the aeroplane's drag model at the reference CG and test CG positions, respectively.

3.2.2.7 *Aeroelastics.* Wing aeroelastics may cause a variation in drag as a function of aeroplane wing mass distribution. Aeroplane wing mass distribution will be affected by the fuel load distribution in the wings and the presence of any external stores.

3.2.2.7.1 There are no simple analytical means to correct for different wing structural loading conditions. If necessary, corrections to the reference condition should be developed by flight test or a suitable analysis process.

3.2.2.7.2 The reference condition for the wing structural loading is to be selected by the applicant based on the amount of fuel and/or removable external stores to be carried by the wing, based on the aeroplane's payload capability and the manufacturer's standard fuel management practices. The reference to the aeroplane's payload capability is to establish the zero fuel mass of the aeroplane, while the reference to the manufacturer's standard fuel management practices is to establish the distribution of that fuel and how that distribution changes as fuel is burned.

3.2.2.7.3 The reference condition for the wing structural loading reference condition should be based on an operationally representative empty weight and payload which defines the zero fuel mass of the aeroplane. The total amount of fuel loaded for each of the three reference masses would be the reference mass minus the zero fuel mass. Standard fuel management practices will determine the amount of fuel present in each fuel tank. An example of standard fuel management practice is to load the main (wing) fuel tanks before loading the centre (body) fuel tanks and to first empty fuel from the centre tanks before using the fuel in the main tanks. This helps keep the CG aft and reduces trim drag.

3.2.2.7.4 Commercial freighters may be designed from scratch, but more often are derivatives of, or are converted from, passenger models. For determining aeroelastic effects, it is reasonable to assume that the reference loading for a freighter is the same as the passenger model it was derived from. If there is no similar passenger model, the reference zero-fuel-mass of a freighter can be based on its payload design density. The payload design density is defined by the full use of the volumetric capacity of the freighter and the highest mass it is designed to carry, expressed in kg/m^3. For example, a typical payload design density for large commercial freighters is 160 kg/m^3.

3.2.2.7.5 Using a reference payload significantly lower than the passenger interior limits or structural limited payload could potentially provide a more beneficial aeroelastic effect. An applicant would need to justify the reference payload assumptions in the context of the capability of the aeroplane and what could be considered typical for the type design.

3.2.2.8 *Fuel lower heating value.* The fuel lower heating value defines the energy content of the fuel. The lower heating value directly affects the fuel flow at a given test condition.

The fuel flow measured during the flight test is corrected to the fuel flow for the reference lower heating value as follows:

$$\Delta Fuel\ Flow_{Corr\ LHV} = Fuel\ Flow_{test\ LHV}\left(\frac{LHV_{test}}{LHV_{Ref}}\right) - Fuel\ Flow_{test\ LHV}$$

where:

ΔFuel $Flow_{Cor\ r\ LHV}$ is the fuel flow correction in kilograms/hour due to the fuel lower heating value being different than the reference fuel lower heating value;

Fuel $Flow_{test\ LHV}$ is the measured fuel flow in kilograms/hour during the test (at the test fuel lower heating value);

LHV_{test} is the fuel lower heating value of the fuel used for the test in MJ/kg; and

LHV_{Ref} is the reference fuel lower heating value = 43.217 MJ/kg.

3.2.2.9 *Altitude.* The altitude at which the aeroplane is flown affects the fuel flow. As noted in Table 3-1, since testing at any altitude other than the reference optimum altitude penalizes the CO_2 MV, this correction may always be considered optional. For applicants choosing to make this correction, the selection of the reference altitude depends on the method used for determining SAR for the three reference aeroplane masses specified in Annex 16, Volume III, 2.3.1. For SAR data clustered around each of the three reference masses (see Annex 16, Volume III, Appendix 1, 6.2), a reference altitude is selected by the applicant for each of the specified reference masses. If SAR data is obtained over a range of masses (see Annex 16, Volume III, Appendix 1, 6.3), the applicant should select a reference altitude for each test point (since none of the test points are directly associated with any of the reference masses). For applicants using first principle models (see 3.4.4), the applicant should select a reference altitude for each target Weight/δ value (see Figure 3-10).

3.2.2.9.1 The engine model should be used to determine the fuel flow at the test altitude (Fuel $Flow_{test\ alt}$) and the fuel flow at the reference altitude (Fuel $Flow_{ref\ alt}$). The fuel flow correction for altitude is determined as follows:

$$\Delta \text{Fuel Flow}_{alt} = \text{Fuel Flow}_{ref\ alt} - \text{Fuel Flow}_{test\ alt}.$$

3.2.2.10 *Temperature.* The ambient temperature affects the fuel flow. The reference temperature is the standard day temperature from the ICAO standard atmosphere at the reference altitude.

3.2.2.10.1 The engine model should be used to determine the difference between the fuel flow at the test temperature (Fuel $Flow_{test\ temp}$) and the fuel flow at the reference temperature (Fuel $Flow_{ref\ temp}$). The fuel flow correction for temperature is determined as follows:

$$\Delta \text{Fuel Flow}_{temp} = \text{Fuel Flow}_{ref\ temp} - \text{Fuel Flow}_{test\ temp}.$$

3.2.2.11 *Engine deterioration level.* When first used, engines undergo a rapid, initial deterioration in fuel efficiency. Thereafter, the rate of deterioration significantly decreases. Engines with less than the reference deterioration level may be used, subject to the approval of the certification authority. As stated in Annex 16, Volume III, Appendix 1, 5.2.1, in such a case, the fuel flow shall be corrected to the reference engine deterioration level using an approved method. Engines with more deterioration than the reference engine deterioration level may be used. In this case, a correction to the reference condition shall not be permitted in accordance with that same paragraph of Annex 16, Volume III.

3.2.2.11.1 As stated above, a correction should generally not be made for engine deterioration level. If an applicant proposes to use an engine or engines with less than the reference deterioration level for testing, it may be possible to establish a conservative correction level to apply to the test fuel flow to represent engines at the reference deterioration level. Such a correction should be substantiated by engine fuel flow deterioration data from the same engine type or family.

3.2.2.12 *Electrical and mechanical power extraction and bleed flow.* Electrical and mechanical power extraction, and bleed flow affect the fuel flow.

3.2.2.12.1 The engine model should be used to determine the difference between the fuel flow at the test power extraction ($\text{Fuel Flow}_{\text{test bleed}}$) and bleed flow and the fuel flow at the reference power extraction and bleed flow ($\text{Fuel Flow}_{\text{ref bleed}}$). The fuel flow correction for electrical and mechanical power extraction and bleed flow is determined as follows:

$$\Delta\text{Fuel Flow}_{\text{Corr bleed}} = \text{Fuel Flow}_{\text{ref bleed}} - \text{Fuel Flow}_{\text{test bleed}}.$$

3.3 VALIDITY OF RESULTS — CONFIDENCE INTERVAL

3.3.1 Introduction

Sections 3.3.2 to 3.3.4 provide an insight into the theory of confidence interval evaluation. Application of this theory and some worked examples are provided in 3.3.4. A suggested bibliography is provided in Appendix 2 to this manual for those wishing to gain a greater understanding.

3.3.2 Direct flight testing

If n measurements of SAR ($y_1, y_2, \ldots, y_n$) are obtained under approximately the same conditions, and it can be assumed that they constitute a random sample from a normal population with true population mean, μ, and true standard deviation, σ, then the following statistics can be derived:

$$\overline{y} = estimate\ of\ the\ mean = \frac{1}{n}\left\{\sum_{i=1}^{i=n} y_{(i)}\right\}$$

$$s = estimate\ of\ the\ standard\ deviation\ of\ the\ mean = \sqrt{\frac{\sum_{i=1}^{i=n}(y_i-\bar{y})^2}{n-1}}.$$

From these and the Student's t-distribution, the confidence interval, CI, for the estimate of the mean, $\overline{y}$, can be determined as:

$$\text{CI} = \bar{y} \pm t_{\left(1-\frac{\alpha}{2},\zeta\right)}\frac{s}{\sqrt{n}}$$

where $t_{\left(1-\frac{\alpha}{2},\zeta\right)}$ denotes the $\left(1-\frac{\alpha}{2}\right)$ percentile of the single-sided Student's t-test with ζ degrees freedom (for a clustered data set $\zeta = n-1$) and where α is defined such that $100(1-\alpha)$ per cent is the desired confidence level for the confidence interval. In other words, it denotes the probability with which the interval will contain the unknown mean, μ. For CO_2 certification purposes, 90 per cent confidence intervals are generally desired and thus $t_{.95,\zeta}$ is used.

See Table 3-9 for a listing of values of $t_{.95,\zeta}$ for different values of ζ.

3.3.3 Regression model

If *n* measurements of SAR ($y_1, y_2, \ldots, y_n$) are obtained under significantly varying values of mass ($x_1, x_2, \ldots, x_n$), respectively, then a polynomial can be fitted to the data by the method of least squares. For determining the mean SAR, μ, the following polynomial regression model is assumed to apply:

$$\mu = B_0 + B_1 x + B_2 x^2 + \ldots + B_k x^k.$$

The estimate of the mean line through the data of the SAR is given by:

$$y = b_0 + b_1 x + b_2 x^2 + \ldots + b_k x^k.$$

Each regression coefficient (B_i) is estimated by b_i from the sample data using the method of least squares in a process summarized as follows:

Each observation (x_i, y_i) satisfies the equations:

$$y_i = B_0 + B_1 x_i + B_2 x_i^2 + \ldots + B_k x_i^k + \varepsilon_i$$

$$= b_0 + b_1 x_i + b_2 x_i^2 + \ldots + b_k x_i^k + e_i$$

where ε_i and e_i are, respectively, the random error and residual associated with the SAR. The random error is assumed to be a random sample from a normal population with mean zero and standard deviation σ. The residual (e_i) is the difference between the measured value and the estimate of the value using the estimates of the regression coefficients and x_i. Its root mean square value (s) is the sample estimate for σ. These equations are often referred to as the normal equations.

Table 3-9. Student's t-distribution (for 90 per cent confidence) for various degrees of freedom

Degrees of freedom (ζ)	$t_{.95,\zeta}$
1	6.314
2	2.920
3	2.353
4	2.132
5	2.015
6	1.943
7	1.895
8	1.860
9	1.833
10	1.812

Degrees of freedom (ζ)	$t_{.95,\zeta}$
12	1.782
14	1.761
16	1.746
18	1.734
20	1.725
24	1.711
30	1.697
60	1.671
>60	1.645

The n data points of measurements (x_i,y_i) are processed as follows:

Each elemental vector ($\underline{x}_i$) and its transpose ($\underline{x}'_i$) are formed such that:

$$\underline{x}_i = (1 \quad x_i \quad x_i^2 \quad . \quad . \quad x_i^k), \text{ a row vector; and}$$

$$x_i' = \begin{pmatrix} 1 \\ x_i \\ x_i^2 \\ \cdot \\ \cdot \\ x_i^k \end{pmatrix}, \text{ a column vector.}$$

A matrix $\underline{X}$ is formed from all the elemental vectors $\underline{x}_i$ for i = 1, ….., n. $\underline{X}'$ is the transpose of $\underline{X}$. A matrix $\underline{A}$ is defined such that $\underline{A} = \underline{X}'\underline{X}$ and a matrix $\underline{A}^{-1}$ is the inverse of $\underline{A}$. In addition, $\underline{y} = (y_1\ y_2\ \ldots\ y_n)$ and $\underline{b} = (b_0\ b_1\ \ldots\ b_2)$, with $\underline{b}$ determined as the solution of the normal equations:

$$\underline{y} = \underline{Xb} \text{ and } \underline{X}'\underline{y} = \underline{X}'\underline{Xb} = \underline{Ab}$$

to give

$$\underline{b} = \underline{A}^{-1}\underline{X}'\underline{y}.$$

The 90 per cent confidence interval CI_{90} for the mean value of the SAR estimated with the associated value of the mass x_0 is then defined as:

$$CI_{90} = \bar{y}\,(x_0) \pm t_{.95,\zeta}\, s\ v(x_0)$$

where $v(x_0) = \sqrt{\underline{x}_0\ \underline{A}^{-1}\ \underline{x}_0'}$.

Thus, $CI_{90} = \bar{y}\,(x_0) \pm t_{.95,\zeta}\ s\ \sqrt{\underline{x}_0\ \underline{A}^{-1}\ \underline{x}_0'}$, where:

$\underline{x}_0 = (1\ x_0\ x_0^2 \ldots x_0^k)$;

$\underline{x}'_0$ is the transpose of $\underline{x}_0$;

$\bar{y}\,(x_0)$ is the estimate of the mean value of the SAR at the associated value of the mass x_o;

$t_{.95,\zeta}$ is obtained for ζ degrees of freedom. For the general case of a multiple regression analysis involving K independent variables (i.e. $K + 1$ coefficients), ζ is defined as $\zeta = n - K - 1$ (for the specific case of a polynomial regression analysis, for which k is the order of curve fit, there are k variables independent of the dependent variable, and so $\zeta = n - k - 1$); and

$s = \sqrt{\frac{\sum_{i=1}^{i=n}(y_i - \bar{y}(x_i))^2}{n-K-1}}$ is the estimate of σ, the true standard deviation.

3.3.4 Worked examples of the determination of 90 per cent confidence intervals

3.3.4.1 *Direct flight testing*

3.3.4.1.1 Example 1: the confidence interval is less than the confidence interval limit.

Consider the following set of 6 independent measurements of SAR obtained by flight test around one of the three reference masses of the CO_2 emissions evaluation metric. After correction to reference conditions, the following clustered data set of SAR values is obtained:

Table 3-10. Measurements of SAR — Example 1

Measurement number	*Corrected SAR (km/kg)*
1	0.38152
2	0.38656
3	0.37988
4	0.38011
5	0.38567
6	0.37820

The number of data points (n) = 6

The degrees of freedom (n-1) = 5

The Student's t-distribution for 90 per cent confidence and 5 degrees of freedom ($t_{(.95\ ,5)}$) = 2.015 (see Table 3-9)

Note.— 6 is the minimum number of test points requested, as stated in Annex 16, Volume III, Appendix 1, 6.2.

Estimate of the mean SAR ($\overline{SAR}$) for the clustered data set:

$$\overline{SAR} = \frac{1}{n}\left\{\sum_{i=1}^{i=n} SAR_{(i)}\right\} = 0.38282 \text{ km/kg.}$$

Estimate of the standard deviation(s):

$$s = \sqrt{\frac{\sum_{i=1}^{i=n}\left(SAR_{(i)} - \overline{SAR}\right)^2}{n-1}} = 0.00344 \text{ km/kg .}$$

Confidence interval determination

The 90 per cent confidence interval (CI_{90}) is calculated as follows (see 3.3.2):

$$CI_{90} = \overline{SAR} \pm t_{(.95,n-1)}\frac{s}{\sqrt{n}} = 0.38282 \pm 2.015 \text{ x } \frac{0.00344}{\sqrt{6}} = 0.38282 \pm 0.00283 \text{ km/kg.}$$

Check of confidence interval limits

The confidence interval extends to ±0.00283 km/kg around the mean SAR value of the clustered data set (0.38282 km/kg). This represents ±0.74 per cent of the mean SAR value, which is below the confidence interval limit of 1.5 per cent, as defined in Annex 16, Volume III, Appendix 1, 6.4.

As a result, the SAR value of 0.38282 km/kg associated to one of the reference masses of the CO_2 emissions evaluation metric can be used for the metric determination.

3.3.4.1.2 Example 2: the confidence interval exceeds the confidence interval limit.

Consider the following set of 6 independent measurements of SAR obtained by flight test around one of the three reference masses of the CO_2 emissions evaluation metric. After correction to reference conditions, the following clustered data set of SAR values is obtained:

Table 3-11. Measurements of SAR — Example 2

Measurement number	*Corrected SAR (km/kg)*
1	0.15208
2	0.15795
3	0.15114
4	0.15225
5	0.15697
6	0.15834

The number of data points (n) = 6

The degrees of freedom (n-1) = 5

The Student's t-distribution for 90 per cent confidence and 5 degrees of freedom ($t_{(.95\,,5)}$) = 2.015 (see Table 3-9)

Estimate of the mean SAR ($\overline{SAR}$) for the clustered data set:

$$\overline{SAR} = \frac{1}{n}\left\{\sum_{i=1}^{i=n} SAR_{(i)}\right\} = 0.15479 \text{ km/kg.}$$

Estimate of the standard deviation(s):

$$s = \sqrt{\frac{\sum_{i=1}^{i=n}\left(SAR_{(i)} - \overline{SAR}\right)^2}{n-1}} = 0.0033 \text{ km/kg.}$$

Confidence interval determination

The 90 per cent confidence interval (CI_{90}) is calculated as follows (see 3.3.2):

$$CI_{90} = \overline{SAR} \pm t_{(.95,n-1)} \frac{s}{\sqrt{n}} = 0.15479 \pm 2.015 \text{ x } \frac{0.0033}{\sqrt{6}} = 0.15479 \pm 0.00271 \text{ km/kg.}$$

Check of confidence interval limits

The confidence interval extends to ±0.00271 km/kg around the mean SAR value of the clustered data set (0.15479 km/kg). This represents ±1.75 per cent of the mean SAR value, which is above the confidence interval limit of 1.5 per cent, as defined in Annex 16, Volume III, Appendix 1, 6.4.

In such case, a penalty equal to the amount that the 90 per cent confidence interval exceeds ±1.5 per cent shall be applied to the mean SAR value, i.e. (1.75-1.50) = 0.25 per cent. The mean SAR value shall therefore be penalized by an amount of 0.25 per cent as follows:

$$\overline{SAR} = \left(1 - \frac{0.25}{100}\right) x\ 0.15479 = 0.15440 \text{ km/kg.}$$

As a result, the SAR value of 0.15440 km/kg associated to one of the reference masses of the CO_2 emissions evaluation metric can be used for the metric determination.

3.3.4.2 *Regression model*

3.3.4.2.1 Example 3: the confidence interval at each of the three reference masses of the CO_2 emissions evaluation metric is less than the confidence interval limit.

Consider the following set of 12 measurements of SAR obtained by flight test at optimum speed and optimum altitude as a function of the aeroplane gross mass. After SAR correction to reference conditions, the following data set is obtained:

Table 3-12. Measurements of SAR — Example 3

Measurement number and reference mass	*Gross mass (m_i) (kg)*	*Corrected SAR (SAR_i) (km/kg)*
1	17 800	0.928
Low mass (*)	17 825	
2	17 970	0.905
3	18 400	0.908
4	18 850	0.884
5	19 500	0.850
6	19 950	0.845
Mid mass (*)	19 953	
7	20 180	0.833
8	20 350	0.818
9	21 000	0.792
10	21 500	0.781
11	21 870	0.779
High mass (*)	22 080	
12	22 150	0.771

(*) Low, mid and high mass represent the reference masses of the CO_2 emissions evaluation metric, as defined in Annex 16, Volume III, Part II, Chapter 2, 2.3.

The number of data points (n) = 12

Note.— 12 is the minimum number of test points requested, as stated in Annex 16, Volume III, Appendix 1, 6.3.

A representation of the above measurement points is proposed in Figure 3-4.

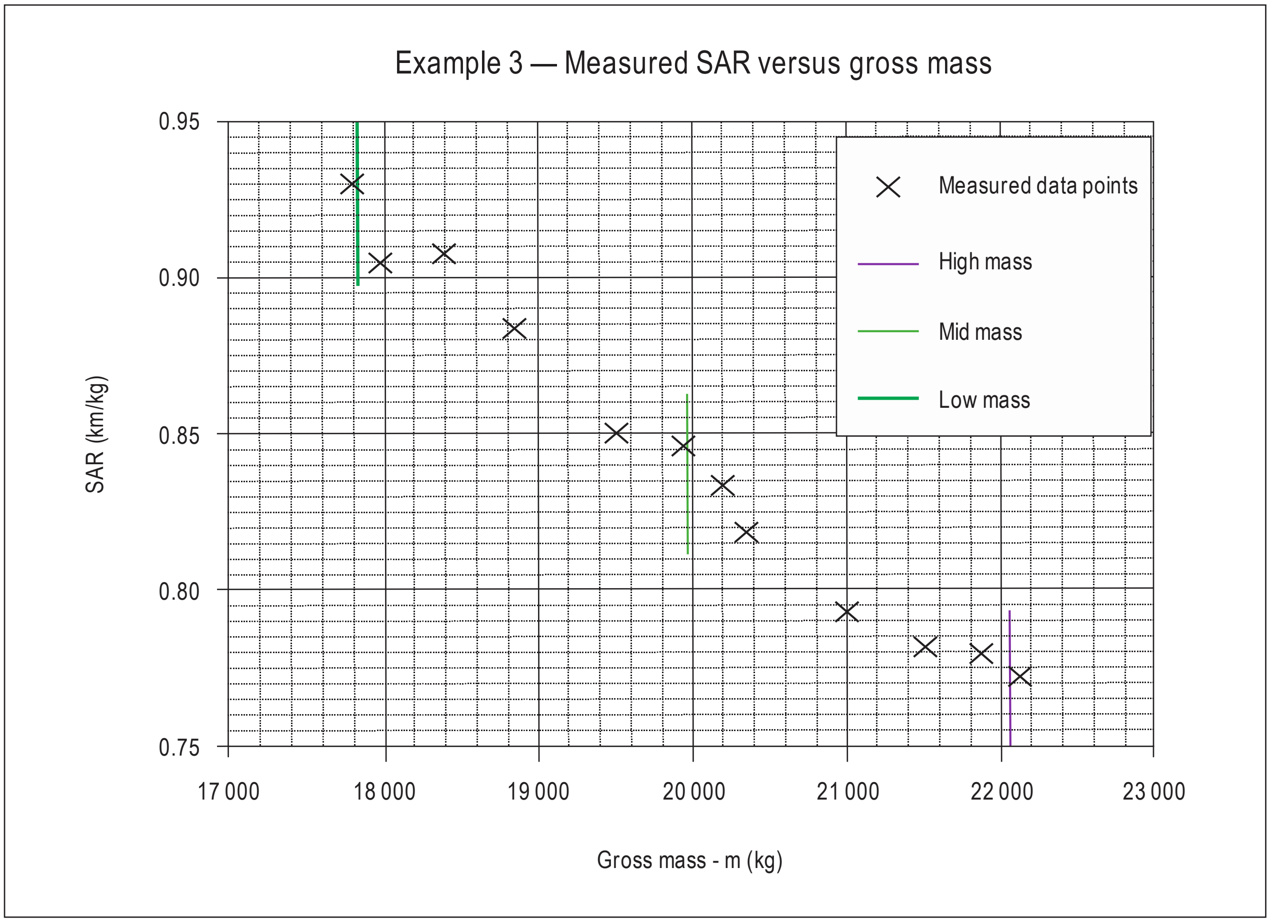

Figure 3-4. Measured SAR versus gross mass — Example 3

Estimate of the mean SAR model by polynomial regression

In order to estimate the SAR model (SAR_{av}) as a function the aeroplane gross mass (m), a polynomial regression of second order is proposed so that:

$$SAR_{av} = B_0 + B_1\, m + B_2\, m^2$$

Each observation (m_i, SAR_i), for i = 1, …, 12 satisfies the equation:

$$SAR_{(i)} = b_0 + b_1\, m_i + b_2\, m_i^2 + e_i$$

where e_i = residual error (difference between the measured SAR value and its estimate).

Under a matrix form this gives:

$$\begin{pmatrix} SAR_1 \\ SAR_2 \\ SAR_3 \\ \vdots \\ SAR_{12} \end{pmatrix} = \begin{pmatrix} 1 & m_1 & m_1^2 \\ 1 & m_2 & m_2^2 \\ 1 & m_3 & m_3^2 \\ \vdots & \vdots & \vdots \\ 1 & m_{12} \cdots & m_{12}^2 \end{pmatrix} \begin{pmatrix} b_0 \\ b_1 \\ b_2 \end{pmatrix} + \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ \vdots \\ e_{12} \end{pmatrix}$$

$$\underline{SAR} = \underline{M}\ \underline{b} + \underline{e}$$

where:

$$\underline{SAR} = \begin{pmatrix} 0.928 \\ 0.905 \\ 0.908 \\ 0.884 \\ 0.850 \\ 0.845 \\ 0.833 \\ 0.818 \\ 0.792 \\ 0.781 \\ 0.779 \\ 0.771 \end{pmatrix} \quad \underline{M} = \begin{pmatrix} 1 & 17800 & 17800^2 \\ 1 & 17970 & 17970^2 \\ 1 & 18400 & 18400^2 \\ 1 & 18850 & 18850^2 \\ 1 & 19500 & 19500^2 \\ 1 & 19950 & 19950^2 \\ 1 & 20180 & 20180^2 \\ 1 & 20350 & 20350^2 \\ 1 & 21000 & 21000^2 \\ 1 & 21500 & 21500^2 \\ 1 & 21870 & 21870^2 \\ 1 & 22150 & 22150^2 \end{pmatrix} \quad \underline{b} = \begin{pmatrix} b_0 \\ b_1 \\ b_2 \end{pmatrix} \quad \underline{e} = \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ \vdots \\ e_n \end{pmatrix}.$$

The least square principle consists in looking for the parameter values of vector B, minimizing the sum of the squares of residuals, i.e.:

$$\text{Min} \sum_{i=1}^{i=12} e_i^2 = \min \sum_{i=1}^{i=12} (SAR_{(i)} - b_0 - b_1\, m_i - b_2\, m_i^2)^2.$$

It is equivalent to look for the solutions of $\frac{\partial(\sum e_i^2)}{\partial b_j} = 0$ for j = (0, 1, 2).

The solution $\underline{B} = \begin{pmatrix} B_0 \\ B_1 \\ B_2 \end{pmatrix}$ is given by:

$$\underline{A^{-1}}\ \underline{M'}\ \underline{SAR} \text{ (see 3.3.3)}$$

where:

$\underline{M'}$ = transpose of $\underline{M}$; and

$\underline{A^{-1}} = (\underline{M'}\ \underline{M})^{-1}$ = inverse of $(\underline{M'}\ \underline{M})$.

Finally, $\underline{B} = \begin{pmatrix} 2.402921963 \\ -0.000120515 \\ 2.10695\ x10^{-09} \end{pmatrix}$; and

$$SAR_{av} = 2.402921963 - 0.000120515\ m + 2.10695.10^{-9}\ m^2.$$

Figure 3-5 provides a representation of the mean SAR model as a function of the aeroplane gross mass.

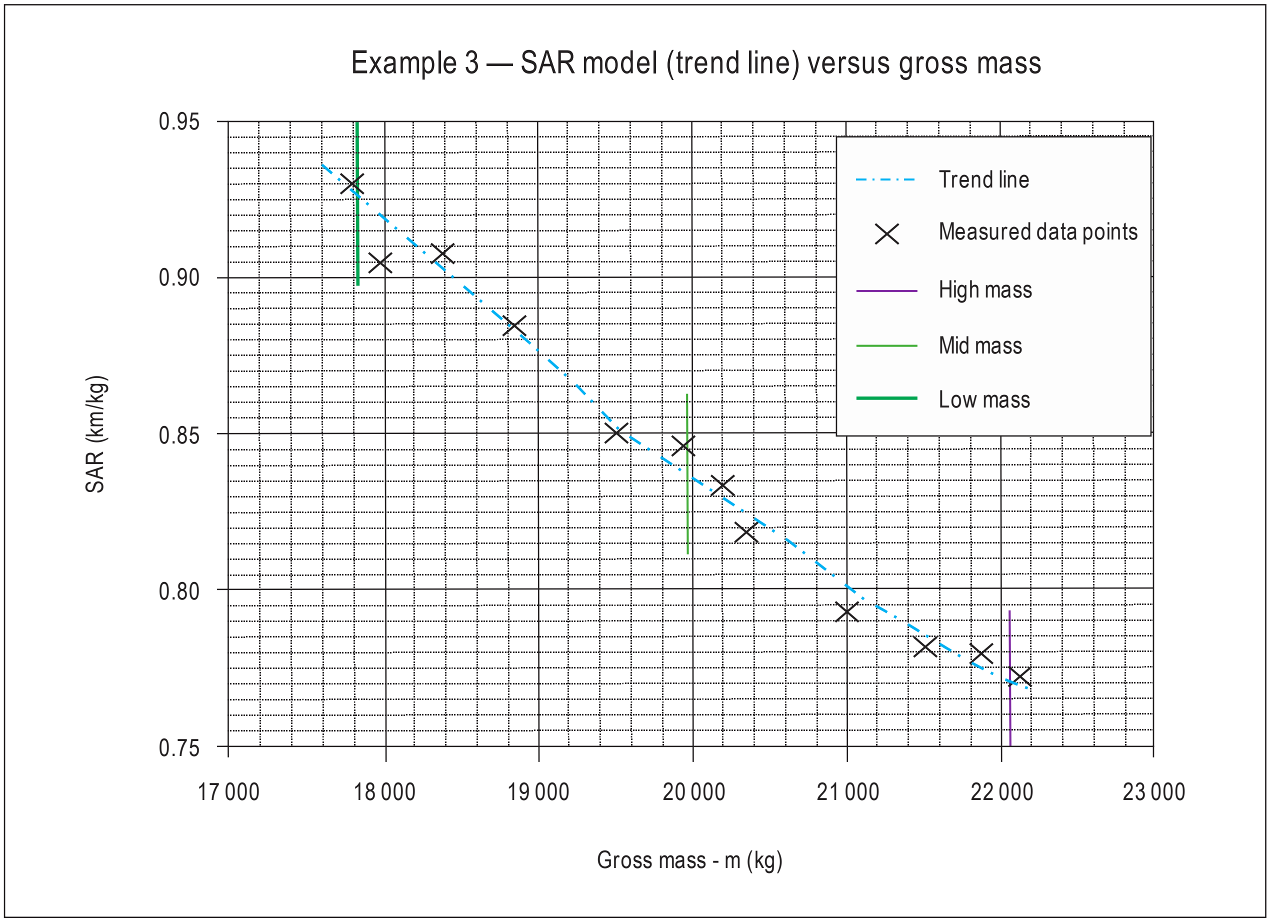

Figure 3-5. SAR model (trend line) versus gross mass — Example 3

The mean SAR values at each of the three reference gross masses of the CO_2 emissions evaluation metric are as follows:

Table 3-13. Mean SAR values — Example 3

Reference mass	*Mass value (kg)*	*Mean SAR value (km/kg)*
Low mass	17 825	0.92418
Mid mass	19 953	0.83710
High mass	22 080	0.76914

Estimate of the standard deviation(s):

$$s = \sqrt{\frac{\sum_{i=1}^{i=n}(SAR_i - SAR_{av(i)})^2}{n-K-1}} = 0.00765\ km/kg$$

where:

the number of data points (n) = 12;

K = 2 for a second order polynomial regression (see 3.3.3); and

the degrees of freedom (n-K-1) = 9.

Confidence interval determination

The 90 per cent confidence interval (CI_{90}) at an aeroplane gross mass m_0 is calculated as follows (see 3.3.3):

$$CI_{90} = SAR_{av}(m_0) \pm t_{(.95,n-K-1)}\, s \sqrt{\underline{m_0}\ \underline{A^{-1}}\ \underline{m_0'}}$$

where:

the Student's t-distribution for 90 per cent confidence and 9 degrees of freedom $t_{(.95,9)}$ = 1.833 (see Table 3-9);

$m_0 = (1\ m_0\ m_0^2)$ and $m_0' = \begin{pmatrix} 1 \\ m_0 \\ m_0^2 \end{pmatrix}$; and

$A^{-1} = (M'\ M)^{-1}$ = inverse of (M' M).

Figure 3-6 provides a representation of the 90 per cent confidence interval as a function of aeroplane gross mass.

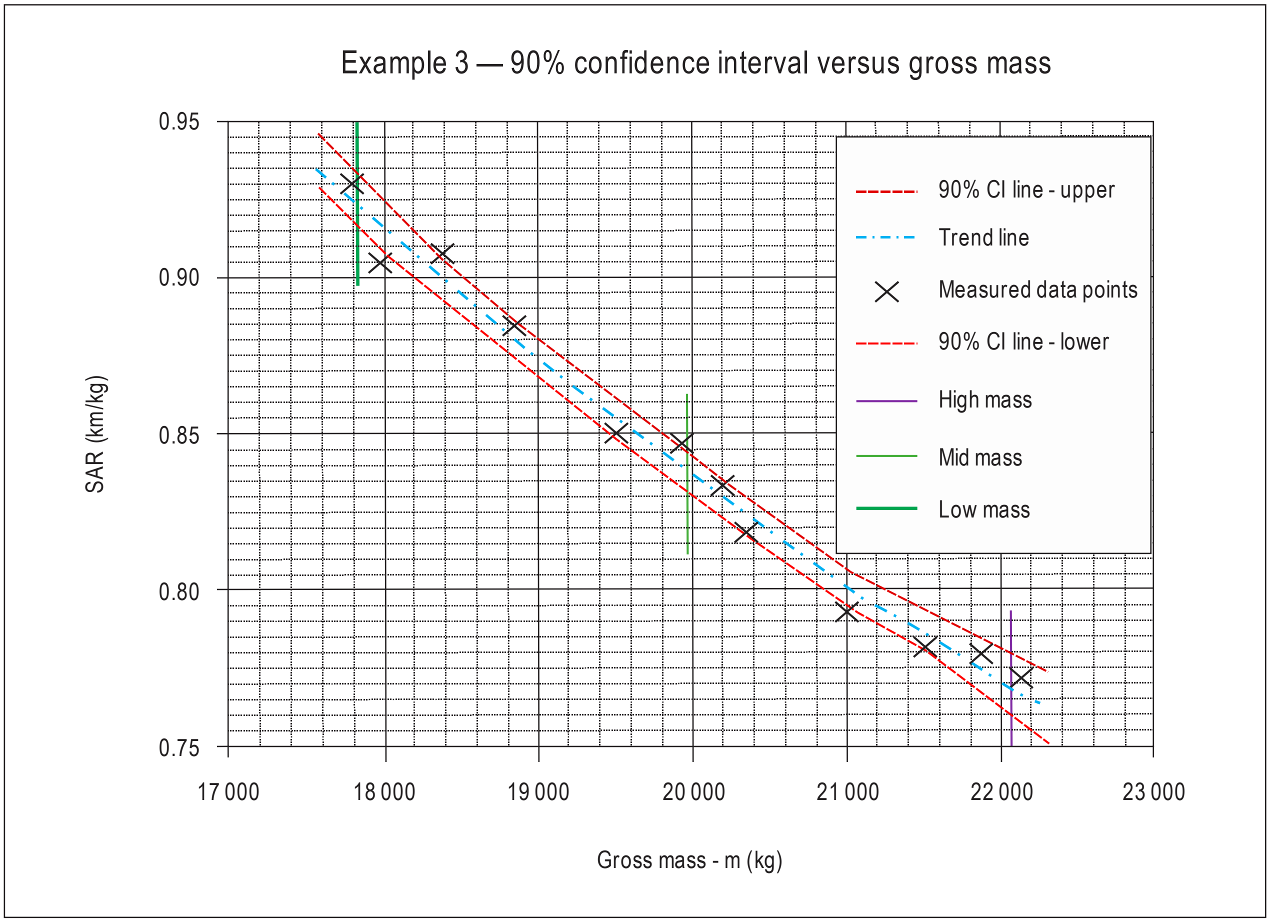

Figure 3-6. 90 per cent confidence interval versus gross mass — Example 3

The 90 per cent confidence intervals at each of the three reference gross masses of the CO_2 emissions evaluation metric are as follows:

Table 3-14. Confidence intervals — Example 3

Reference mass	*Mass value (kg)*	*90% confidence interval (kg/km)*
Low mass	17 825	CI_{90} = 0.92418 ± 0.00915
Mid mass	19 953	CI_{90} = 0.83710 ± 0.00619
High mass	22 080	CI_{90} = 0.76914 ± 0.00925

Check of confidence interval limits

For each of the three reference gross masses of the CO_2 emissions evaluation metric, the confidence interval extends around the mean SAR value to an amount in per cent, provided in Table 3-15.

Table 3-15. Check of confidence intervals — Example 3

Reference mass	*Mass value (kg)*	*90% confidence interval (percentage of mean SAR)*
Low mass	17 825	(0.00915/0.92418) x 100 = 0.99%
Mid mass	19 953	(0.00619/0.83710) x 100 = 0.74%
High mass	22 080	(0.00925/0.76914) x 100 = 1.2%

The 90 per cent confidence intervals at each of the three reference gross masses of the CO_2 emissions evaluation metric are all below the confidence interval limit of 1.5 per cent, as defined in Annex 16, Volume III, Appendix 1, 6.4.

As a result, the following mean SAR values associated to each of the three reference masses of the CO_2 emissions evaluation metric can be used for the metric determination.

Table 3-16. Mean SAR values — Example 3

Reference mass	*Mass value (kg)*	*Mean SAR value (km/kg)*
Low mass	17 825	0.92418
Mid mass	19 953	0.83710
High mass	22 080	0.76914

3.3.4.2.2 Example 4: the confidence interval of at least one of the three reference masses of the CO_2 emissions evaluation metric exceeds the confidence interval limit.

Consider the following set of 12 measurements of SAR obtained by flight test at optimum speed and optimum altitude as a function of the aeroplane gross mass. After SAR correction to reference conditions, the following data set is obtained:

Table 3-17. Measurements of SAR — Example 4

Measurement number and reference mass	*Gross mass (m_i) (kg)*	*Corrected SAR (SAR_i) (km/kg)*
1	17 800	0.932
Low mass (*)	17 825	
2	18 200	0.925
3	18 620	0.913
4	18 890	0.889
5	19 350	0.868
6	19 610	0.848
Mid mass (*)	19 953	
7	19 920	0.838
8	20 510	0.830
9	20 790	0.806
10	21 220	0.815
11	21 480	0.779
High mass (*)	22 080	
12	22 100	0.788

(*) *Low, mid and high mass represent the reference masses of the CO_2 emissions evaluation metric, as defined in Annex 16, Volume III, Part II, Chapter 2, 2.3.*

The number of data points (n) = 12 (minimum requested, as stated in Annex 16, Volume III, Appendix 1, 6.3).

A representation of the above measurement points is proposed in Figure 3-7.

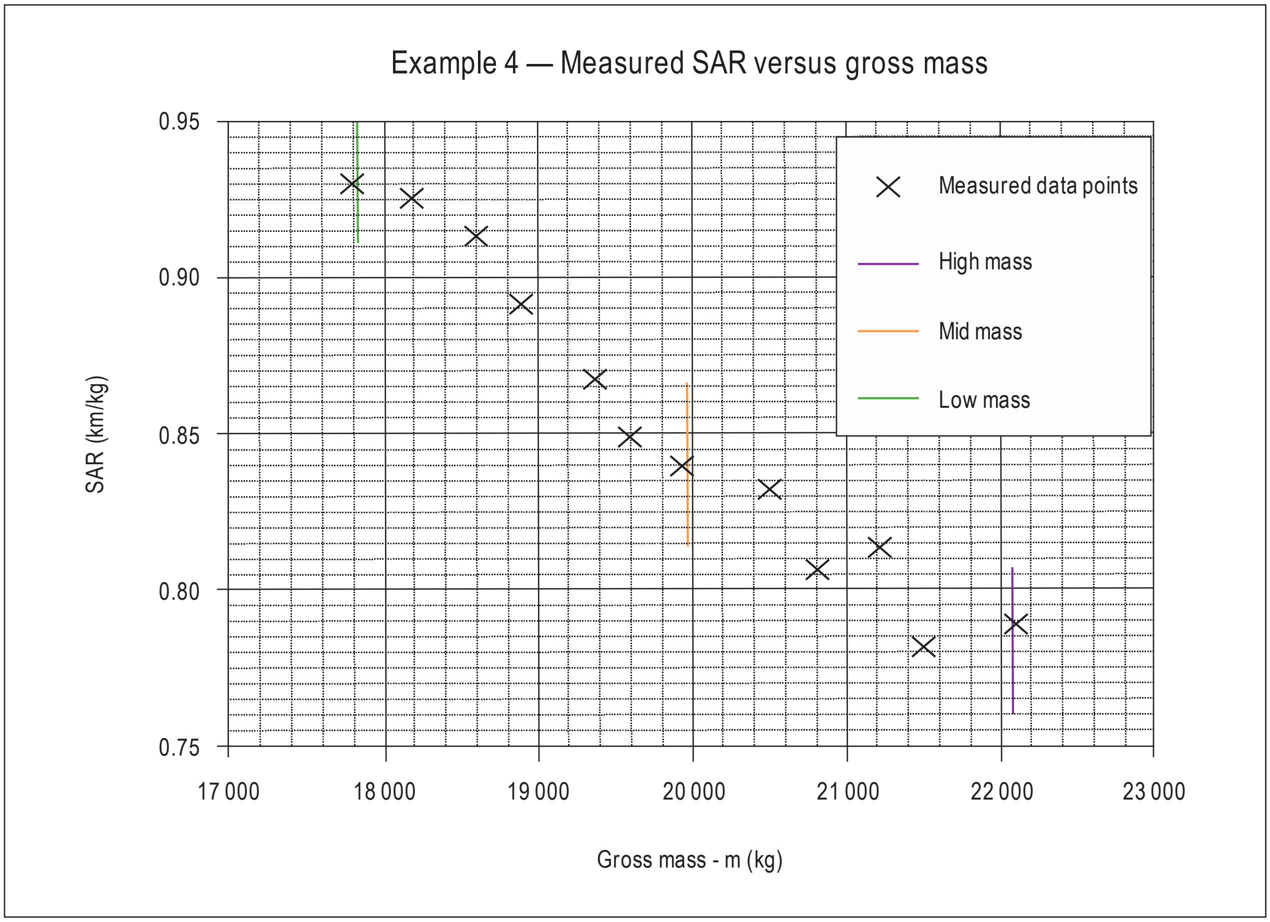

Figure 3-7. Measured SAR versus gross mass — Example 4

Estimate of the mean SAR model by polynomial regression

In order to estimate the SAR model (SAR_{av}) as a function the aeroplane gross mass (m), a polynomial regression of second order is proposed, so that:

$$SAR_{av} = B_0 + B_1\, m + B_2\, m^2.$$

Each observation (m_i, SAR_i), for i = 1, …, 12 satisfies the equation:

$$SAR_{(i)} = b_0 + b_1\, m_i + b_2\, m_i^2 + e_i$$

with e_i = residual error (difference between the measured SAR value and its estimate).

Under a matrix form this gives:

$$\begin{pmatrix} SAR_1 \\ SAR_2 \\ SAR_3 \\ \vdots \\ SAR_{12} \end{pmatrix} = \begin{pmatrix} 1 & m_1 & m_1^2 \\ 1 & m_2 & m_2^2 \\ 1 & m_3 & m_3^2 \\ \vdots & \vdots & \vdots \\ 1 & m_{12} \cdots & m_{12}^2 \end{pmatrix} \begin{pmatrix} b_0 \\ b_1 \\ b_2 \end{pmatrix} + \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ \vdots \\ e_{12} \end{pmatrix}$$

$$\underline{SAR} = \underline{M}\ \underline{b} + \underline{e}$$

where:

$$SAR = \begin{pmatrix} 0.932 \\ 0.925 \\ 0.913 \\ 0.889 \\ 0.868 \\ 0.848 \\ 0.838 \\ 0.830 \\ 0.806 \\ 0.815 \\ 0.779 \\ 0.788 \end{pmatrix} \quad M = \begin{pmatrix} 1 & 17800 & 17800^2 \\ 1 & 18200 & 18200^2 \\ 1 & 18620 & 18620^2 \\ 1 & 18890 & 18890^2 \\ 1 & 19350 & 19350^2 \\ 1 & 19610 & 19610^2 \\ 1 & 19920 & 19920^2 \\ 1 & 20510 & 20510^2 \\ 1 & 20790 & 20790^2 \\ 1 & 21220 & 21220^2 \\ 1 & 21480 & 21480^2 \\ 1 & 22100 & 22100^2 \end{pmatrix} \quad b = \begin{pmatrix} b_0 \\ b_1 \\ b_2 \end{pmatrix} \quad e = \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ \vdots \\ e_n \end{pmatrix}.$$

The least square principle consists in looking for the parameter values of vector B, minimizing the sum of the squares of residuals, i.e.:

$$\text{Min} \sum_{i=1}^{i=12} e_i^2 = \min \sum_{i=1}^{i=12} (SAR_{(i)} - b_0 - b_1\, m_i - b_2\, m_i^2)^2.$$

It is equivalent to look for the solutions of $\frac{\partial(\sum e_i^2)}{\partial b_j} = 0$ for j = (0, 1, 2).

The solution $B = \begin{pmatrix} B_0 \\ B_1 \\ B_2 \end{pmatrix}$ is given by:

$$A^{-1}\, M'\, SAR \text{ (see 3.3.3)}$$

where:

M' = transpose of M; and

$A^{-1} = (M'\, M)^{-1}$ = inverse of (M' M).

Finally, $B = \begin{pmatrix} 3.26727172 \\ -0.000205692 \\ 4.21798\ x10^{-09} \end{pmatrix}$; and

$$SAR_{av} = 3.26727172 - 0.000205692\ m + 4.21798.10^{-9}\ m^2.$$

Figure 3-8 provides a representation of the mean SAR model as a function of the aeroplane gross mass.

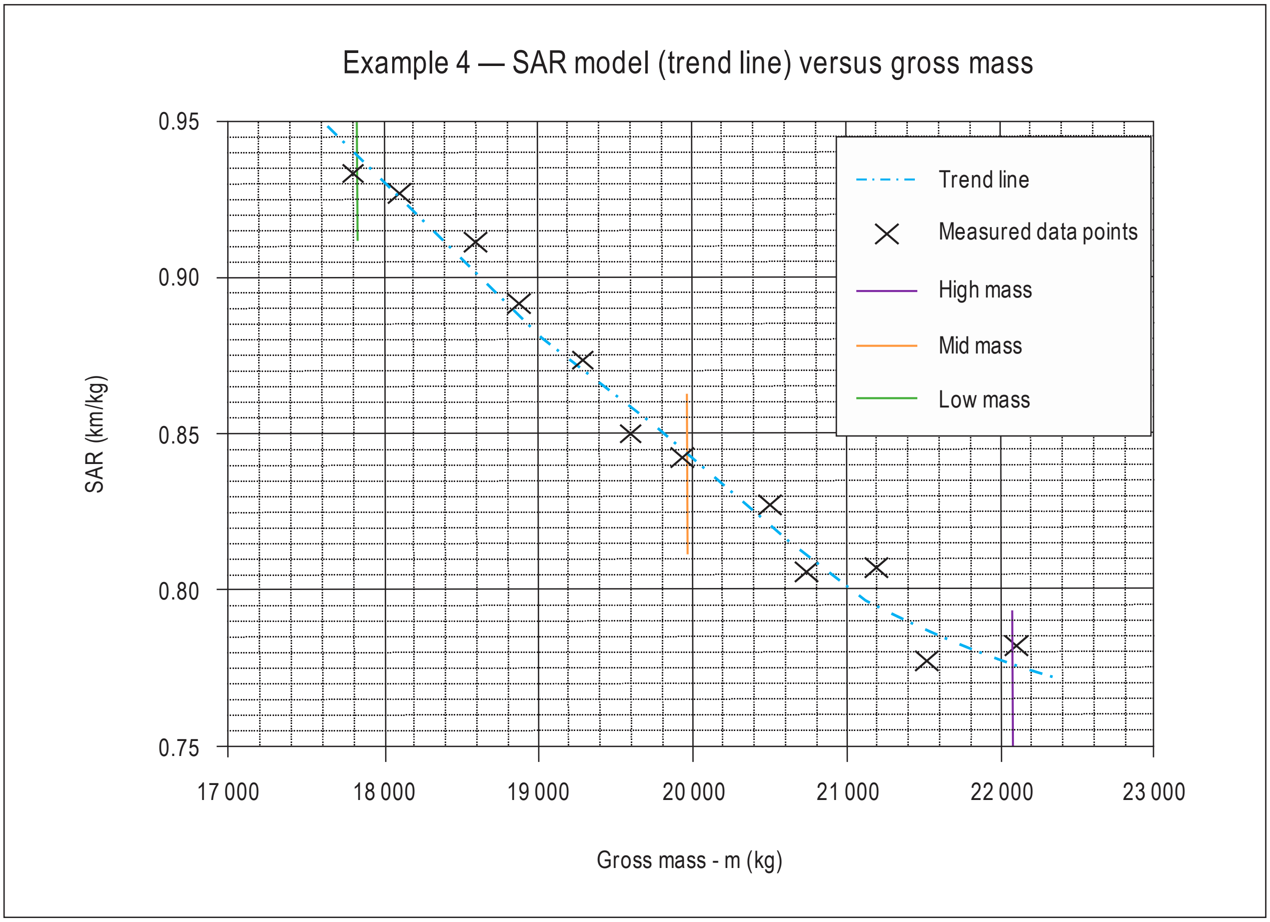

Figure 3-8. SAR model (trend line) versus gross mass — Example 4

The mean SAR values at each of the three reference gross masses of the CO_2 emission evaluation metric are as follows:

Table 3-18. Mean SAR value — Example 4

Reference mass	*Mass value (kg)*	*Mean SAR value (km/kg)*
Low mass	17 825	0.94100
Mid mass	19 953	0.84238
High mass	22 080	0.78198

Estimate of the standard deviation(s):

$$s = \sqrt{\frac{\sum_{i=1}^{i=n}(SAR_i - SAR_{av(i)})^2}{n-K-1}} = 0.01050 \text{ km/kg}$$

where:

the number of data points (n) = 12;

K = 2 for a second order polynomial regression (see 3.3.3); and

the degrees of freedom (n-K-1) = 9.

Confidence interval determination

The 90 per cent confidence interval (CI_{90}) at an aeroplane gross mass m_0 is calculated as follows (see 3.3.3):

$$CI_{90} = SAR_{av}(m_0) \pm t_{(.95, n-K-1)}\, s \sqrt{\underline{m_0}\, \underline{A^{-1}}\, \underline{m_0}'}$$

where:

the Student's t-distribution for 90 per cent confidence and 9 degrees of freedom $t_{(.95, 9)}$ = 1.833 (see Table 3-9);

$m_0 = (1\ m_0\ m_0^2)$ and $m_0' = \begin{pmatrix} 1 \\ m_0 \\ m_0^2 \end{pmatrix}$; and

$A^{-1} = (M'\ M)^{-1}$ = inverse of (M' M).

Figure 3-9 provides a representation of the 90 per cent confidence interval as a function of aeroplane gross mass.

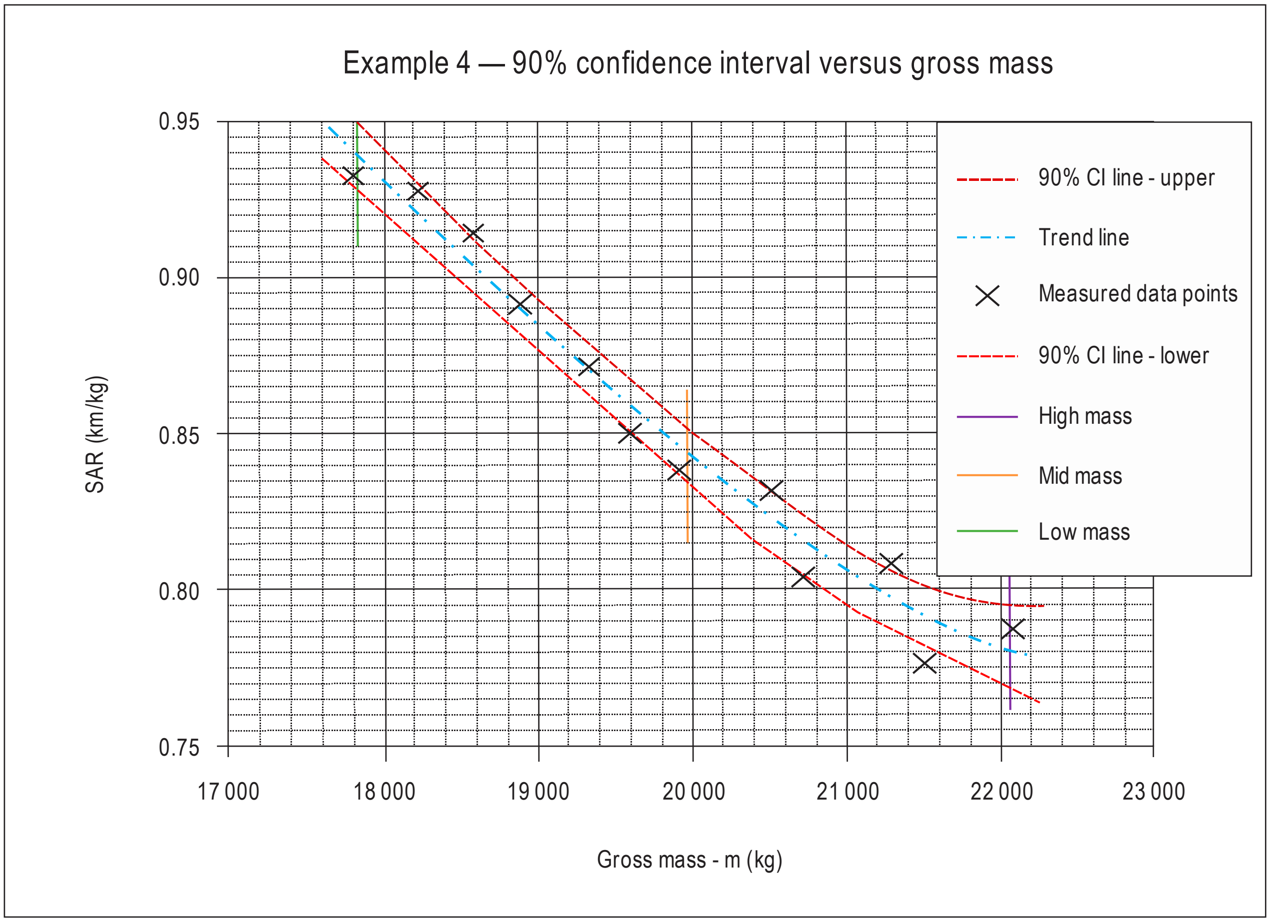

Figure 3-9. 90 per cent confidence interval versus gross mass — Example 4

The 90 per cent confidence intervals at each of the three reference gross masses of the CO_2 emissions evaluation metric are as follows:

Table 3-19. Confidence intervals — Example 4

Reference mass	*Mass value (kg)*	*90% confidence interval (kg/km)*
Low mass	17 825	$CI_{90} = 0.94100 \pm 0.01399$
Mid mass	19 953	$CI_{90} = 0.84238 \pm 0.00823$
High mass	22 080	$CI_{90} = 0.78198 \pm 0.01505$

Check of confidence interval limits

For each of the three reference gross masses of the CO_2 emissions evaluation metric, the confidence interval extends around the mean SAR value to an amount provided in Table 3-20.

Table 3-20. Check of confidence intervals — Example 4

Reference mass	*Mass value (kg)*	*90% confidence interval (percentage of mean SAR)*
Low mass	17 825	(0.01399/0.94100) x 100 = 1.52%
Mid mass	19 953	(0.00823/0.84238) x 100 = 0.98%
High mass	22 080	(0.01505/0.78198) x 100 = 1.93%

The 90 per cent confidence intervals at the low and high reference gross masses of the CO_2 emissions evaluation metric are above the confidence interval limit of 1.5 per cent, as defined in Annex 16, Volume III, Appendix 1, 6.4.

In such case, a penalty equal to the amount that the 90 per cent confidence interval exceeds ±1.5 per cent shall be applied to the mean SAR values as follows:

Table 3-21. Corrected SAR values — Example 4

Reference mass	*Mass value (kg)*	*Corrected SAR value (km/kg)*
Low mass	17 825	0.94100 x [1 - (1.52-1.5)/100] = 0.94081
High mass	22 080	0.78198 x [1 - (1.93-1.5)/100] = 0.77862

As a result, the following mean SAR values associated to each of the three reference masses of the CO_2 emissions evaluation metric can be used for the metric determination.

Table 3-22. Mean SAR values — Example 4

Reference mass	*Mass value (kg)*	*Mean SAR value (km/kg)*
Low mass	17 825	0.94081
Mid mass	19 953	0.84238
High mass	22 080	0.77862

3.4 EQUIVALENT PROCEDURES

Annex 16, Volume III establishes the methods of compliance as being direct flight test or by use of a validated performance model. The procedures presented in this section may be used in lieu of those procedures in the Annex, subject to approval by the certificating authority.

3.4.1 Approval based on existing data

3.4.1.1 The use of existing data as an equivalent procedure may be requested by applicants, and it can be utilized in the below approach of equivalent procedures, or other approaches approved by the certificating authority according to their technical judgement.

a) Develop a regression curve approach for SAR values across the gross weight (MTOM) range using existing data.

3.4.1.2 The information typically needed to use existing data for demonstrating compliance as an equivalent procedure is as follows:

a) The existing model used company test data that was not witnessed by an authority and/or the aeroplane type design was not conformed by an authority, but the data obtained is deemed acceptable by the certificating authority.

b) The accuracy of the instrumentation and the data reduction processes may not have been documented to the quality standard desired for certification, or the original documentation may not have been retained, but the data available is deemed acceptable by the certificating authority.

3.4.2 Approval of a change based on back-to-back testing

The use of back-to-back test data may be requested by applicants as an equivalent procedure for determining the CO_2 evaluation metric value for a relatively small type design change (e.g., antenna installations or other simple drag changes). This approach will typically not be appropriate for engine changes where the specific fuel consumption (SFC) of the engine may change due to internal changes. This compliance approach will likely be especially useful for supplemental Type Certificate (STC) modifiers who do not have access to the original flight test data from the aeroplane manufacturer.

a) Back-to-back testing should be accomplished on the same aeroplane and engines with the modification installed and not installed.

b) Instrumentation adequate to provide data meeting the accuracy requirements of the standard should be installed in the test aeroplane.

c) The data reduction and comparison processes should be acceptable to the certificating authority.

3.4.3 Approval of a change based on analysis

3.4.3.1 The use of analytical processes may be requested by an applicant to evaluate a change to the CO_2 emissions evaluation metric value of a previously approved aeroplane type design, and establish compliance with Annex 16, Volume III, provided those processes are approved by the certificating authority according to their technical judgement. Paragraphs 3.4.3.2 and 3.4.3.3 provide a non-exhaustive list of possible analytical methods, which do not preclude the use of other equivalent procedures.

3.4.3.2 *Changes affecting aeroplane aerodynamics characteristics*

3.4.3.2.1 Possible methods available to evaluate the effect of aerodynamic changes on aeroplane drag are:

a) semi-empirical methods (SEM): these methods (ESDU, Hörner, NACA or NASA for instance) can be found in the bibliography;

b) computational fluid dynamics (CFD): these numerical analysis methods do not take into account aeroplane structural deformation for drag determination;

c) computational fluid dynamics combined with computational structural mechanics (CFD-CSM): these methods combine classic CFD with an analysis of structural deformation whose effect on drag is taken into account; and

d) wind tunnel testing.

3.4.3.2.2 The use of above methods may depend on the type of change listed below.

a) For design changes affecting parasitic drag and/or profile drag (i.e. friction drag and/or viscous pressure drag), SEM, CFD or wind tunnel testing could be used. In this case, it is assumed that aeroplane aeroelastics are not affected.

b) For all other design changes, CFD could be used for items not affecting aeroplane aeroelastics, CFD-CSM could be used for items affecting aeroplane aeroelastics, and wind tunnel testing could be used in all cases.

3.4.3.3 *Changes affecting propulsion system specific fuel consumption (SFC)*

3.4.3.3.1 Possible methods available to evaluate the effect of propulsion system design changes on specific fuel consumption (SFC) are:

a) thermo dynamical models (engine performance decks);

b) computational fluid dynamics (CFD);

c) component testing;

d) full engine testing; and

e) any combination of a) to d).

3.4.4 Approval based on first principle models

3.4.4.1 *Introduction*

3.4.4.1.1 Annex 16, Volume III, Appendix 1, section 2 authorizes the Specific Air Range to be determined by the use of a performance model approved by the certificating authority.

3.4.4.1.2 A performance model is defined in Annex 16, Volume III, Part 1, Chapter 1 as *an analytical tool or method validated from corrected flight test data that can be used to determine the SAR values for calculating the* CO_2 *emissions evaluation metric value at the reference conditions.*

3.4.4.1.3 The objective of this chapter is to address the approval of SAR values based on specific types of performance models known as "First Principle Models". A First Principle Model can be defined as a tool that enables the derivation of a SAR value, in given flight conditions, from flight mechanics equations using aeroplane aerodynamics and engine performance data.

3.4.4.1.4 First Principle Models shall be validated by actual SAR flight test data, acquired in accordance with the procedures defined in the Annex Standard, and their validity need only to be shown for the test points and conditions relevant to showing compliance with the Standard.

3.4.4.1.5 The method proposed in 3.4.4 represents one possible method to show compliance with the Standard.

3.4.4.2 *Selection of relevant flight test points for the model validation*

3.4.4.2.1 First principle SAR models developed by aeroplane manufacturers typically cover most of the flight envelope to provide many different operational fuel planning requirements. The range of flight test data from these models typically spans large Mach and Weight/δ ranges.

3.4.4.2.2 The example in below Figure 3-10 illustrates the wide distribution of the flight test data across the model and also that none of the test points generally fall exactly at the three SAR conditions of the Standard.

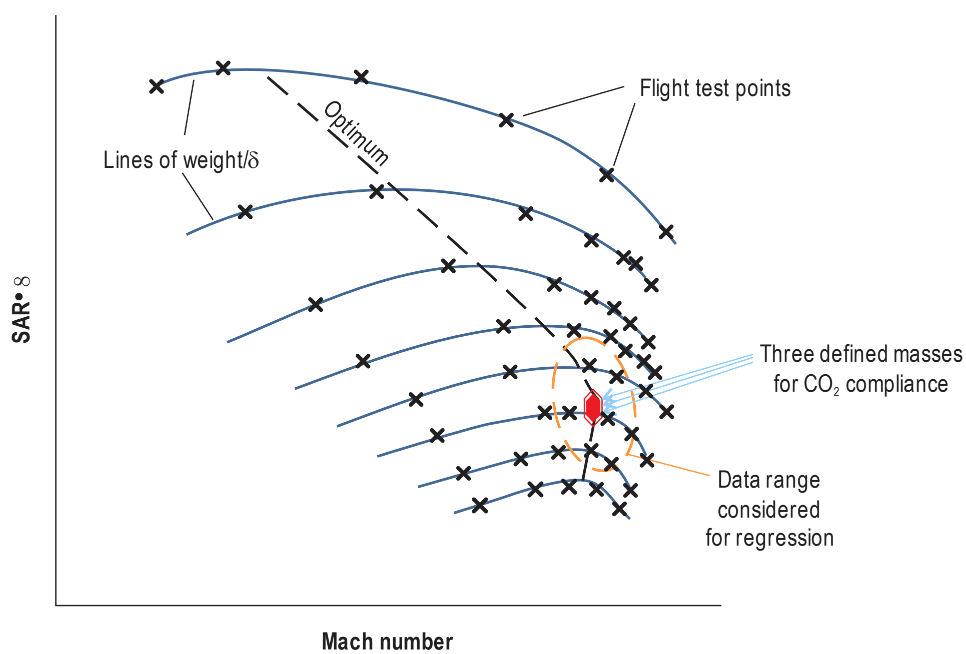

Figure 3-10. Test points positions

3.4.4.2.3 Figure 3-10 shows that the three SAR conditions of the Standard are relatively close to each other in terms of Mach and Weight/δ since they represent the same optimum aerodynamic conditions. Although these three SAR conditions may not be specifically flight tested, they are generally well framed by other relevant flight test data in a local Mach and Weight/δ range.

3.4.4.2.4 In order to validate the performance model in conditions relevant to showing compliance with the Standard, flight test points around optimum Mach number and optimum Weight/δ shall be selected based on the following criteria:

a) Weight/δ between ±5 per cent of the optimum Weight/δ; and

b) Mach number between -0.02 and +0.015 of the optimum Mach number.

Note 1.— The above ±5 per cent value aims at capturing flight test points on Weight/δ lines above and below optimum Weight/δ according to the aeroplane manufacturer's best flight test practices.

Note 2.— The density of test points is generally lower below the optimum Mach number than above where it constitutes the operational Mach range. Additionally, the Mach effect on SAR is generally flatter below the optimum Mach number than above, and a sharp SAR decrease may sometimes be experienced when approaching high Mach numbers. This is the reason why the Mach range for the test point selection is larger below the optimum Mach number (-0.02) than above (+0.015).

Note 3.— Testing target Weight/δ values nondimensionalizes the pressure altitude so that Weight/δ varies as $C_L.M^2$. Therefore, spanning Weight/δ at a given Mach has the same effect as spanning C_L at that Mach number.

3.4.4.3 *SAR model justification*

3.4.4.3.1 *Flight test SAR versus SAR model*

3.4.4.3.1.1 In order to justify the use of first principle models to determine the SAR values for calculating the CO_2 emissions evaluation metric value, manufacturers need to explain to certificating authorities their modelling validation principles and to show that their modelling correctly matches flight test data acquired, in accordance with the procedures defined in the Annex 16, Volume III Standard.

3.4.4.3.1.2 One possible method is to present plots of ΔSAR (i.e. Measured SAR minus Computed SAR) in per cent versus Mach and versus Weight/δ (or C_L) in the Mach and Weight/δ (or C_L) range of the cruise envelope (Figures 3-11 and 3-12 below).

3.4.4.3.1.3 The objective is to take credit of an increased number of test points, when available, to qualitatively show that the test point selection, as per the criteria of section 3.4.4.2 above (red points in Figures 3-11 and 3-12 below), is not differently scattered or biased than other test points in the cruise envelope (green points).

Figure 3-11. ΔSAR versus Mach number

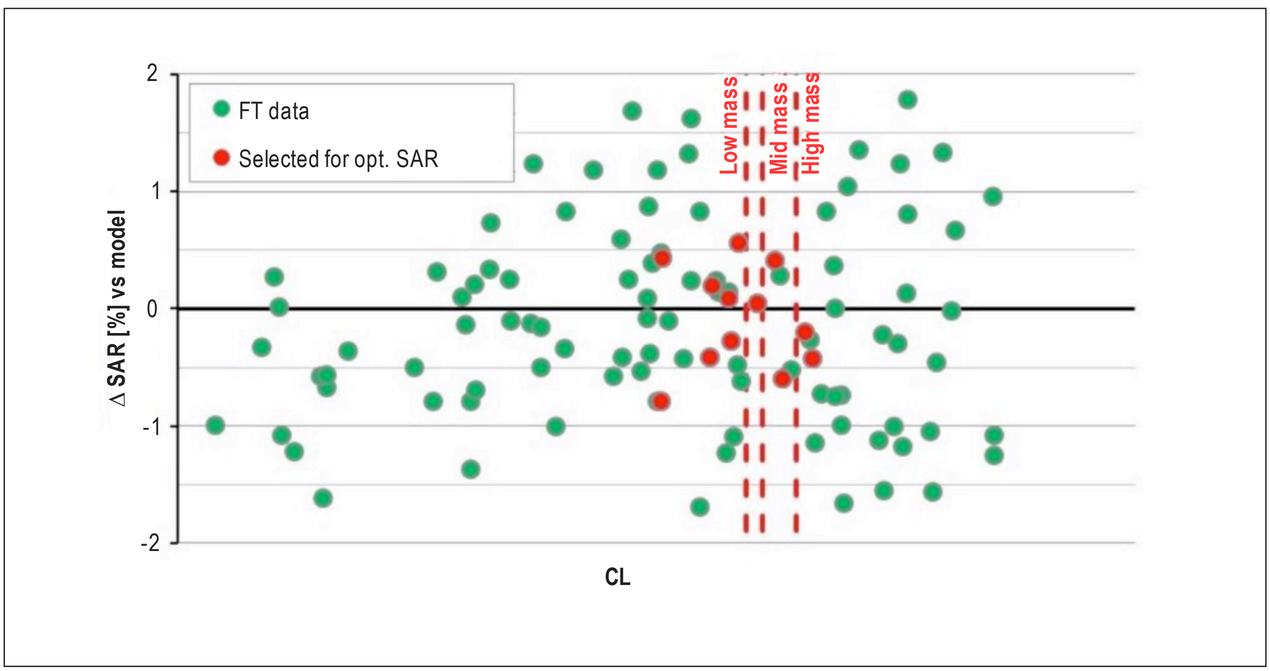

Figure 3-12. ΔSAR versus C_L

3.4.4.3.2 SAR model presentation

3.4.4.3.2.1 Since the comparison of flight test points with the model does not validate how the model is built and how the optimum conditions represent the CO_2 emissions evaluation metric, the SAR model used to show compliance with the Annex16, Volume III Standard should be presented to the certificating authority.

3.4.4.3.2.2 One possible method is to present plots of computed SAR versus Mach and versus C_L for the three masses of the Annex16, Volume III Standard (see Figures 3-13 and 3-14 below), and to show how the SAR model is supported by flight test data.

3.4.4.3.2.3 The objective of these plots is to show that there are no abnormal discontinuities in the Mach and C_L ranges selection around SAR conditions used for the CO_2 emissions evaluation metric determination.

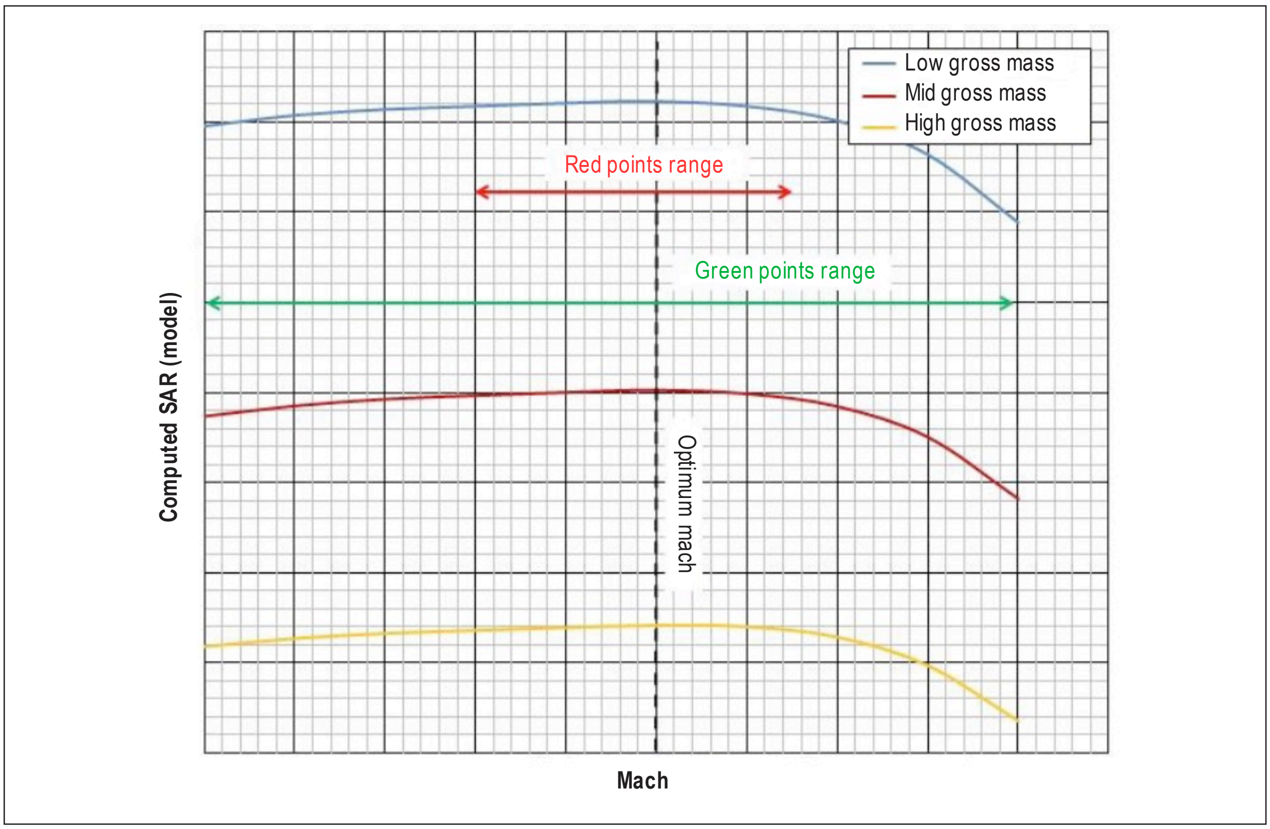

Figure 3-13. Computed SAR (model) versus Mach at optimum altitude

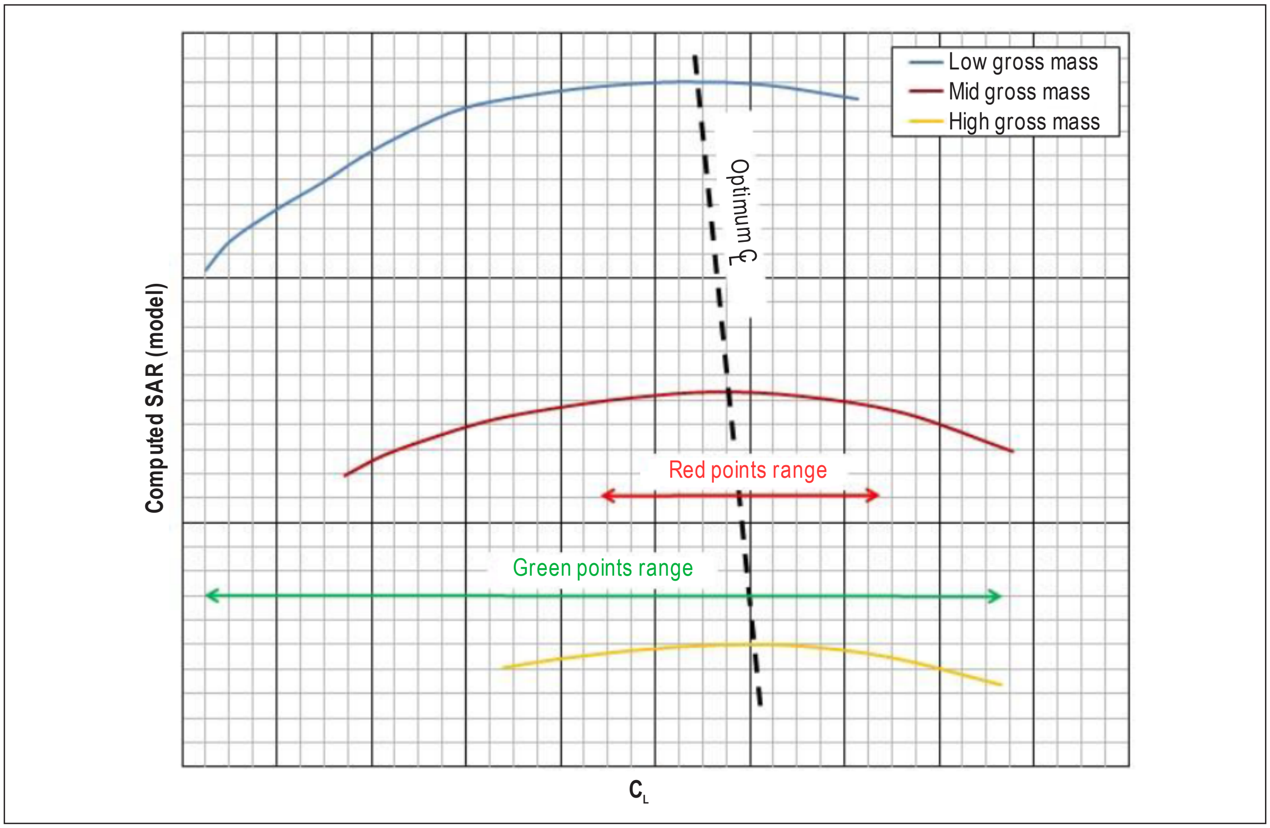

Figure 3-14. Computed SAR (model) versus C_L at optimum Mach

3.4.4.4 *SAR model validation – confidence interval determination*

3.4.4.4.1 As explained in section 3.4.4.2 above, the SAR model needs to be validated in the regions with conditions relevant to showing compliance with the Standard. For that purpose, flight test points should be selected, as per the criteria of section 3.4.4.2, and the number of these selected flight test points will represent n measurements of SAR (SAR_1, SAR_2, …, SAR_n) obtained in approximately the same conditions of Mach and C_L.

3.4.4.4.2 *Step 1 — Estimate of the mean ΔSAR value and of the standard deviation*

3.4.4.4.2.1 For each flight test point (i), the SAR difference $\Delta SAR_{(i)}$ between the measured SAR and the computed SAR can be determined as follows (in per cent):

$$\Delta SAR_{(i)}(\%) = [(Measured\ SAR_{(i)} - Computed\ SAR_{(i)}) / Computed\ SAR_{(i)}] \times 100\ (\%).$$

3.4.4.4.2.2 The estimate of the mean ΔSAR value in per cent, $\Delta SAR_{AVG}(\%)$, can then be determined for the n measurements as follows:

$$\Delta SAR_{AVG}(\%) = estimate\ of\ the\ mean\ \Delta SAR\ (\%) = \frac{1}{n}\left\{\sum_{i=1}^{i=n} \Delta SAR_{(i)}\right\}(\%).$$

3.4.4.4.2.3 And the estimate of the standard deviation of the mean as follows:

$$s = estimate\ of\ the\ standard\ deviation\ of\ the\ mean = \sqrt{\frac{\sum_{i=1}^{i=n}\left(\Delta SAR_{(i)} - \Delta SAR_{AVG}\right)^2}{n-1}}\ (\%).$$

3.4.4.4.3 *Step 2 — Confidence interval determination*

3.4.4.4.3.1 Due to the low number of test points in a small range of Mach and C_L, it is assumed that the statistical population follows the Student law, and the 90 per cent confidence interval can be defined as per ETM section 3.3.2 "Direct Flight Testing" methodology. The objective being to determine an average ΔSAR value (%) and its associated 90 per cent confidence interval, the minimum acceptable sample size is twelve points as per Annex 16, Volume III, Appendix 1, section 6.3.

Note.— Based on additional justifications for the representativeness or conservatism of the modelling, and overall modelling behaviour, this minimum acceptable sample size may be reduced down to six data points upon agreement by the certificating authorities.

3.4.4.4.3.2 From above and from the Student's t-distribution, the 90 per cent confidence interval (CI_{90}) for the estimate of the mean ΔSAR_{AVG} can be determined as:

$$CI_{90}\ (\%) = \left[\Delta SAR_{AVG} \pm t_{(.95,n-1)} \frac{s}{\sqrt{n}}\right] (\%)$$

where $t_{(.95,\ n-1)}$ = Student's t-distribution (for 90 per cent confidence) for n degrees of freedom (see Table 3-9).

3.4.4.4.4 *Step 3 — Validity of results*

3.4.4.4.4.1 Annex 16, Volume III, Appendix 1, section 6.4 states "*If the 90 per cent confidence interval of the SAR value at any of the three reference aeroplane masses exceeds ±1.5 per cent, the SAR value at that reference mass may be used, subject to the approval of the certificating authority, if a penalty is applied to it. The penalty shall be equal to the amount that the 90 per cent confidence interval exceeds ±1.5 per cent. If the 90 per cent confidence interval of the SAR value is less than or equal to ±1.5 per cent no penalty need be applied.*"

3.4.4.4.4.2 Similarly, for a first principle model:

a) If the 90 per cent confidence interval remains within the ±1.5 per cent limits, then the ΔSAR_{AVG} (%) value can be retained as the reference deviation for model correction.

b) If the 90 per cent confidence interval exceeds the ±1.5 per cent limits, then a penalty equal to the amount that the 90 per cent confidence interval exceeds ±1.5 per cent shall be applied to the ΔSAR_{AVG}(%) value, subject to the approval of the certificating authority. The corrected ΔSAR_{AVG}(%) value then becomes the reference deviation for the model correction.

3.4.4.4.5 *Step 4 — SAR model correction*

3.4.4.4.5.1 To determine the CO_2 emissions evaluation metric, the SAR values computed with a first principle model at each of the three reference aeroplane masses shall be corrected by an amount in per cent equal to the reference deviation determined in Step 3.

3.4.4.5 Worked example of the determination of 90 per cent confidence intervals

3.4.4.5.1 Let's consider the following set of 12 flight test points meeting the selection criteria of section 3.4.4.2, and for which the difference between measured SAR and computed SAR has been established as follows:

Table 3-23. Test point selection

Selected test point number	*ΔSAR (%)*
1	0.08
2	-0.6
3	-0.42
4	0.19
5	-0.43
6	0.23
7	-0.28
8	0.45
9	0.10
10	-0.28
11	-0.80
12	-0.64

a) The number of data points (n) = 12.

b) The degrees of freedom (n-1) = 11.

c) The Student's t-distribution for 90 per cent confidence and 11 degrees of freedom ($t_{(.95\,,11)}$) = 1.797 (see Table 3-9).

3.4.4.5.2 *Step 1 — Estimate of the mean ΔSAR (%) and of the standard deviation(s)*

$$\Delta SAR_{AVG}(\%) = \frac{1}{12}\left\{\sum_{i=1}^{i=12} \Delta SAR_{(i)}\right\} = \text{-0.200 \%}$$

$$s = \sqrt{\frac{\sum_{i=1}^{i=12}\left(\Delta SAR_{(i)} - \Delta SAR_{AVG}\right)^2}{n-1}} = 0.39950\ \%$$

3.4.4.5.3 *Step 2 — 90 per cent confidence interval (CI_{90})*

$$CI_{90} = \Delta SAR_{AVG} \pm t_{(.95,11)}\frac{s}{\sqrt{12}} = -0.200 \pm 0.207\ \%$$

3.4.4.5.4 *Step 3 — Check of confidence interval limits*

3.4.4.5.4.1 The confidence interval extends to ±0.207 per cent around the mean ΔSAR value (-0.200 per cent), which is below the confidence interval limit of ±1.5 per cent. As a result, a ΔSAR value of -0.200 per cent can be retained for model correction.

3.4.4.5.5 *Step 4 — SAR Model correction*

3.4.4.5.5.1 The SAR associated to each reference mass of the CO_2 emissions evaluation metric can be determined by computation with the first principle model and shall be corrected by an amount of -0.200 per cent as follows:

$$SAR_{corrected} = SAR_{computed} \times 0.998$$

Appendix 1

REFERENCES

1. ASTM International D1655-15 entitled "*Standard Specification for Aviation Turbine Fuels*". This ASTM International publication may be obtained from the ASTM International, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA, 19428-2959 USA, www.astm.org.

2. Defence Standard 91-91, Issue 7, Amendment 3, entitled "*Turbine Fuel, Kerosene Type, Jet A-1*". This Ministry of Defence Standard may be obtained from Defence Equipment and Support, UK Defence Standardization, Kentigern House, 65 Brown Street, Glasgow G2 8EX, UK.

3. ASTM International D4809-13 entitled "*Standard Test Method for Heat of Combustion of Liquid Hydrocarbon Fuels by Bomb Calorimeter (Precision Method)*". This ASTM International publication may be obtained from the ASTM International, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA, 19428-2959 USA, www.astm.org.

4. ASTM International D4052-11 entitled "*Standard Test Method for Density and Relative Density of Liquids by Digital Density Meter*". This ASTM International publication may be obtained from the ASTM International, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA, 19428-2959 USA, www.astm.org.

5. ASTM International D445-15 entitled "*Standard Test Method for Kinematic Viscosity of Transparent and Opaque Liquids (and Calculation of Dynamic Viscosity)*". This ASTM International publication may be obtained from the ASTM International, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA, 19428-2959 USA, www.astm.org.

Appendix 2

BIBLIOGRAPHY

1. Kendall, M.G. and A. Stuart. *The Advanced Theory of Statistics.* Volumes 1, 2 and 3. New York: Hafner, 1971.

2. Kendall, M.G. and G.U. Yule. *An Introduction to the Theory of Statistics.* 14th ed. New York: Griffin, 1950.

3. Snedecor, G.W. and W.G. Cochran. *Statistical Methods.* 6th ed. Arnes, Iowa: The Iowa State University Press, 1968.

4. Walpole, R.E. and R.H. Myers. *Probability and Statistics for Engineers and Scientists.* New York: MacMillan, 1972.

5. Wonnacott, T.H. and R.J. Wonnacott. *Introductory Statistics,* 5th ed. N.p.: John Wiley & Sons, 1990.

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